Dark Matter mass map Reconstruction & Analysis

J.-L. Starck

CEA, IRFU, Service d'Astrophysique, France

jstarck@cea.fr

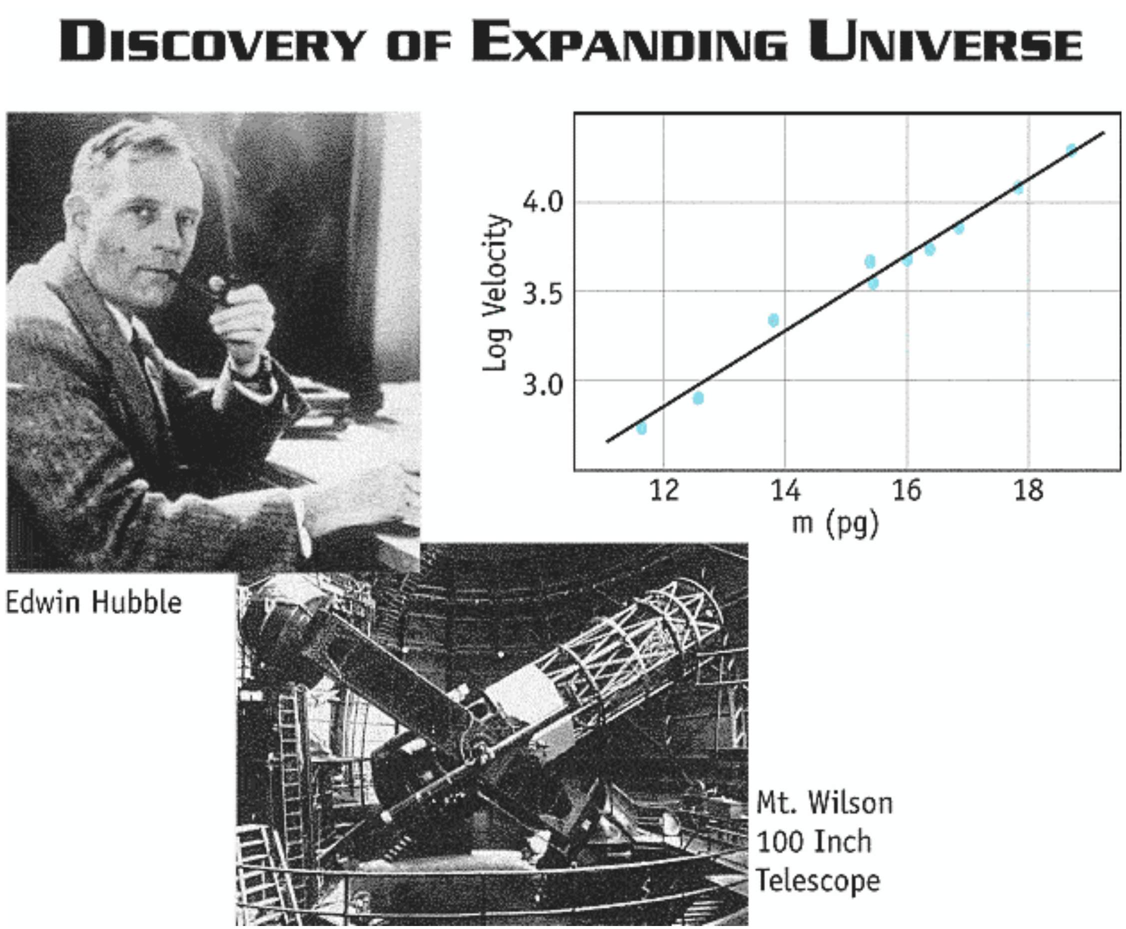
http://jstarck.free.fr

Collaborators: S. Pires, *R. Teyssier, R Massey, A Armara.*

,

S. Pires (CEA) *A . Refregier (CEA)* *R. Massey, Caltech* *A. Armara Zurich*

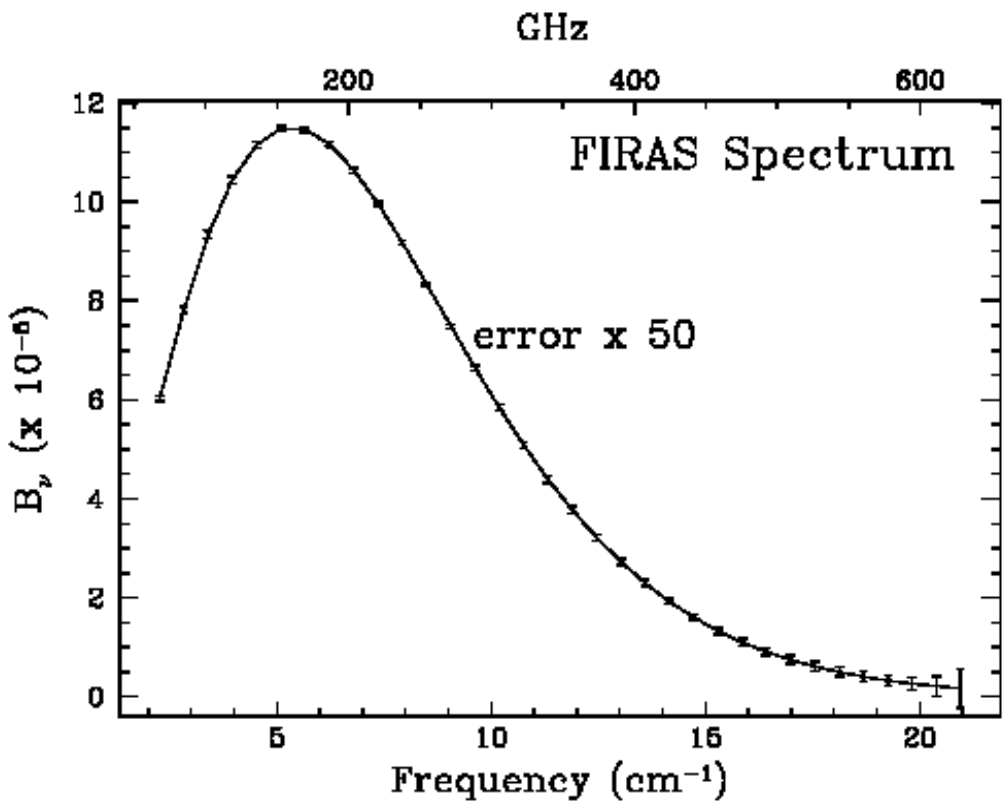
The Big Bang


DISCOVERY OF EXPANDING UNIVERSE
Log Velocity
4.0
3.5
3.0
12
14
16
18
m (pg)
Edwin Hubble
Mt. Wilson
100 Inch
Telescope

The Cosmic Microwave Background

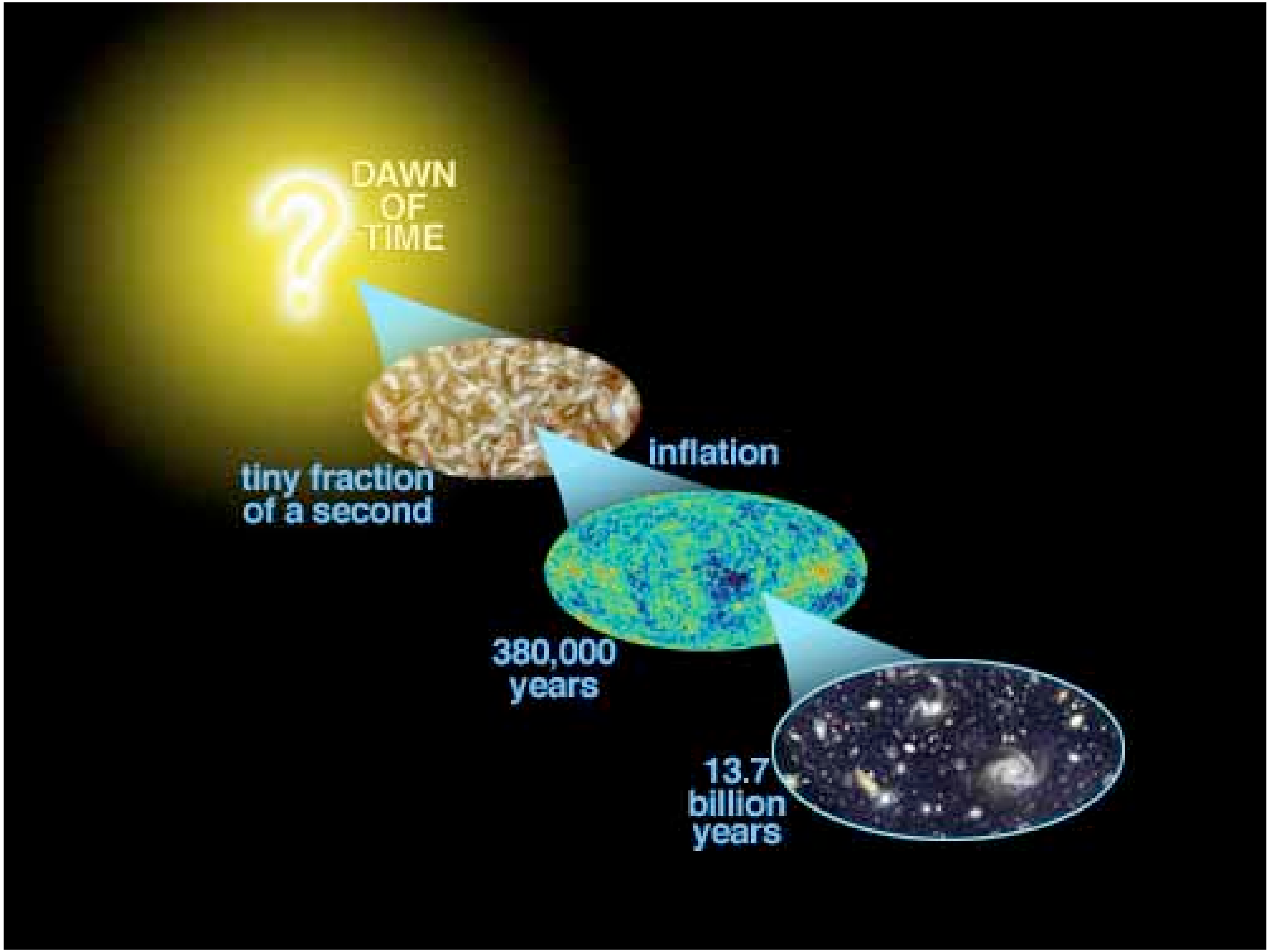
- The Universe is filled with a blackbody radiation field at a temperature of 3K.
- Predicted by G. Gamow in 1948
- Observed for the first time by Penzias and Wilson (1965)
- Confirmed by COBE (1990)

The Cosmic Microwave Background



$T = 2.728 \pm 0.004$ K

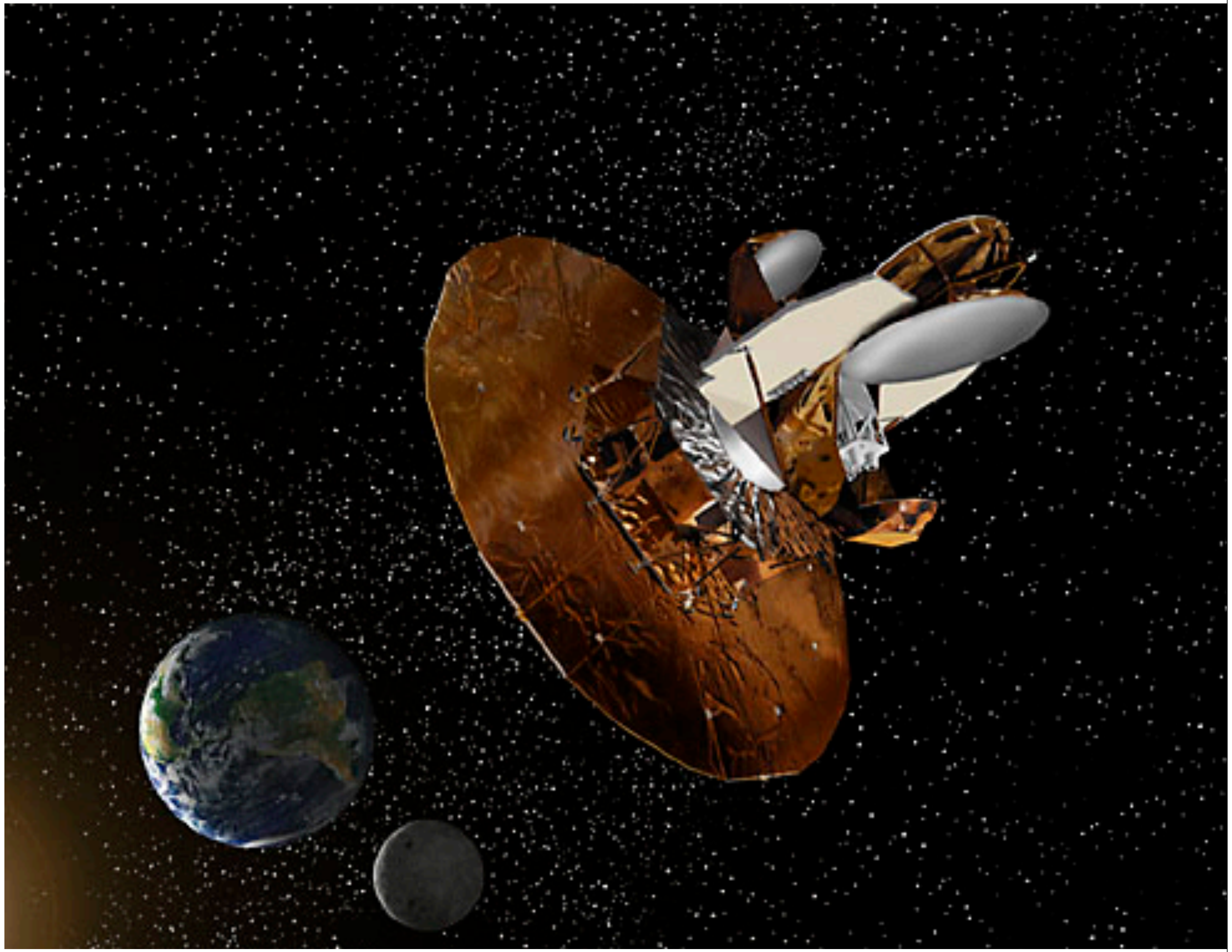
CMB electromagnetic spectrum


DAWN OF TIME
inflation
tiny fraction of a second
380,000 years
13.7 billion years

Wilkinson Microwave Anisotropy Probe

A partnership between NASA/GSFC and Princeton

Science Team:



NASA/GSFC

Chuck Bennett (**PI**)
Michael Greason
Bob Hill
Gary Hinshaw
Al Kogut
Michele Limon
Nils Odegard
Janet Weiland
Ed Wollack

Brown

Greg Tucker

UCLA

Ned Wright

UBC

Mark Halpern

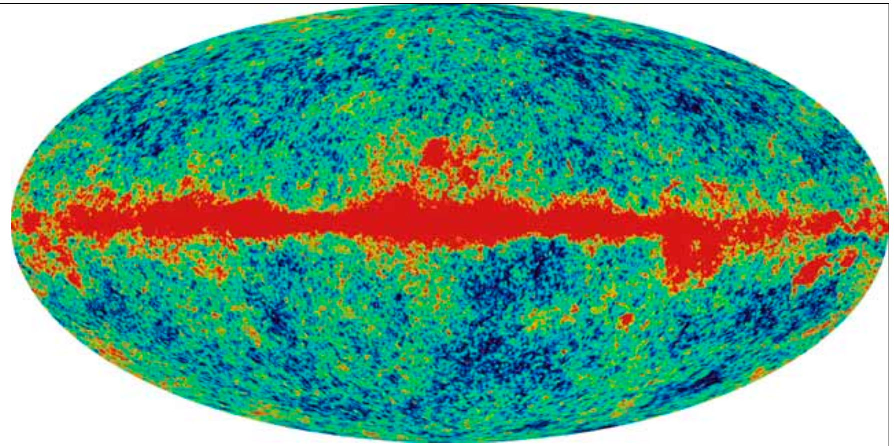
Chicago

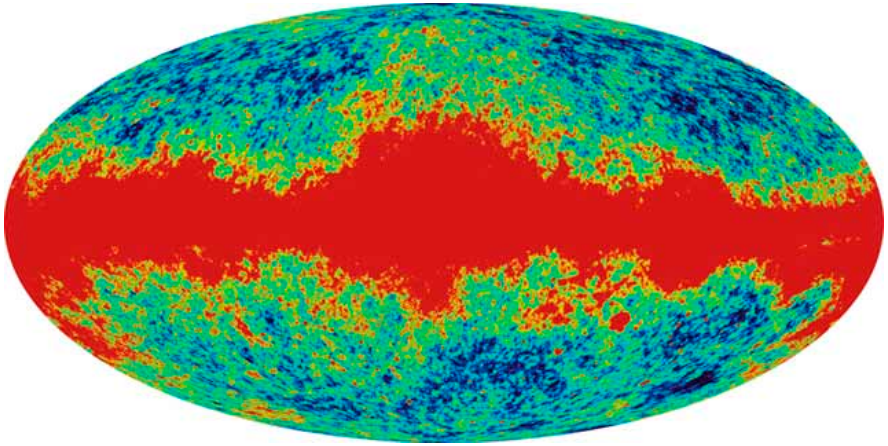
Stephan Meyer

Princeton

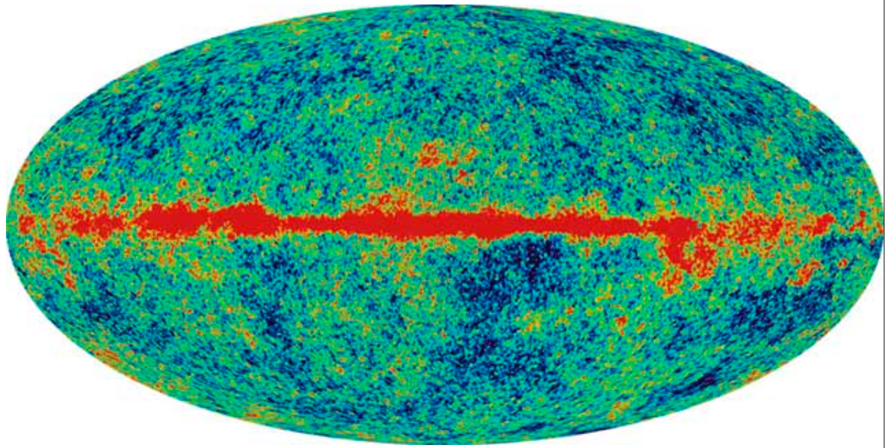
Chris Barnes
Norm Jarosik
Eiichiro Komatsu
Michael Nolta
Lyman Page
Hiranya Peiris
David Spergel
Licia Verde

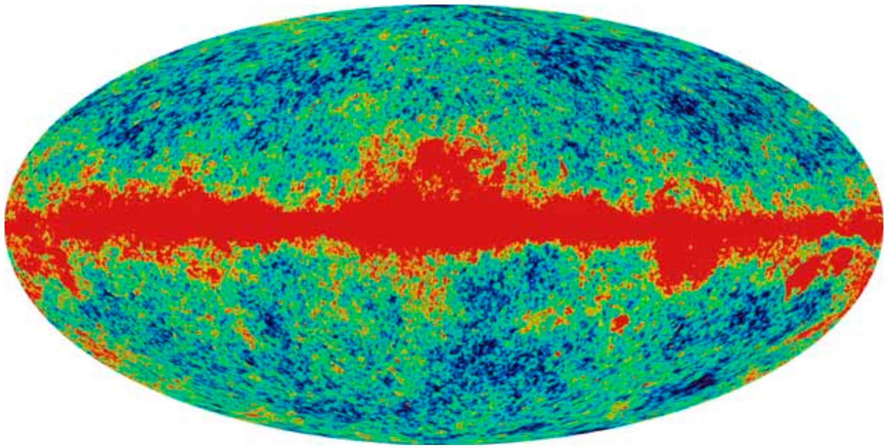
WMAP:five frequency maps

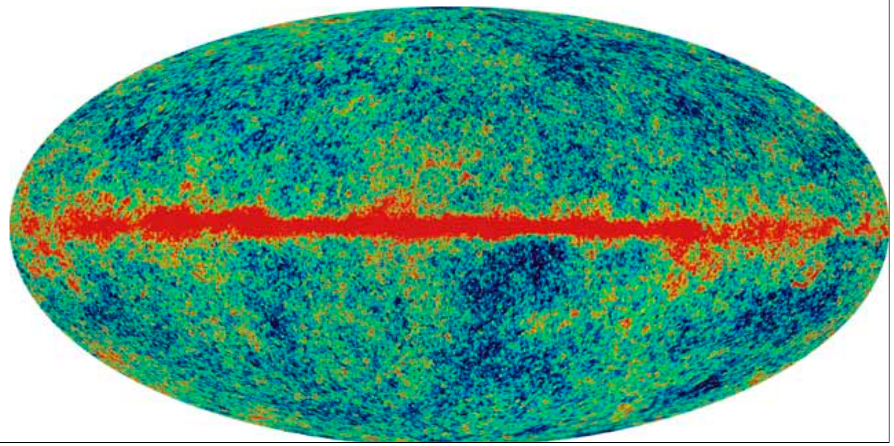


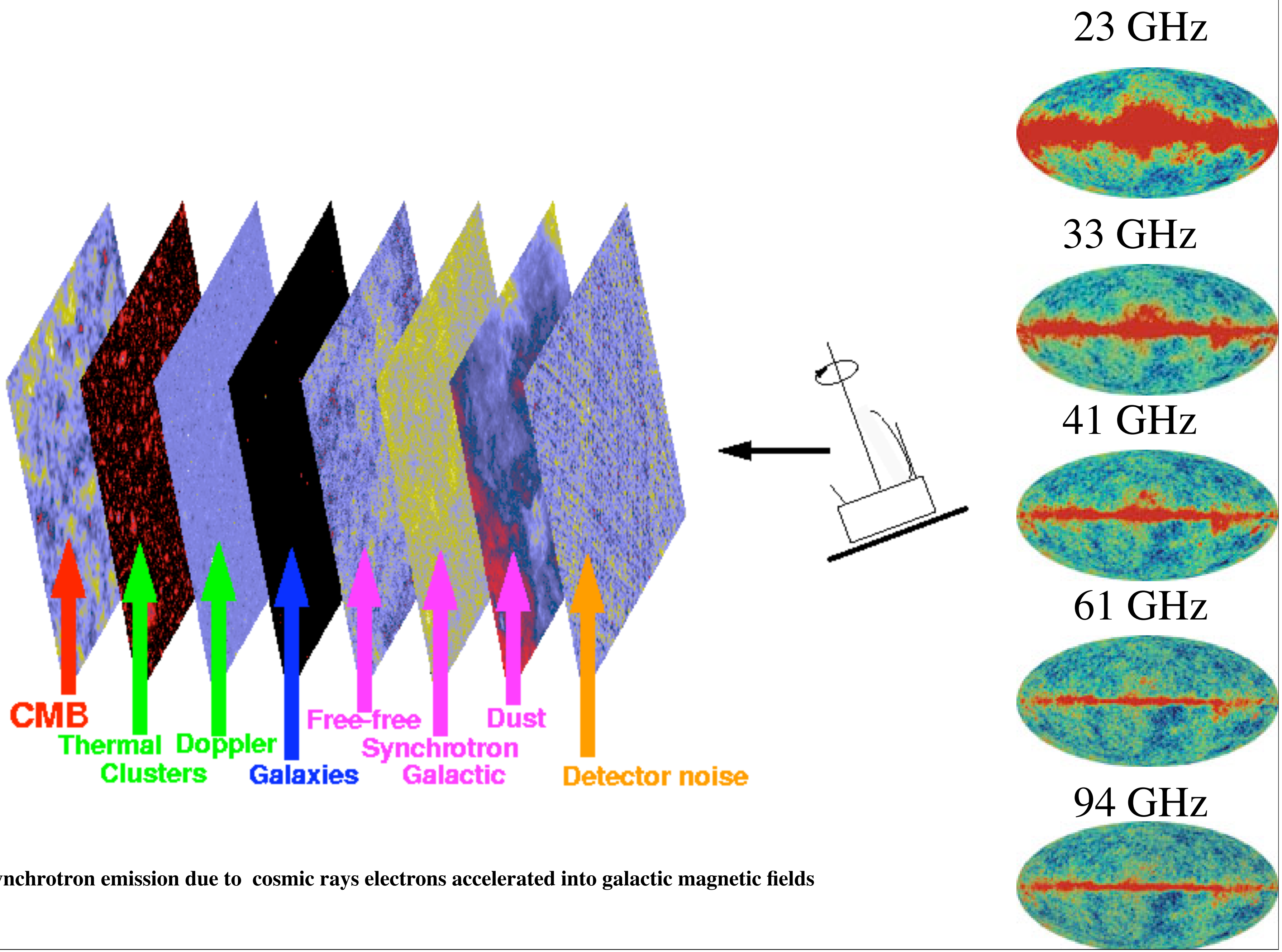


23, 33, 41, 61 and 93 GHz



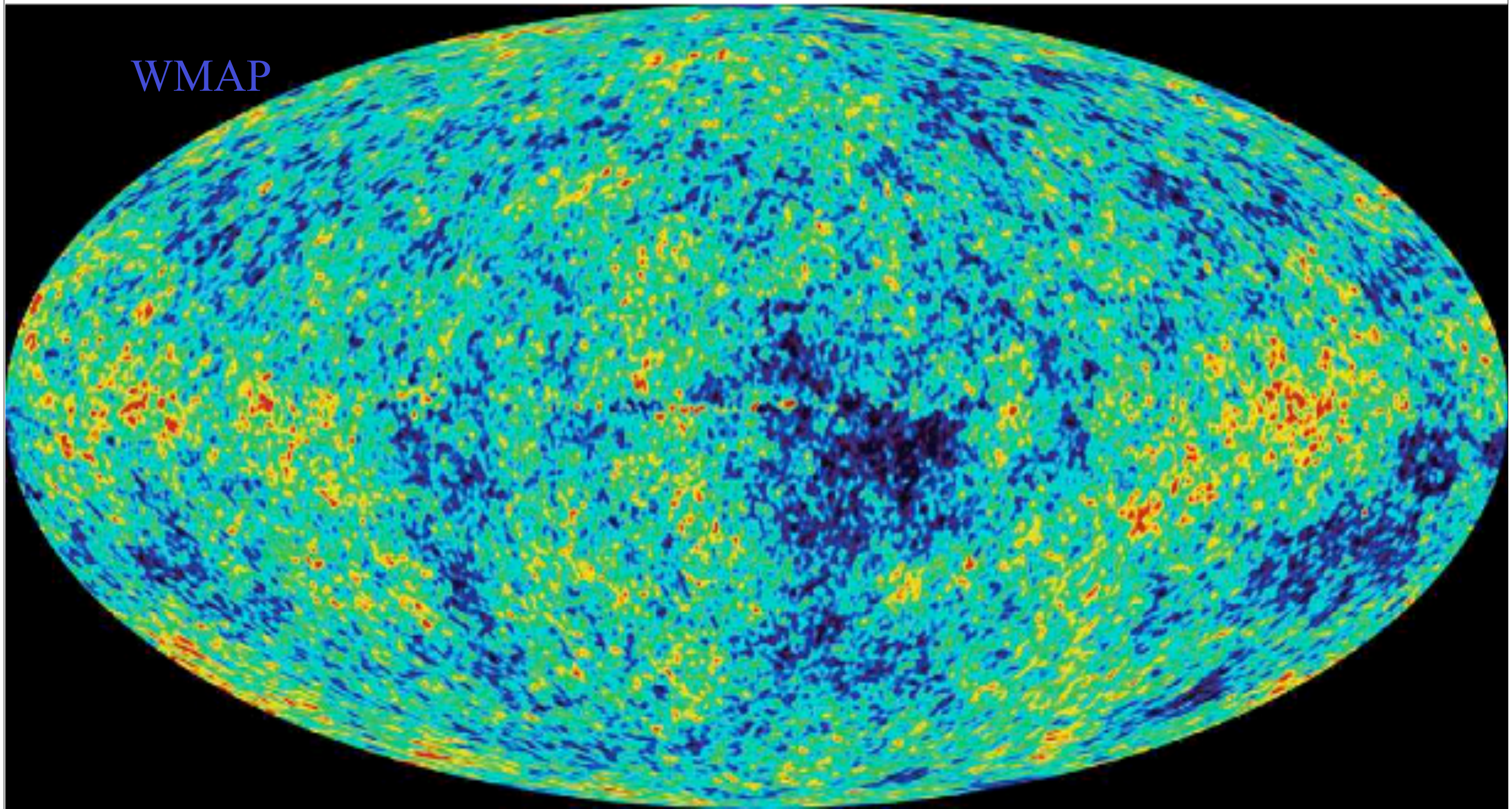




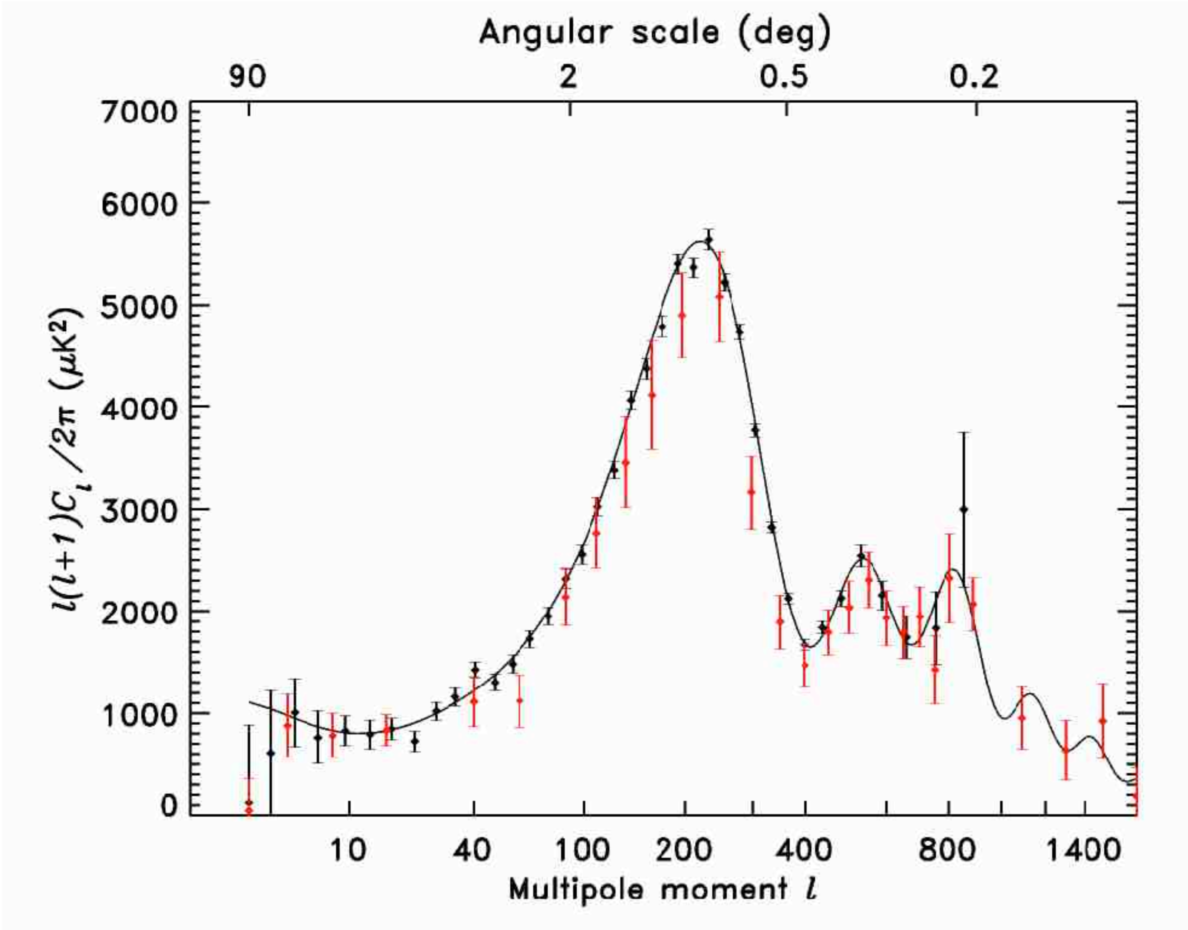


Synchrotron emission due to cosmic rays electrons accelerated into galactic magnetic fields

The CMB exhibits Fluctuations

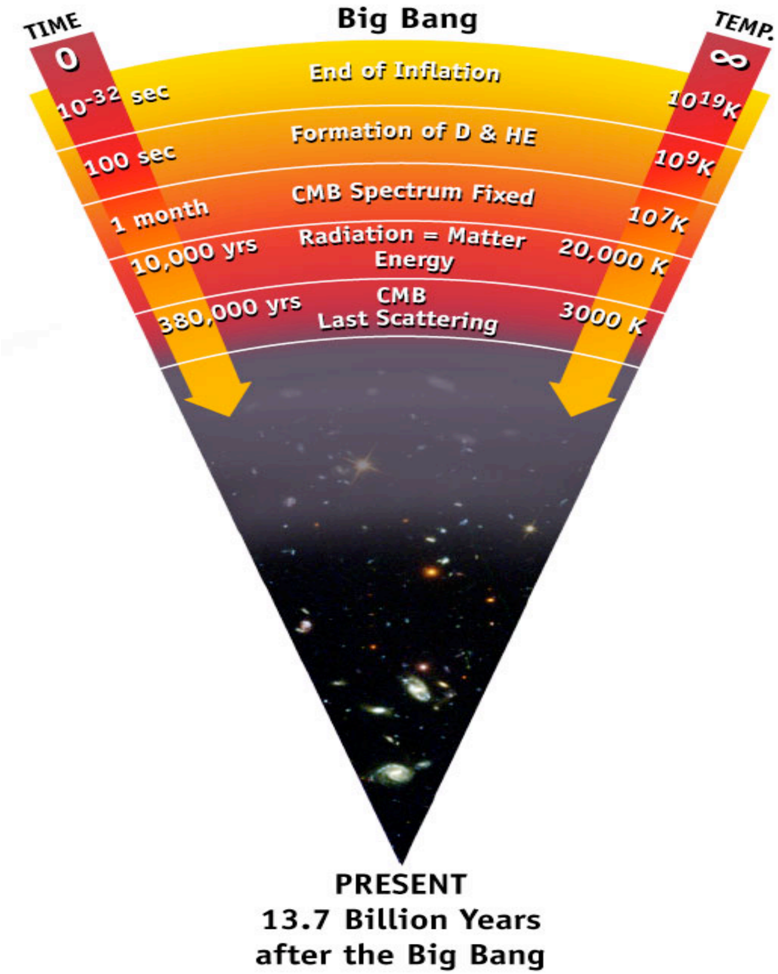


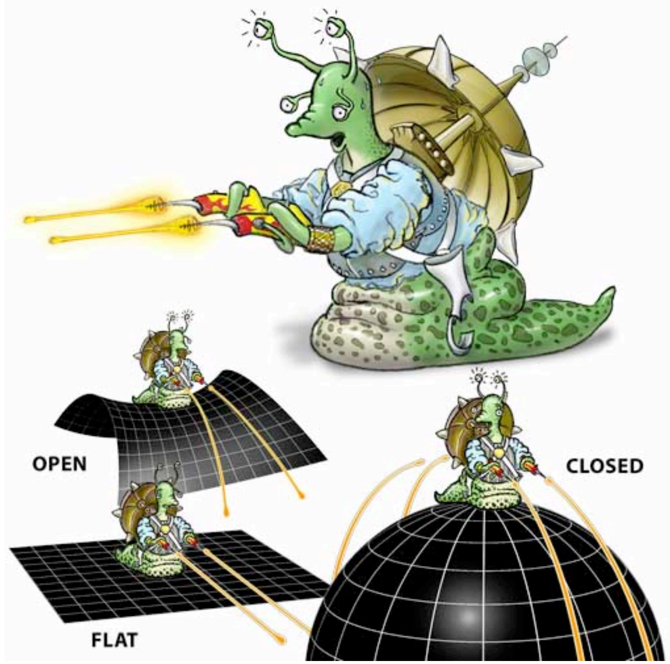
Power spectrum of WMAP

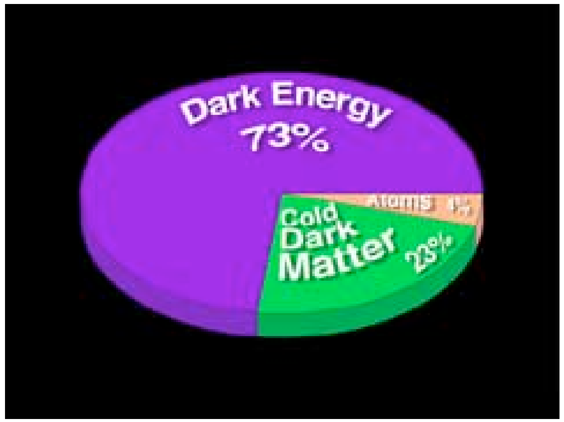


Remarkably consistent with earlier data

COSMOLOGY


TIME
Big Bang
TEMP.
0
∞
End of Inflation
10^{-32} sec
10^{19}K
Formation of D & HE
100 sec
10^{9}K
CMB Spectrum Fixed
1 month
10^{7}K
Radiation = Matter Energy
10,000 yrs
20,000 K
CMB Last Scattering
380,000 yrs
3000 K
PRESENT
13.7 Billion Years after the Big Bang


OPEN
CLOSED
FLAT


Dark Energy
73%
Atoms
Cold Dark Matter
23%

The current situation:
The triumph of dark stuff in cosmology

Visible matter cannot account for dynamics; ***dominated*** by DM

Cosmic energy budget:

$\Omega_{visible} \sim 0.004$

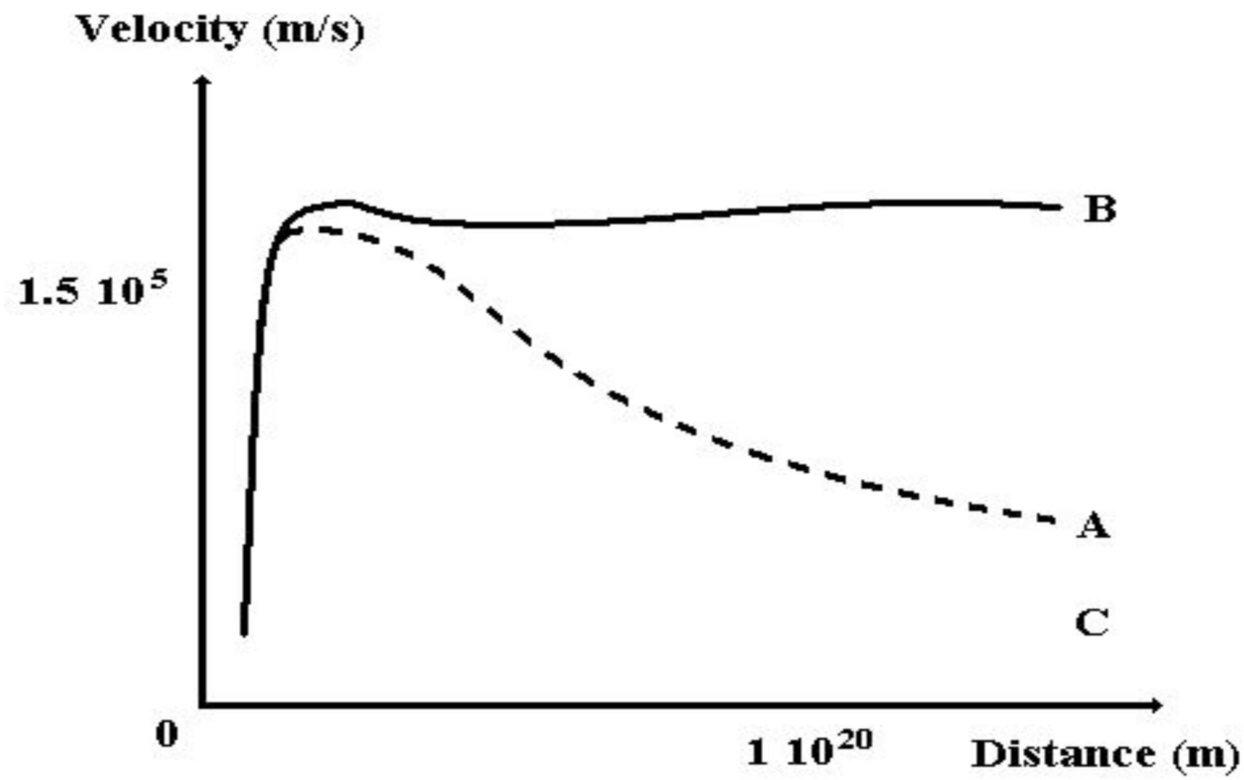
$\Omega_{baryon} \sim 0.04$

$\Omega_{dark\ matter} \sim 0.3$

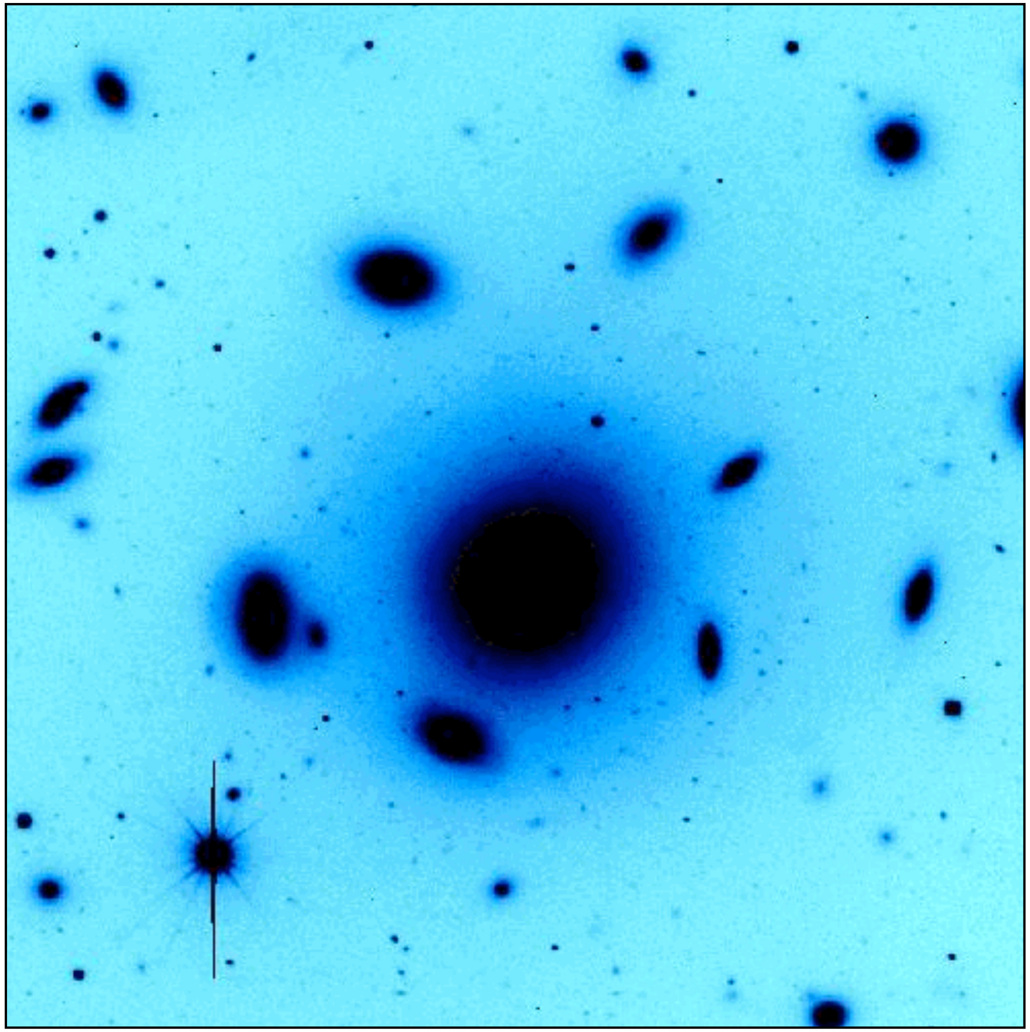
$\Omega_{dark\ energy} \sim 0.7$

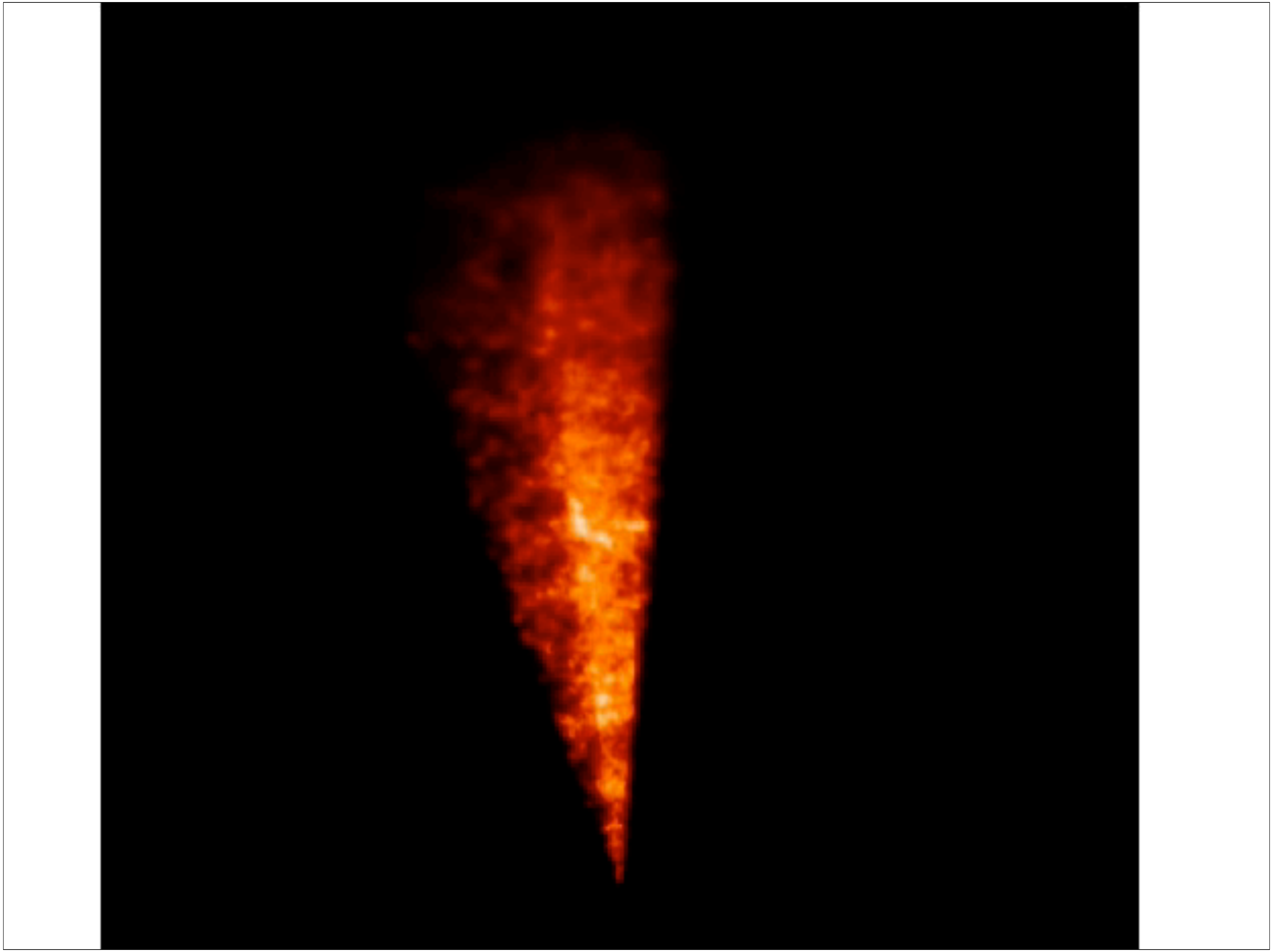
Dark matter/energy, once accepted, works extremely well on large scales.

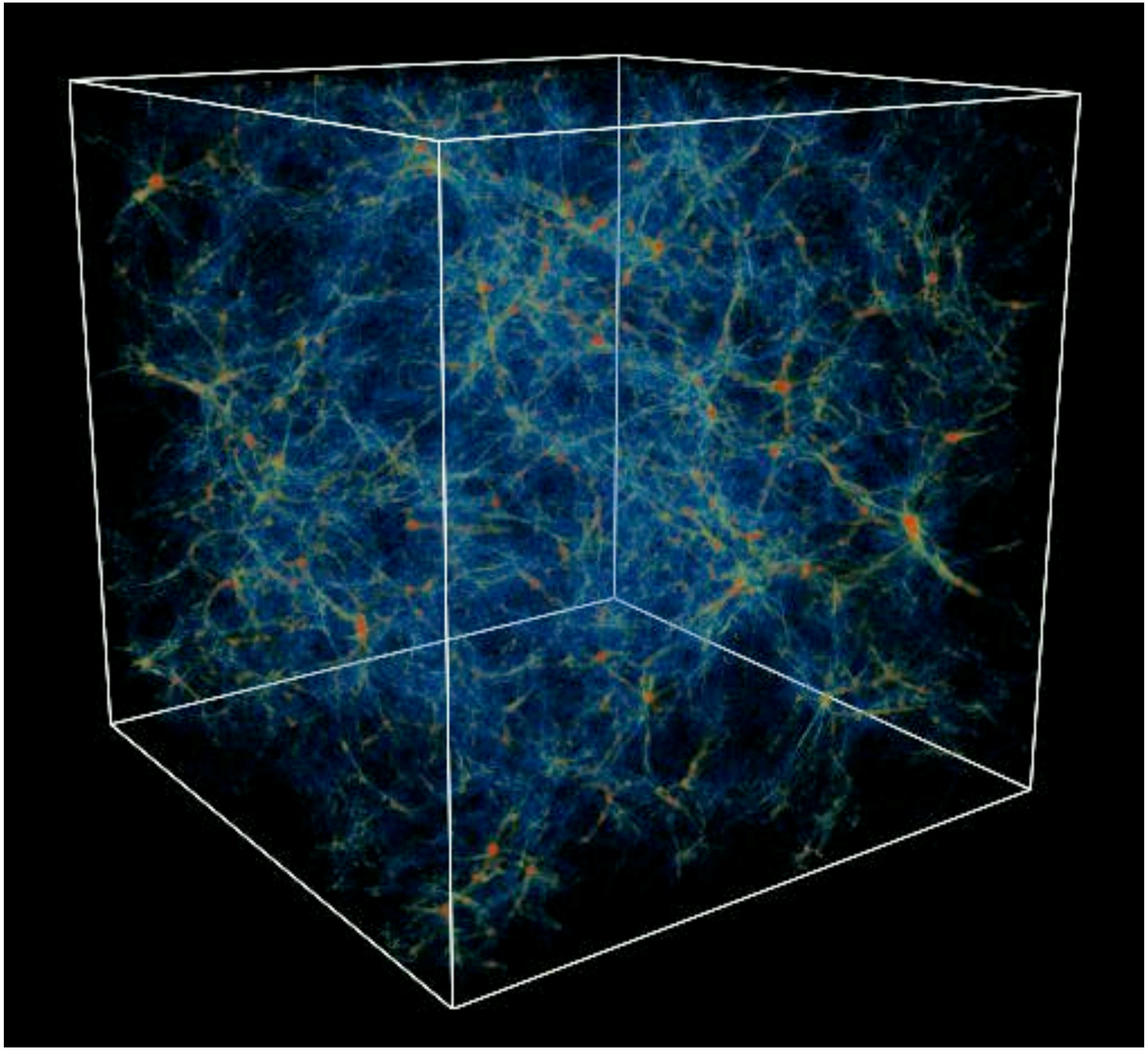
- Flattened Rotation Curves
- Two Explanations
  - CDM
  - Modify gravity



Clusters of galaxies







Evidence for dark matter

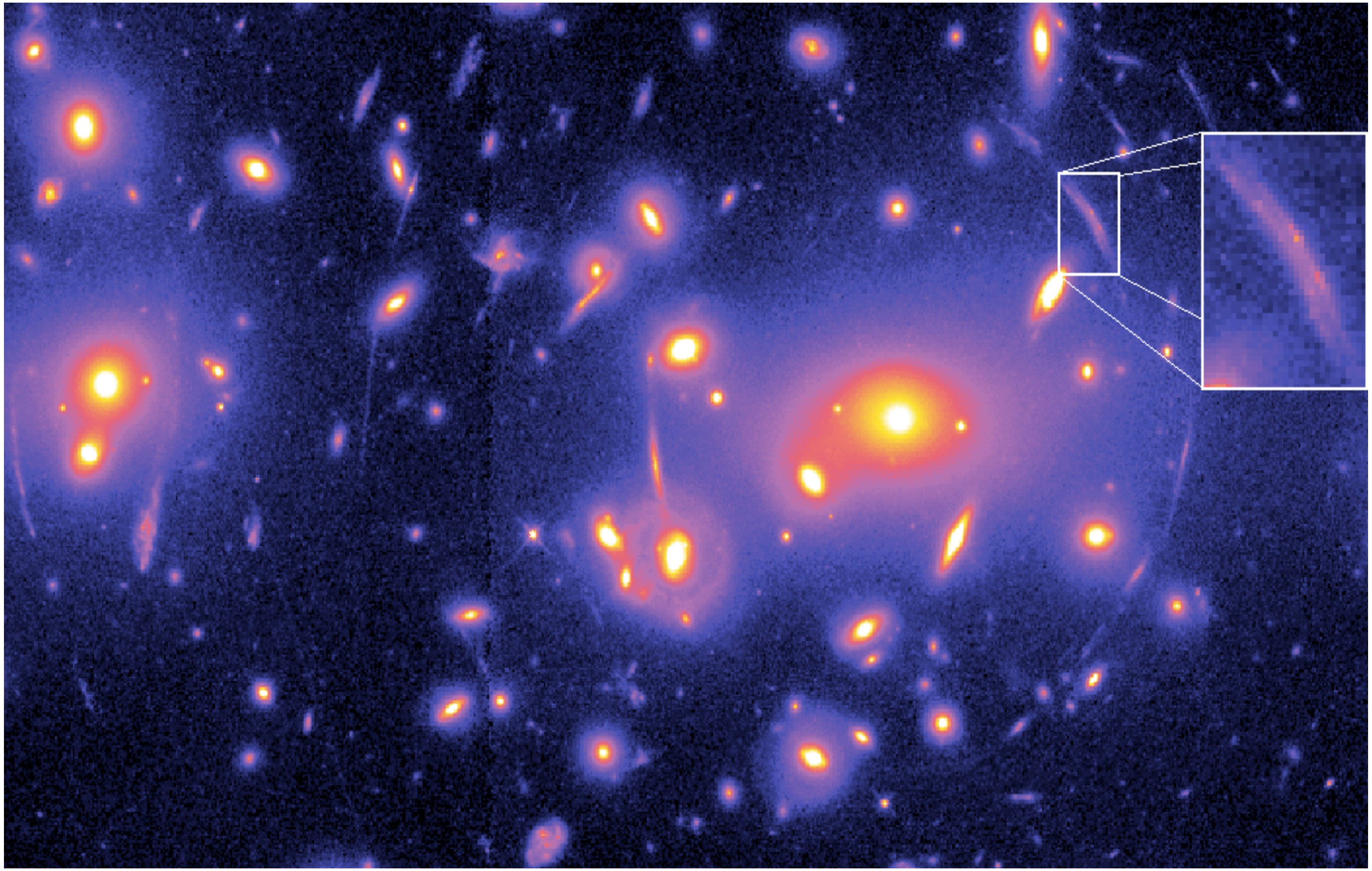
- Rotation curves of galaxies
- Cluster of galaxies
- Inflation predicts Omega=1
- N-body simulation

❑ **MACHOs** - ***baryonic matter***
(brown dwarfs & black holes)
(**M**assive **A**strophysical **C**ompact **H**alo **O**bjects)
=> NOW EXCLUDED

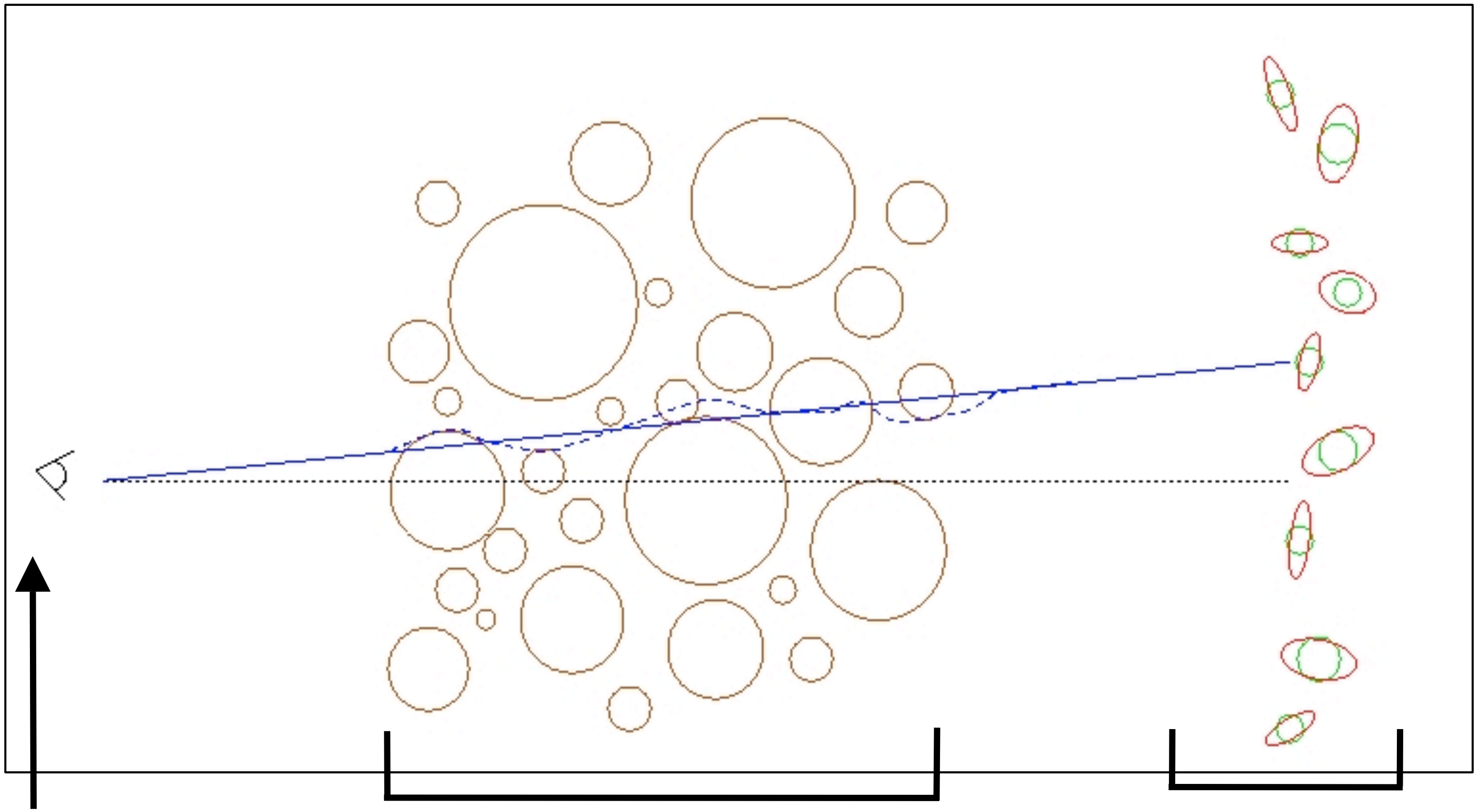
❑ **Neutrinos** - ***non baryonic matter***
=> NOW EXCLUDED

❑ **Les WIMPs** - ***non baryonic matter*** -
(**W**eakly **I**nteractive **M**assive **P**articles)
=> hard to detect

Strong Gravitational Lensing



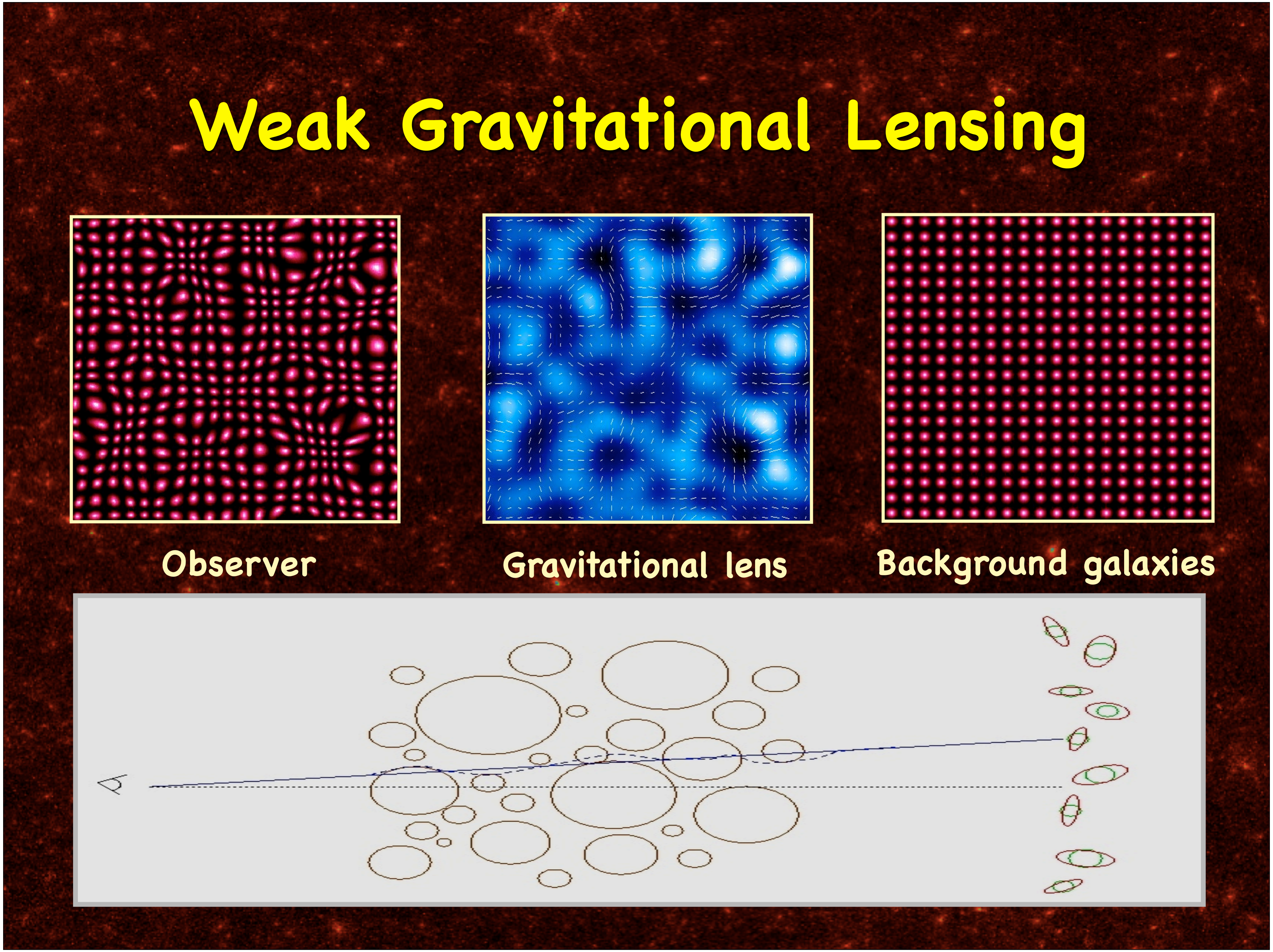
Weak Gravitational Lensing



Telescope

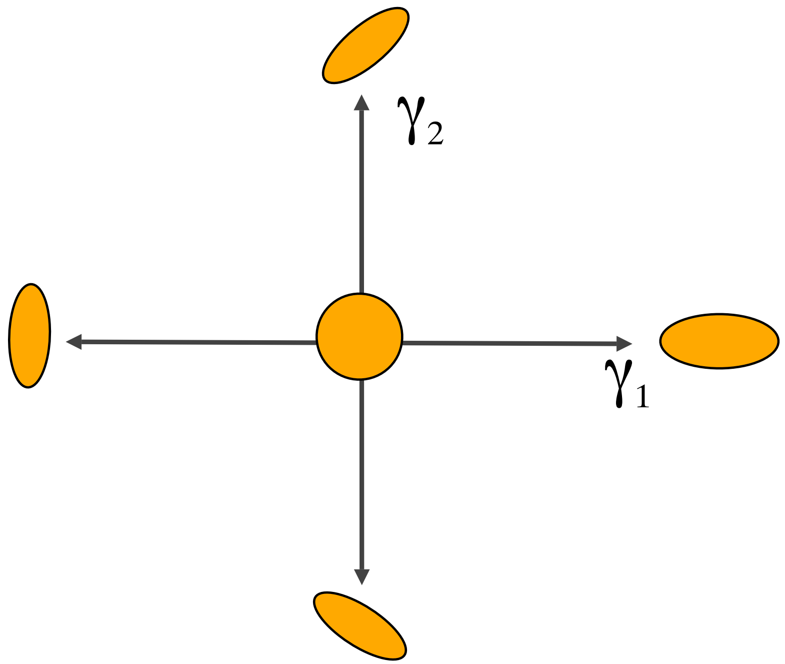
Gravitational Lens

Observed galaxies


Weak Gravitational Lensing
Observer
Gravitational lens
Background galaxies

The shear map (γ_1, γ_2)

γ_1 = deformation along the x-axis,
and γ_2 at 45 degrees from it.



$$\gamma = \gamma_1 + i\gamma_2 = |\gamma| e^{2i\theta}$$

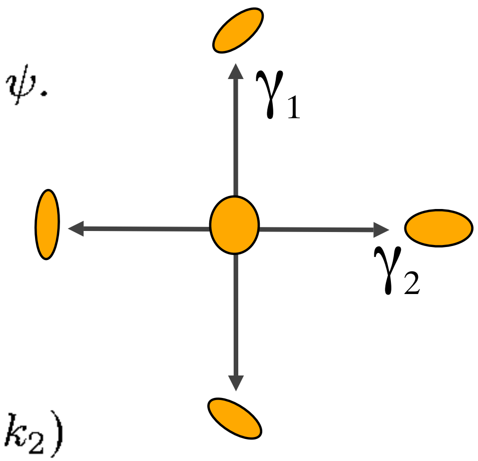
Where the modulus represents the amount of shear and the phase represents its direction.

Relation between the shear maps and the mass map κ

The relation between the weak shear maps γ_1, γ_2 and the mass map κ are:

$$
\begin{aligned}
\gamma_1 &= (\partial_1 - \partial_2)\psi \\
\gamma_2 &= 2\partial_1\partial_2\psi
\end{aligned}
$$

where ψ is the potential function defined by $\kappa = \nabla^2\psi$.



In Fourier Space

$$
\begin{aligned}
\hat{\kappa}(k_1, k_2) &= k^2\hat{\psi}(k_1, k_2) \\
\hat{\gamma_1}(k_1, k_2) &= \frac{k_1^2 - k_2^2}{k^2}\hat{\kappa}(k_1, k_2) \\
\hat{\gamma_2}(k_1, k_2) &= \frac{2k_1k_2}{k^2}\hat{\kappa}(k_1, k_2)
\end{aligned}
$$

with $k^2 = k_1^2 + k_2^2$.

An inverse problem

Noting $\hat{P}_1(k_1, k_2) = \frac{k_1^2 - k_2^2}{k^2}$ (with $\hat{P}_1(k_1, k_2) = 0$ when $k_1^2 = k_2^2$) and $\hat{P}_2(k_1, k_2) = \frac{2k_1k_2}{k^2}$ (with $\hat{P}_2(k_1, k_2) = 0$ when $k_1 = 0$ or $k_2 = 0$), the mass reconstruction consists in **searching κ such that it verifies both $\gamma_1 = P_1 * \kappa$ and $\gamma_2 = P_2 * \kappa$**. In practice, γ_1 and γ_2 are obtained through observations and are contaminated by noise. Then the relations between the observed data γ_{1b}, γ_{2b} and the true mass map κ are given by:

$$
\left\{
\begin{array}{lll}
\gamma_{1b} & = & P_1 * \kappa + N_1 \\
\gamma_{2b} & = & P_2 * \kappa + N_2
\end{array}
\right.
$$

The Inverse Filter: E and B mode

Noticing that $\hat{P}_1^{\,2} + \hat{P}_2^{\,2} = 1$, the least square estimation $\hat{\tilde{\kappa}}_l^{(E)}$ is:

$$\hat{\tilde{\kappa}}_l^{(E)} = \hat{P}_1 \hat{\gamma}_{1b} + \hat{P}_2 \hat{\gamma}_{2b}$$

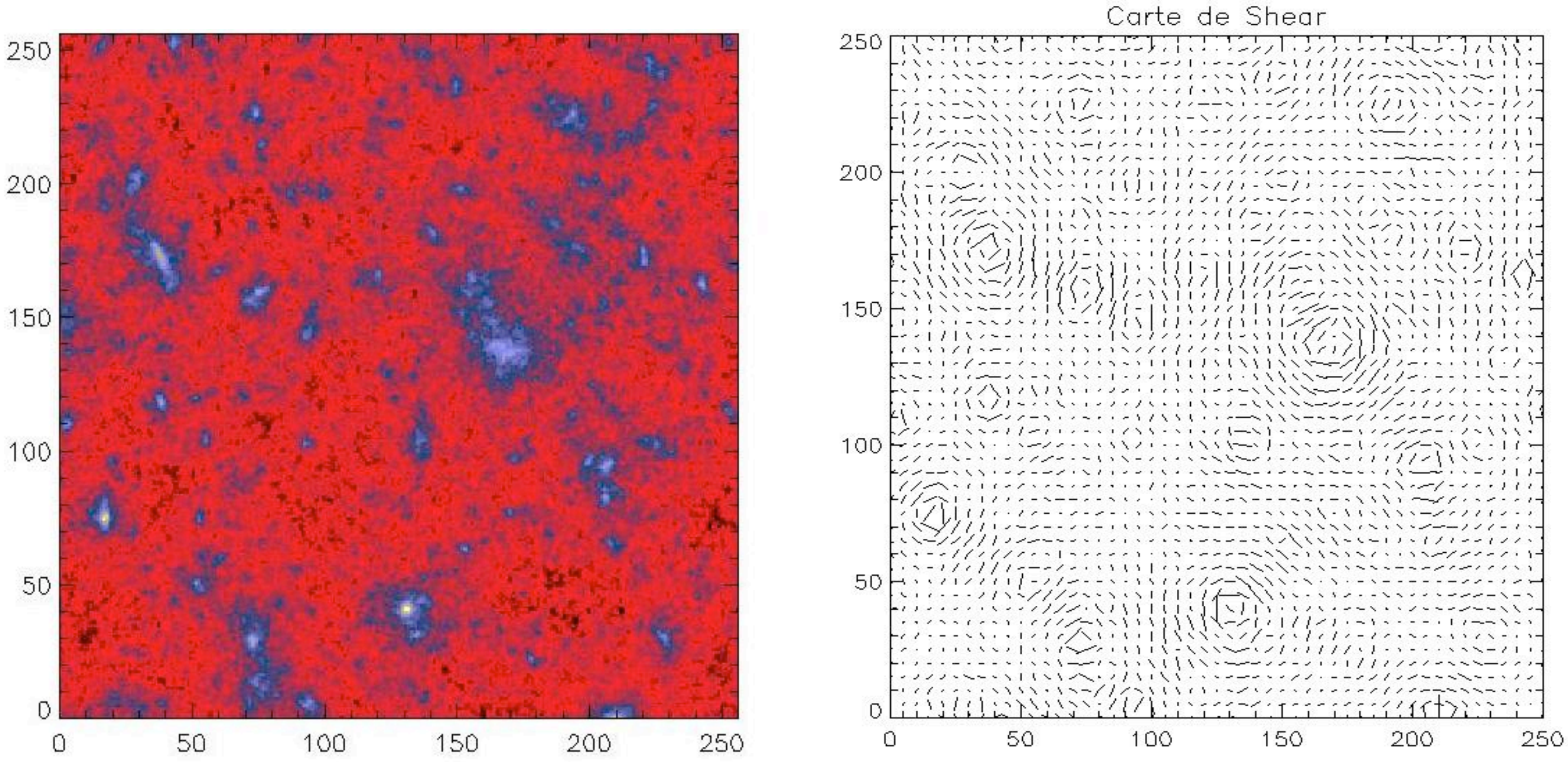
The relation between this estimation and the true mass map is $\hat{\tilde{\kappa}}_l = \hat{\kappa} + \hat{N}$, where $\hat{N} = \hat{P}_1 \hat{N}_1 + \hat{P}_2 \hat{N}_2$. Another interesting feature is the term $\hat{\tilde{\kappa}}_l^{(B)} = P_2 * \gamma_{1b} - P_1 * \gamma_{2b}$. Indeed it should be free of any contamination from κ and can be used as a test for data quality estimation.

Therefore, the **so called E and B mode** are obtained by:

$$\begin{aligned}
\hat{\tilde{\kappa}}_l^{(E)} &= P_1 * \gamma_{1b} + P_2 * \gamma_{2b} \\
\hat{\tilde{\kappa}}_l^{(B)} &= P_2 * \gamma_{1b} - P_1 * \gamma_{2b}
\end{aligned}$$

Both of them are noisy, and must be filtered before being analysed.

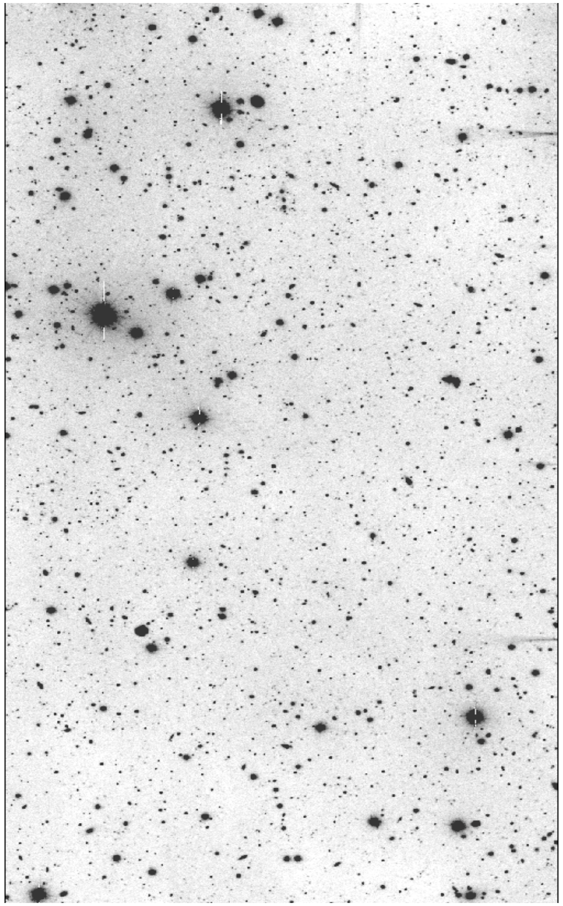
Simulated Mass Map



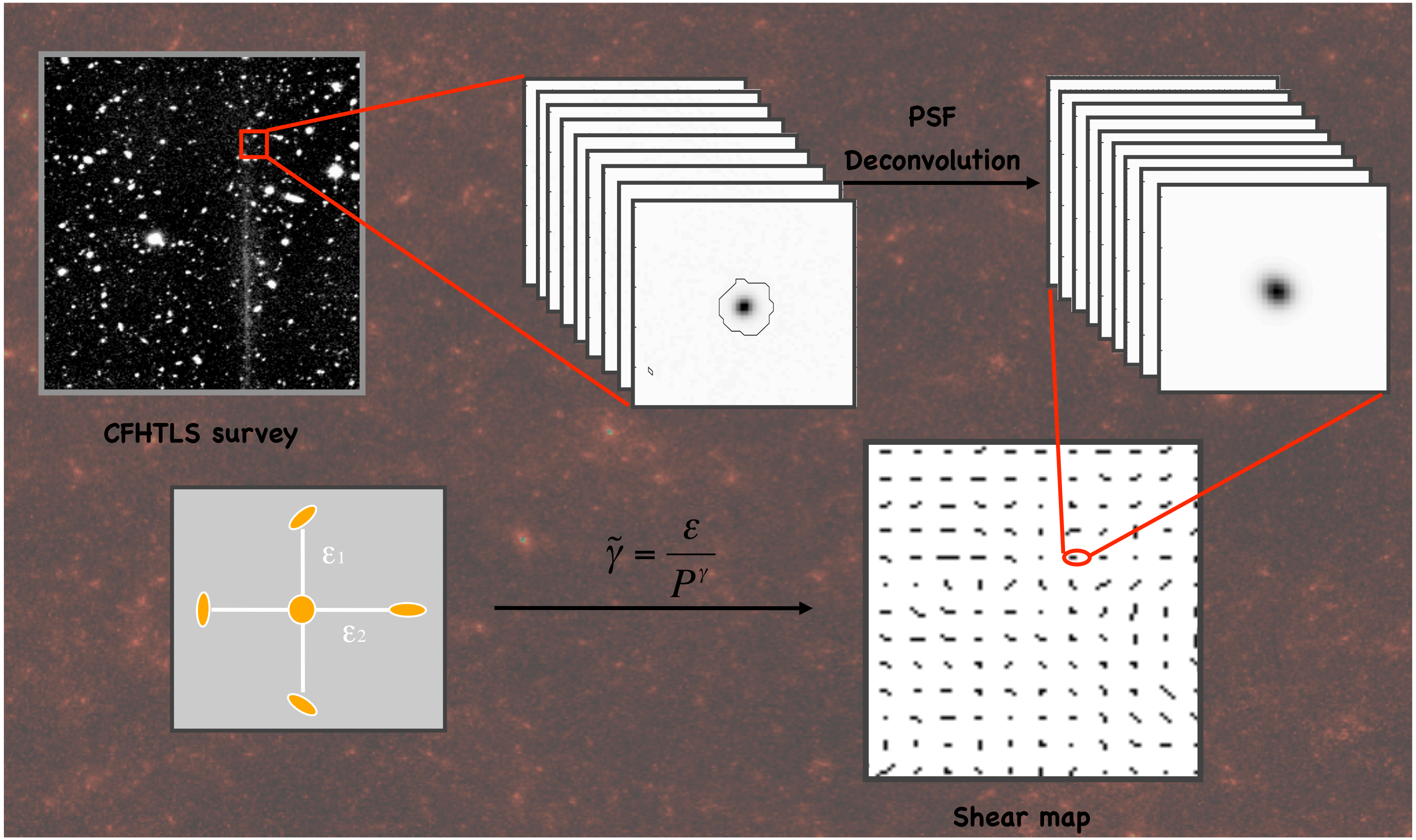
Deep Optical Images

William Herschel Telescope
La Palma, Canaries

16’x8’
R<25.5
30 (15) gals/sq. arcmin



From shear measurements to shear map



Mass inversion

Weak lensing mass map reconstruction using wavelets,

J.L. Starck, S. Pires and A. Réfrégier, A&A, June 2006, Vol. 451, p1139-1150

$$\gamma_1 = \frac{1}{2}\left(\partial_1^2 - \partial_2^2\right)\psi$$

$$\gamma_2 = \partial_1 \partial_2 \psi$$

$$\kappa = \frac{1}{2}\left(\partial_1^2 + \partial_2^2\right)\psi$$

$$\Longrightarrow$$

$$\gamma_i = P_i * \kappa$$

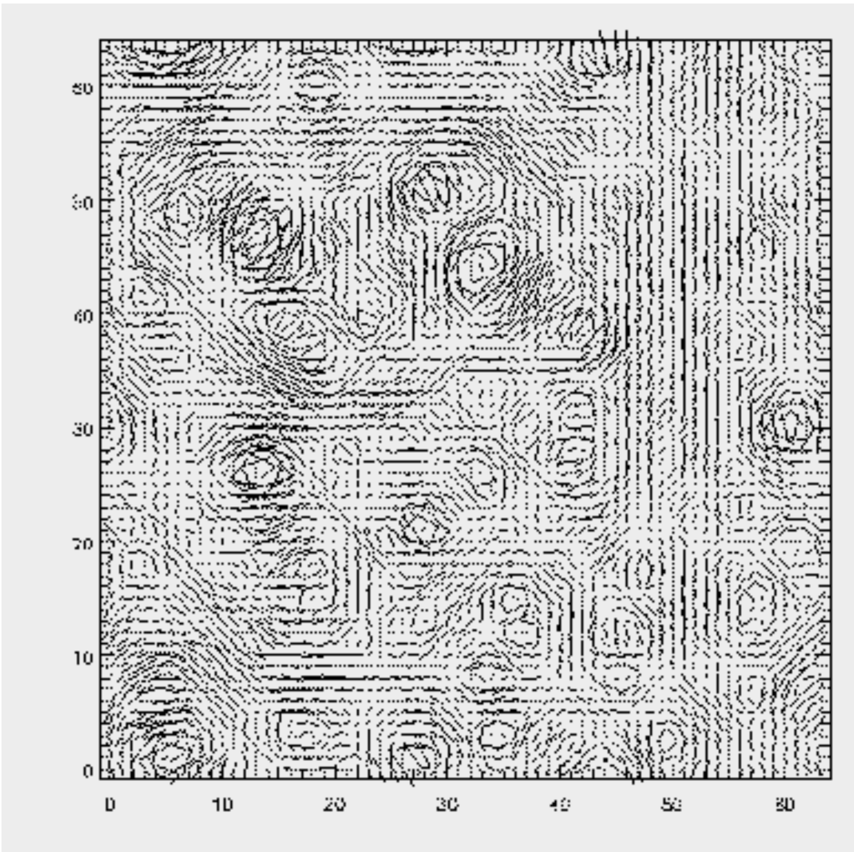
$$\kappa = P_1 * \gamma_1 + P_2 * \gamma_2$$

$$\hat{P}_1(k) = \frac{k_1^2 - k_2^2}{k^2}$$

$$\hat{P}_2(k) = \frac{2 k_1 k_2}{k^2}$$

®**Direct measure of the distribution of mass in the universe, as opposed to the distribution of light, as in other methods (eg. Galaxy surveys)**

From the statistics of the shear field, weak lensing provides:

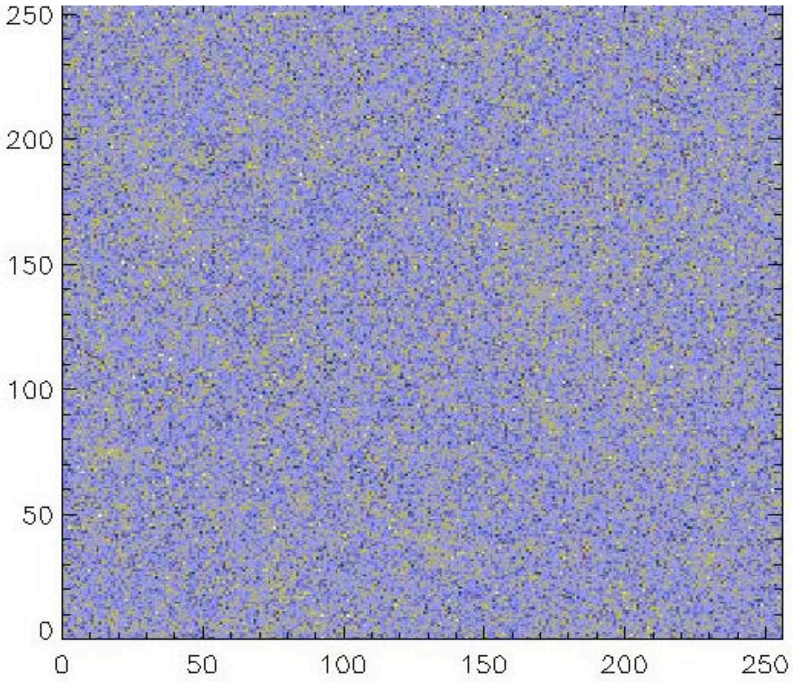


- **Mapping of the distribution of Dark Matter on various scales**
- **Measurement of cosmological parameters.**
- **Measurement of the evolution of structures**
- **a mass-selected cluster catalog**

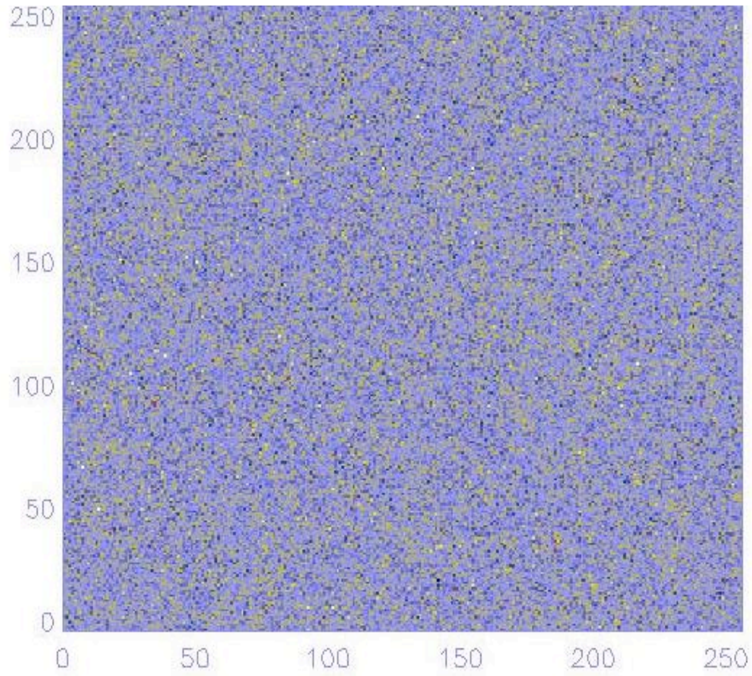
1x1 deg

Problem :
maps are very noisy

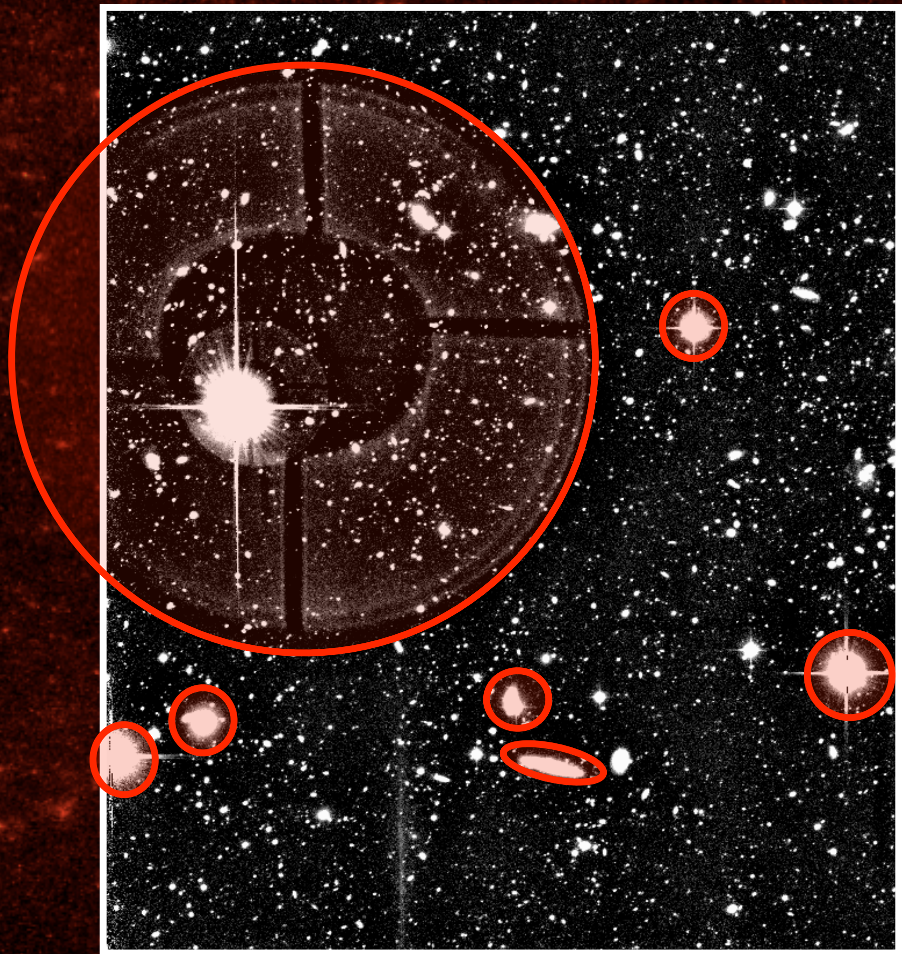
Simulated spatial observation



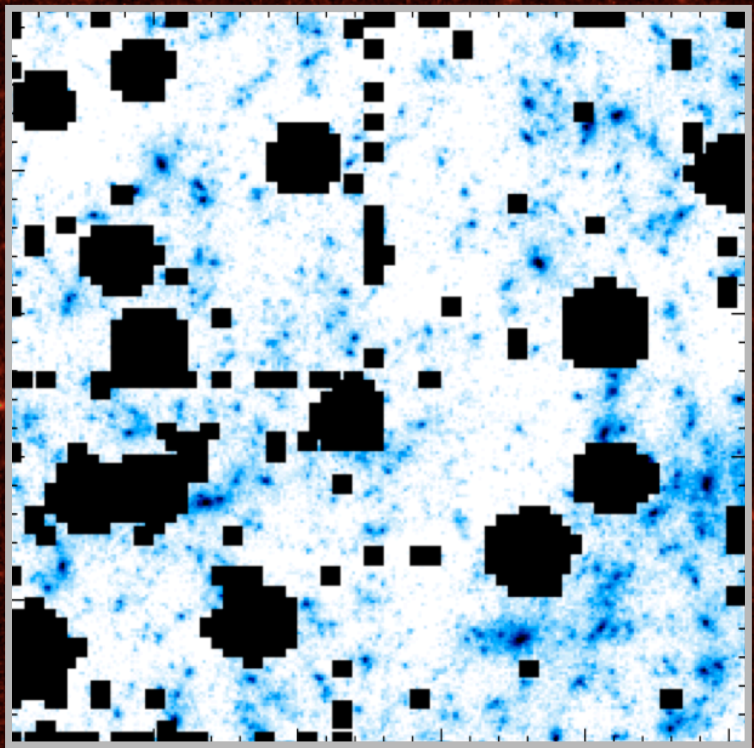
Simulated on ground observation



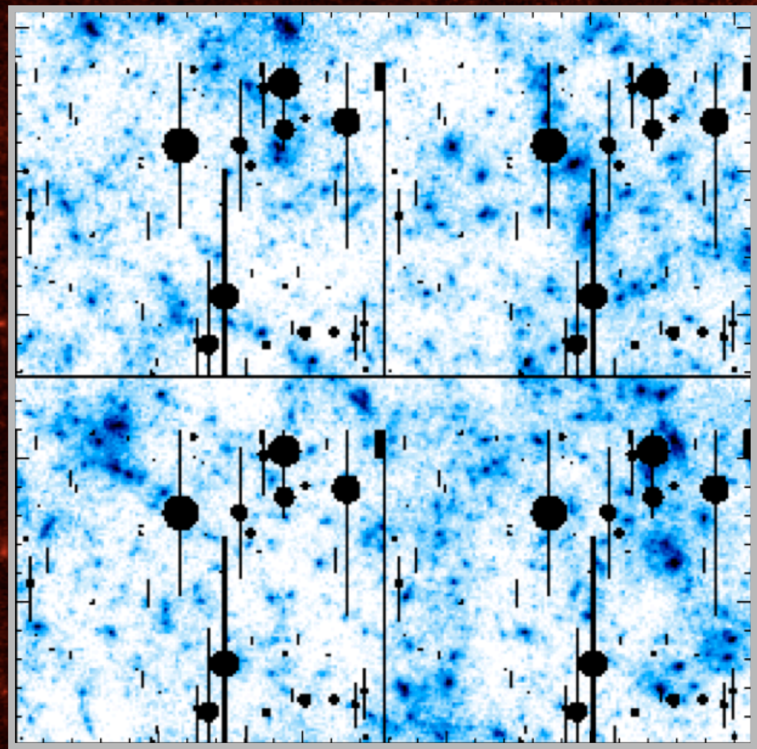
Weak lensing missing data



Masked masks



Mask pattern of CFHTLS survey on 1° x 1° field



Mask pattern of Subaru survey on 1° x 1° field

Interpolation of Missing Data: Inpainting

•*M. Elad, J.-L. Starck, D.L. Donoho, P. Querre, "Simultaneous Cartoon and Texture Image Inpainting using Morphological Component Analysis (MCA)", ACHA, Vol. 19, pp. 340-358, 2005.*
•*M.J. Fadili, J.-L. Starck and F. Murtagh, "Inpainting and Zooming using Sparse Representations", in press.*

$$\mathbf{\Theta}_\Lambda = \mathbf{Id}_\Lambda$$

$$\min_{\alpha} \|\alpha\|_{\ell_0} \text{ s.t. } y = Mx$$

Where M is the mask: M(i,j) = 0 ==> missing data
M(i,j) = 1 ==> good data

$$x^{(n+1)} = \mathcal{S}_{\Phi,\lambda^{(n)}} \left\{ x^{(n)} + M \left(y - x^{(n)} \right) \right\}$$

20%

50%

80%

Jalal Fadili's web page (http://www.greyc.ensicaen.fr/~jfadili).

Weak lensing inpainting algorithm

S. Pires, J.-L. Starck, A. Amara, R. Teyssier, A. Refregier and J. Fadili,
"FASTLens (FAst STatistics for weak Lensing) : Fast method for Weak Lensing Statistics and map making", submitted.

$$\gamma_i \longrightarrow \boxed{\min_{\kappa} \| \Phi^t \kappa \|_{l_0} \text{ subject to } \sum_i \| \gamma_i - M(P_i * \kappa) \|_{l_2}^2 \leq \varepsilon} \longrightarrow \kappa$$

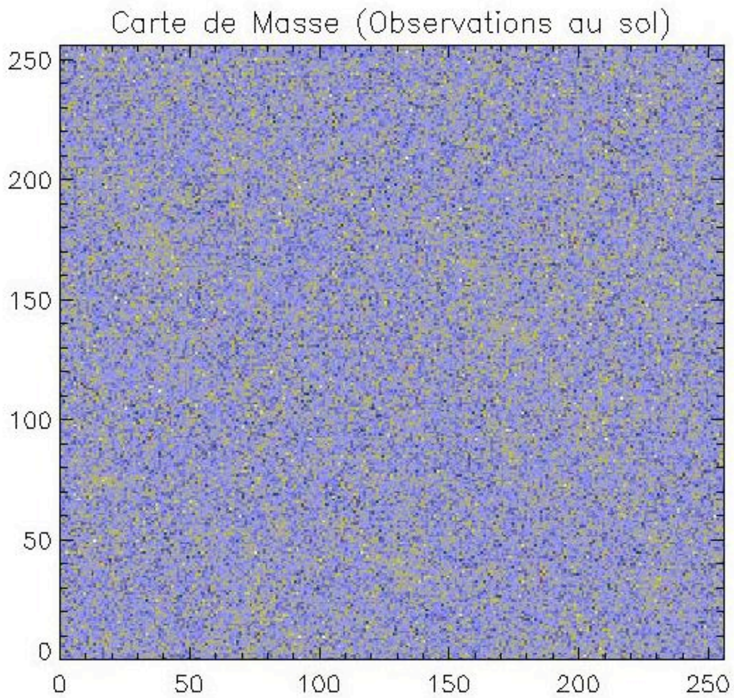
Physical priors

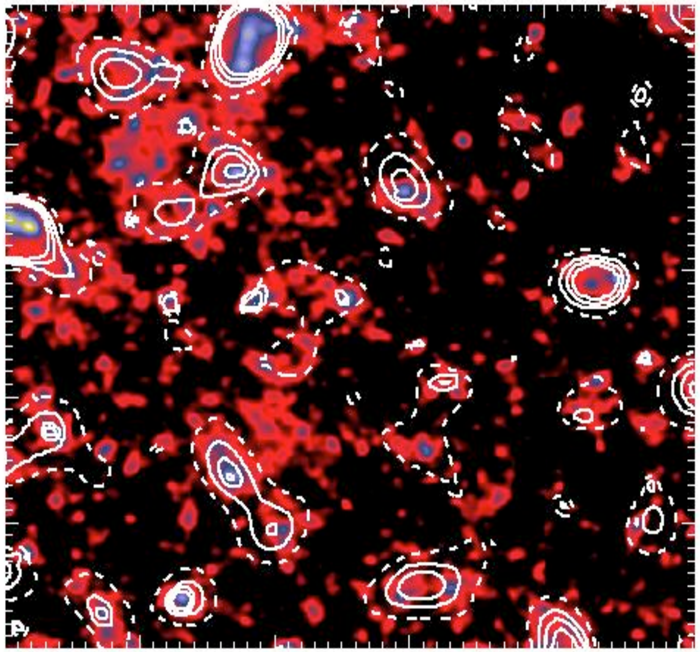
$$\gamma_i^{obs} = M . \gamma_i$$

$$\kappa = P_1 * \gamma_1 + P_2 * \gamma_2$$

Φ^t is the DCT

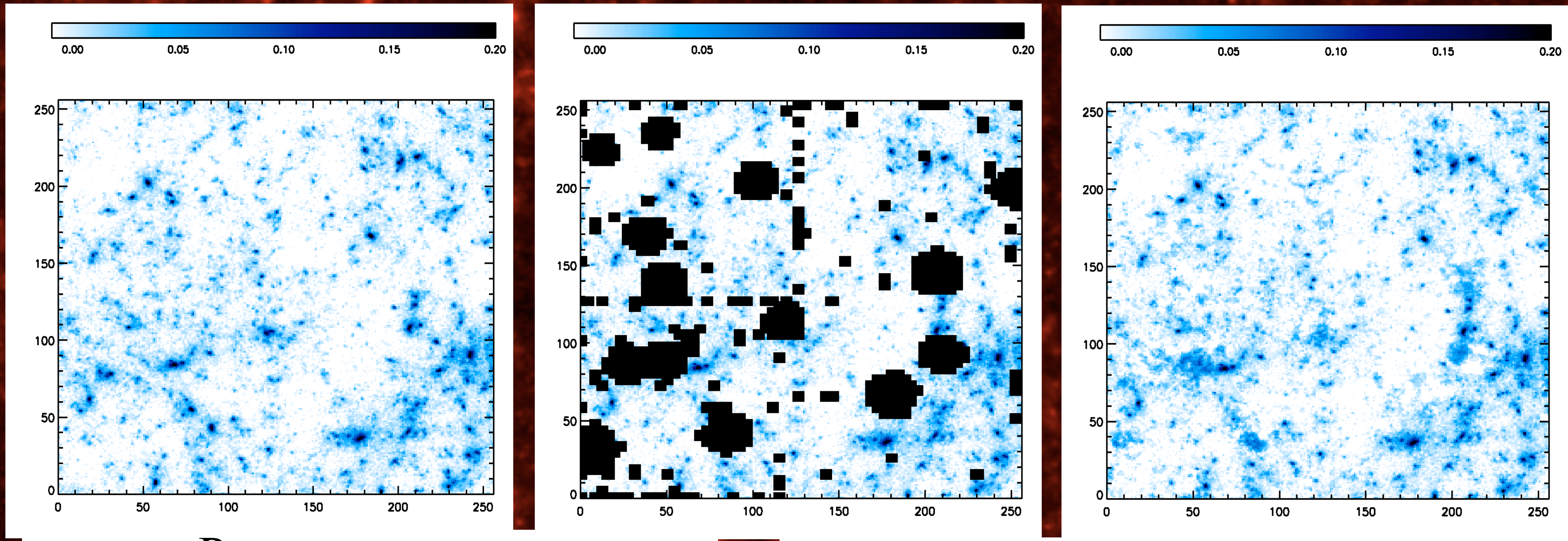
Reconstructed Dark Matter Map



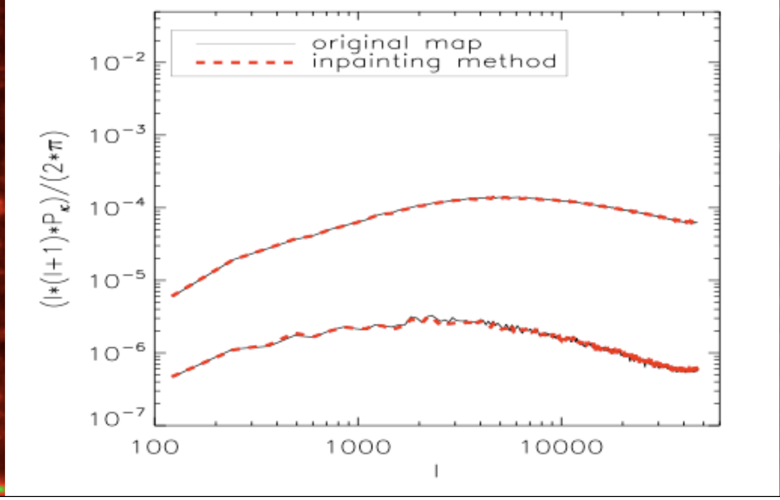


J.-L. Starck, S. Pires and A. Réfrégier, Astronomy and Astrophysics, 451, 3, 2006, pp.1139-1150 , 2006

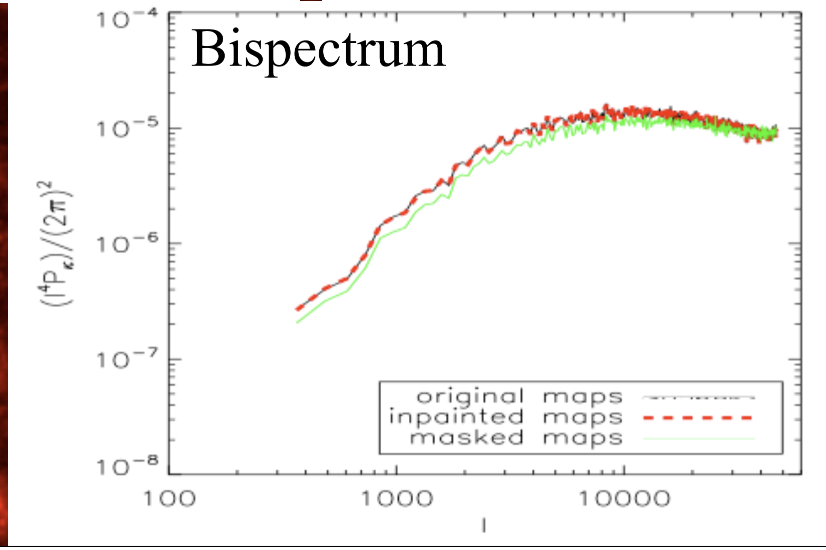
Inpainting :

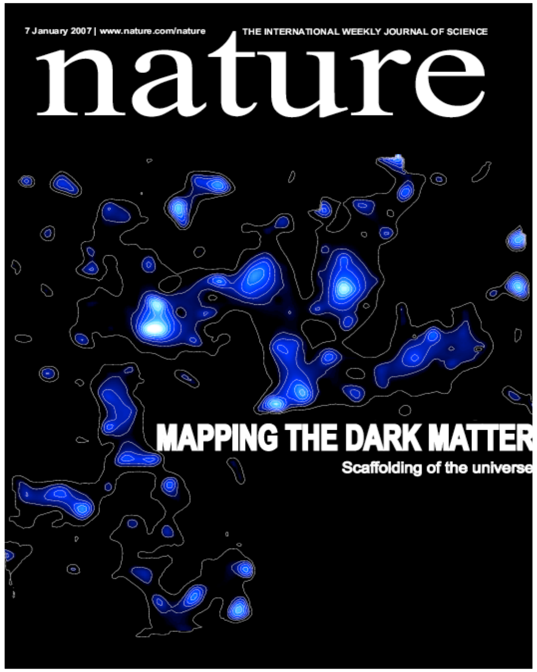


Power spectrum



Bispectrum





COSMOS data :

Maps of the Universe's Dark matter scaffolding, Massey et al, **Nature**,

Vol. 445, pp. 286-290, 2007

Baryonic and non-baryonic matter comparison at large scale

The total projected mass map from WL (dominated by dark matter) is shown as contours. It is compared to 3 independent baryonic tracers : stellar mass (in blue), galaxy number density seen in optical and near-**IR** light (in green) and the hot gas seen in **x-rays** (in red).