Inverse Problems in Astrophysics

•Part 1: Introduction inverse problems and image deconvolution

•Part 2: Introduction to Sparsity and Compressed Sensing

•Part 3: Wavelets in Astronomy: from orthogonal wavelets and to the Starlet transform.

•Part 4: Beyond Wavelets

•Part 5: Inverse problems and their solution using sparsity: denoising, deconvolution, inpainting, blind source separation.

•Part 6: CMB & Sparsity

•Part 7: Perspective of Sparsity & Compressed Sensing in Astrophsyics

Multiscale Transforms

Critical Sampling

(bi-) Orthogonal WT
Lifting scheme construction
Wavelet Packets
Mirror Basis

Redundant Transforms

Pyramidal decomposition (Burt and Adelson)
Undecimated Wavelet Transform
Isotropic Undecimated Wavelet Transform
Complex Wavelet Transform
Steerable Wavelet Transform
Dyadic Wavelet Transform
Nonlinear Pyramidal decomposition (Median)

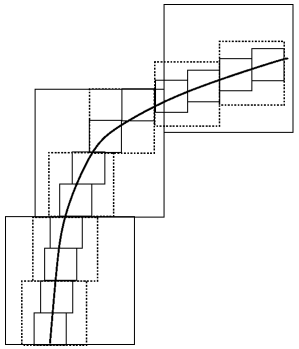
New Multiscale Construction

Contourlet
Bandelet
Finite Ridgelet Transform
Platelet
(W-)Edgelet
Adaptive Wavelet

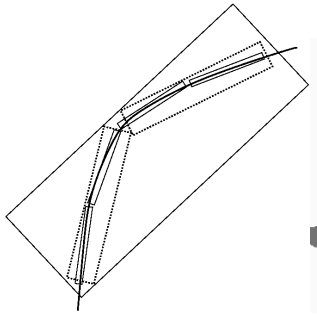
Ridgelet
Curvelet (Several implementations)
Wave Atom

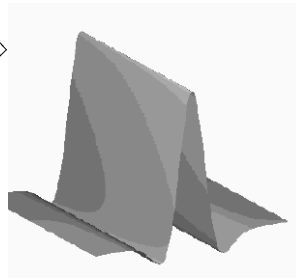
Wavelets and edges

• many wavelet coefficients are needed to account for edges i.e. singularities along lines or curves :



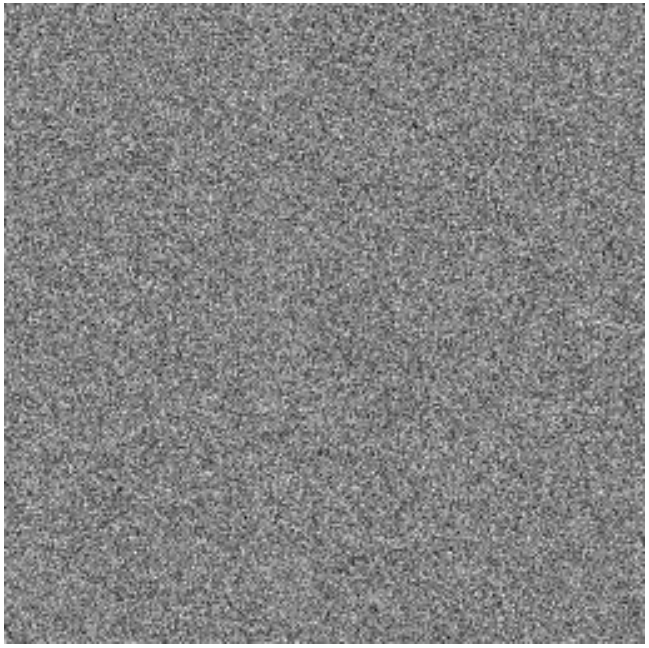
• need dictionaries of strongly anisotropic atoms :

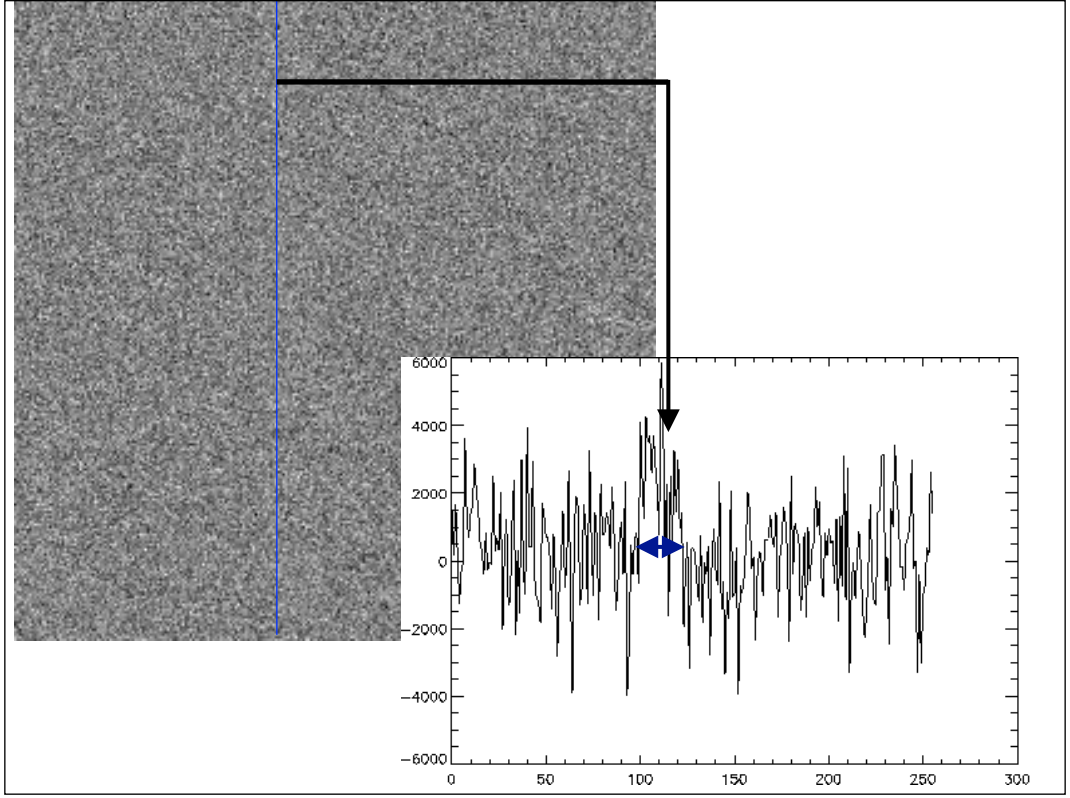




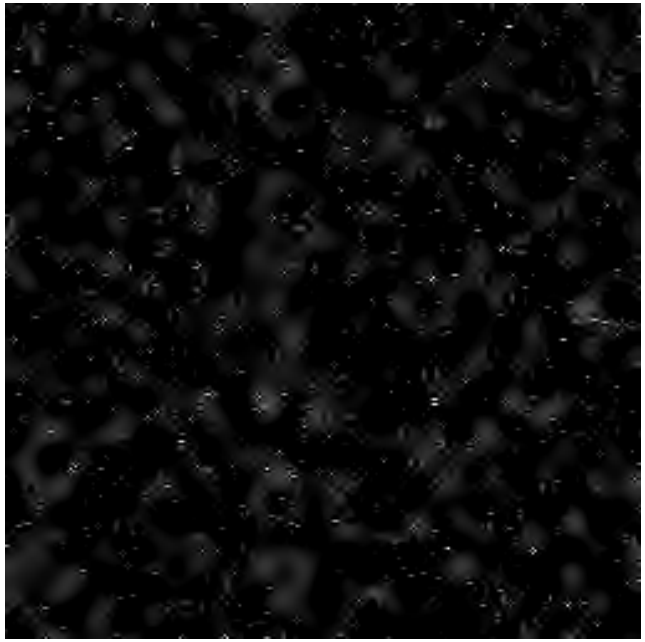
ridgelets, curvelets, contourlets, bandelettes, etc.

SNR = 0.1

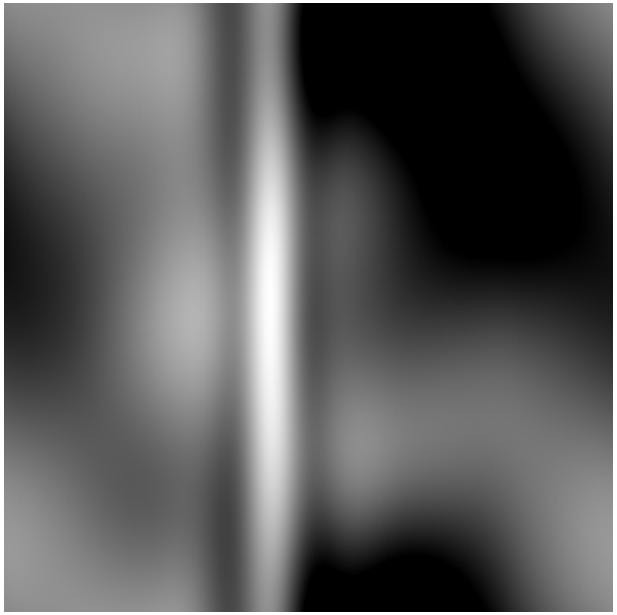


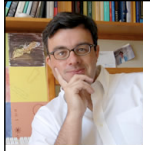

6000
4000
2000
0
−2000
−4000
−6000
0
50
100
150
200
250
300

Undecimated Wavelet Filtering (3 sigma)



Ridgelet Filtering (5sigma)



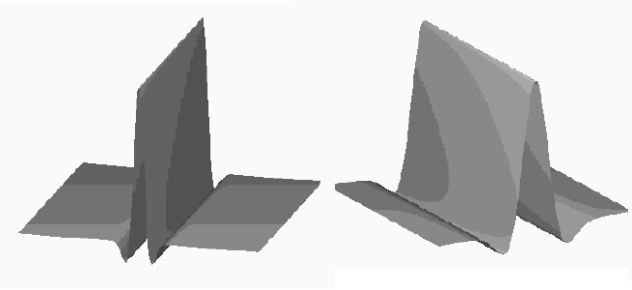


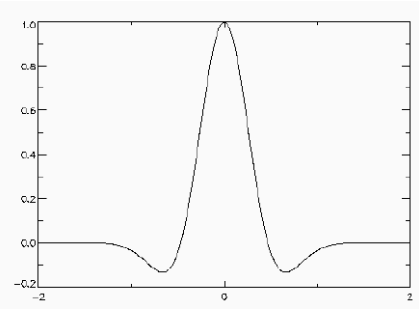
Continuous Ridgelet Transform

Ridgelet Transform (Candes, 1998): $$R_f(a,b,\theta) = \int \psi_{a,b,\theta}(x) f(x) dx$$

Ridgelet function: $$\psi_{a,b,\theta}(x) = a^{\frac{1}{2}} \psi\left(\frac{x_1 \cos(\theta) + x_2 \sin(\theta) - b}{a} \right)$$

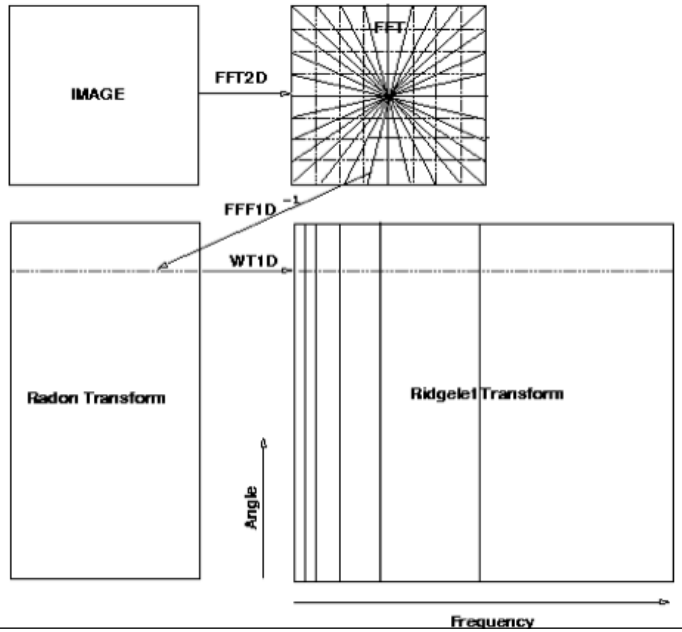
The function is constant along lines. Transverse to these ridges, it is a wavelet.





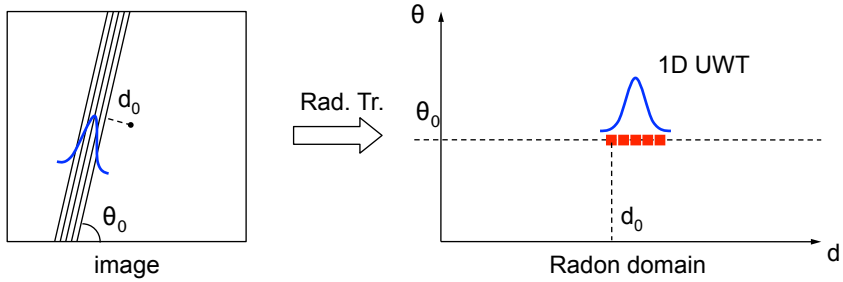
The ridgelet coefficients of an object f are given by analysis

of the Radon transform via: $$R_f(a,b,\theta) = \int Rf(\theta,t)\psi(\frac{t-b}{a})dt$$



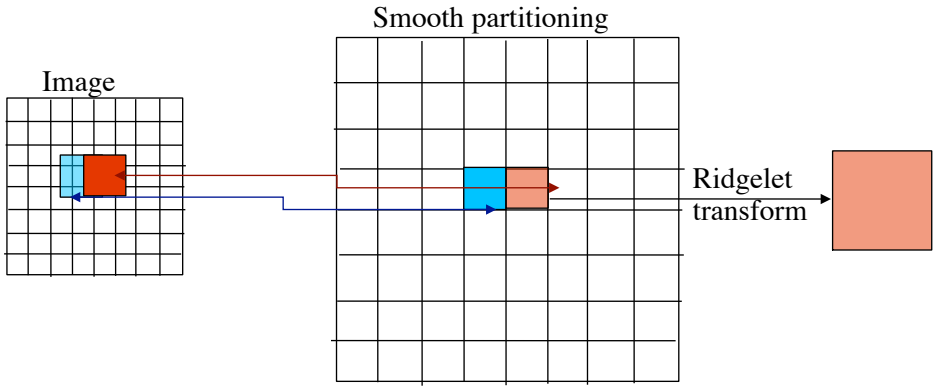
Ridgelet Transform

- Ridgelet transform: Radon + 1D Wavelet



1. Rad. Tr.
2. For each line, apply the same denoising scheme as before

LOCAL RIDGELET TRANSFORM

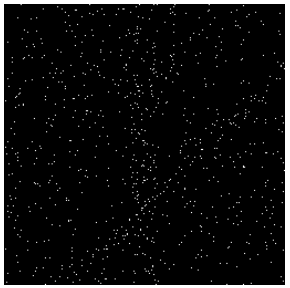


The partitioning introduces a redundancy, as a pixel belongs to 4 neighboring blocks.

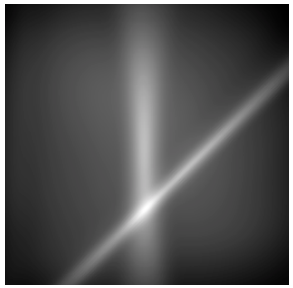
Poisson Noise and Line-Like Sources Restoration (MS-VST + Ridgelet)

B. Zhang, M.J. Fadili and J.-L. Starck, "Wavelets, Ridgelets and Curvelets for Poisson Noise Removal" ,ITIP, Vol 17, No 7, pp 1093--1108, 2008.

underlying intensity image



simulated image of counts

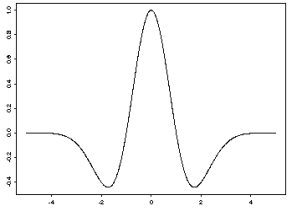


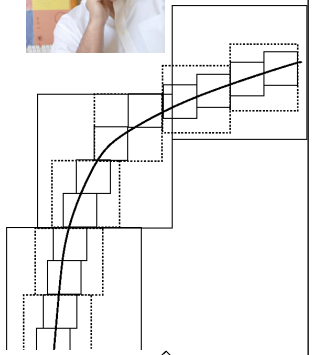
restored image
from the left image of counts

Max Intensity
background = 0.01
vertical bar = 0.03
inclined bar = 0.04

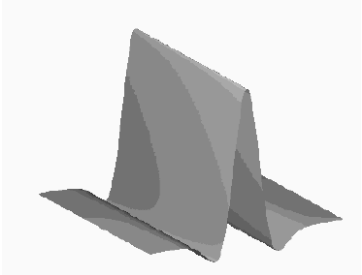
The Curvelet Transform (1999)

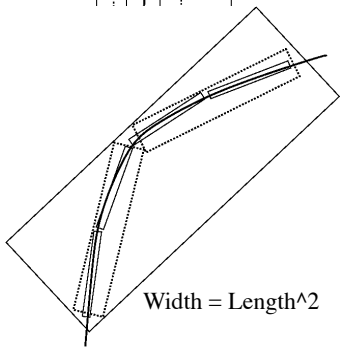
Wavelet

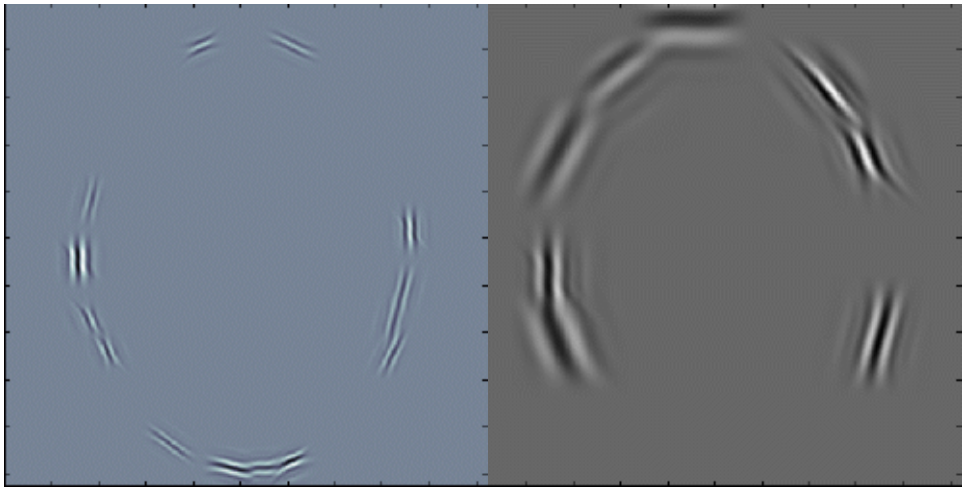




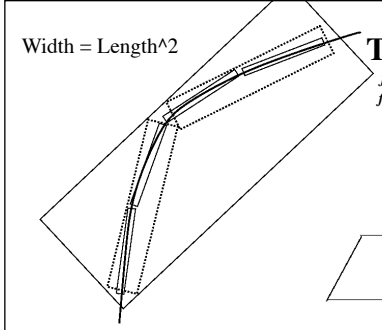
Curvelet





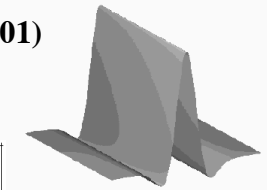


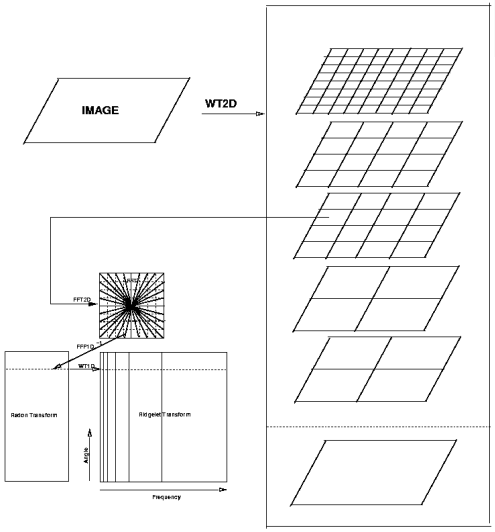
The Curvelet Transform for Image Denoising, *IEEE Transaction on Image Processing, 11, 6, 2002,*
- *2D Wavelet Tranforfm*
- *Local Ridgelet Transform*



The Curvelet Transform (CUR01)

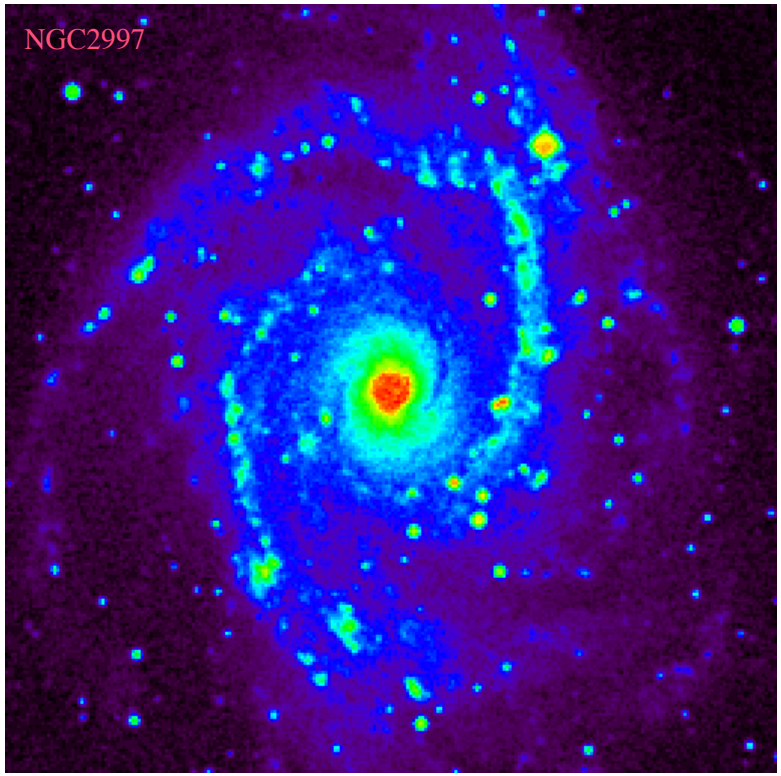
J.-L. Starck, E. Candes, D.L. Donoho ***The Curvelet Transform for Image Denoising****, IEEE Transaction on Image Processing, 11, 6*



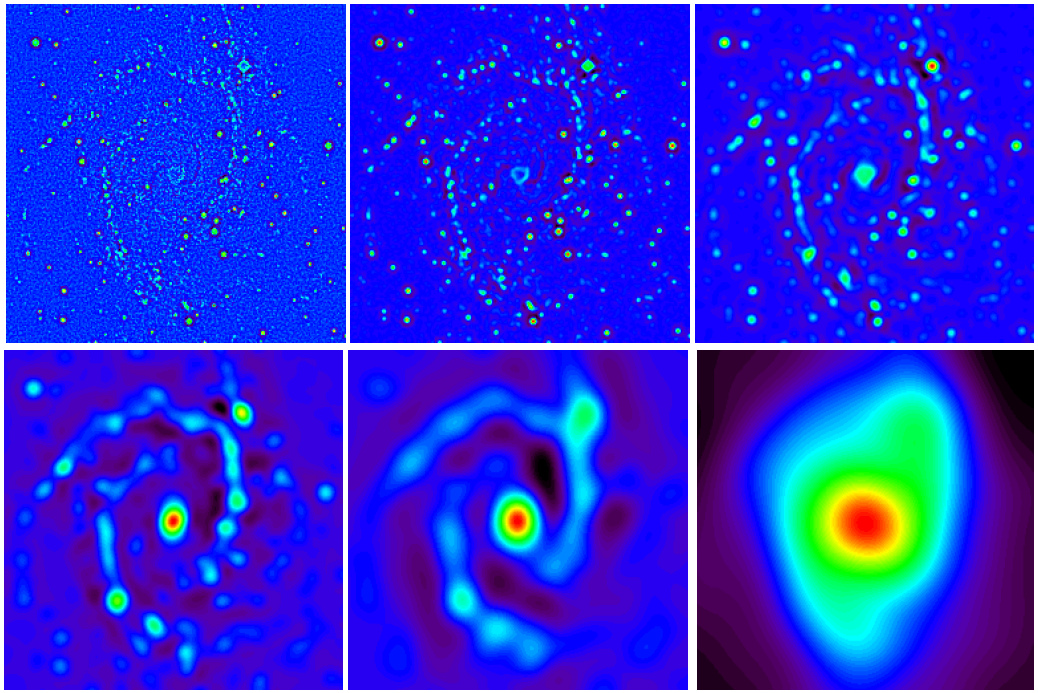


Redundancy $16J + 1$ for J wavelet scales.

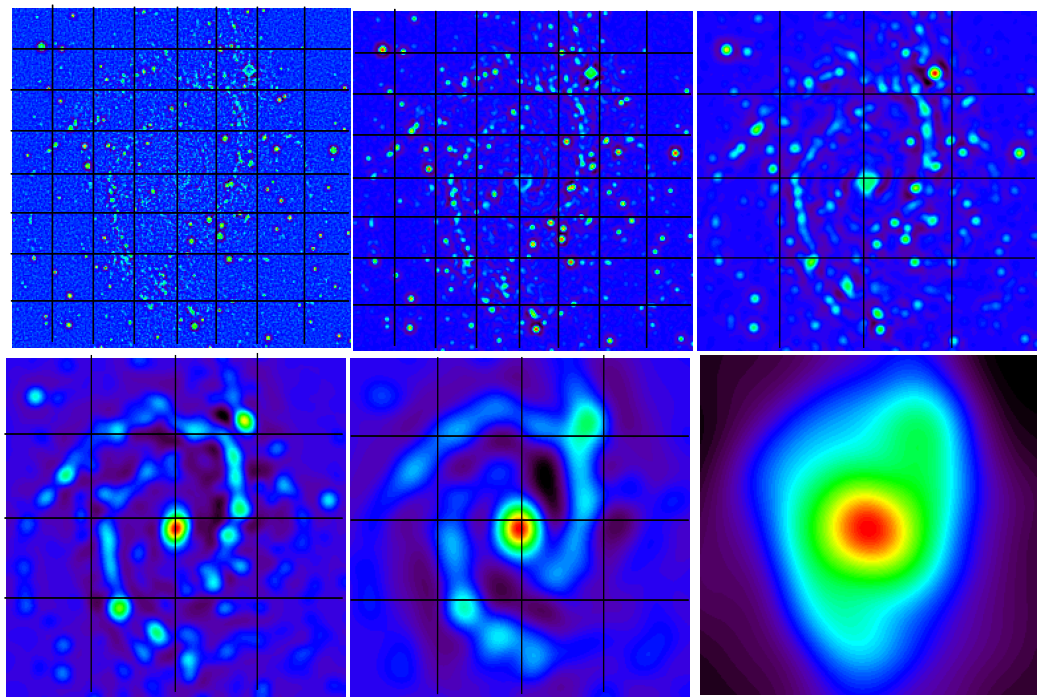
Complexity $O(N^2(\log N)^2)$ for $N \times N$ images.


NGC2997

Undecimated Isotropic WT: $I(k,l) = c_{J,k,l} + \sum_{j=1}^{J} w_{j,k,l}$



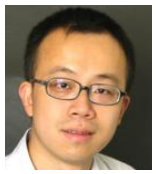
PARTITIONING

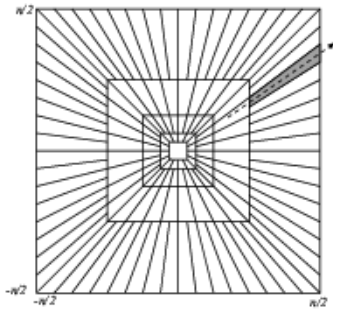


The Fast Curvelet Transform, Candes et al, 2005

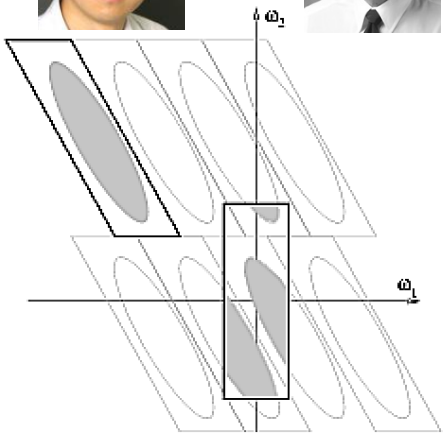
CUR03 - Fast Curvelet Transform using the USFFT
CUR04 - Fast Curvelet Transform using the Wrapping and 2DFFT

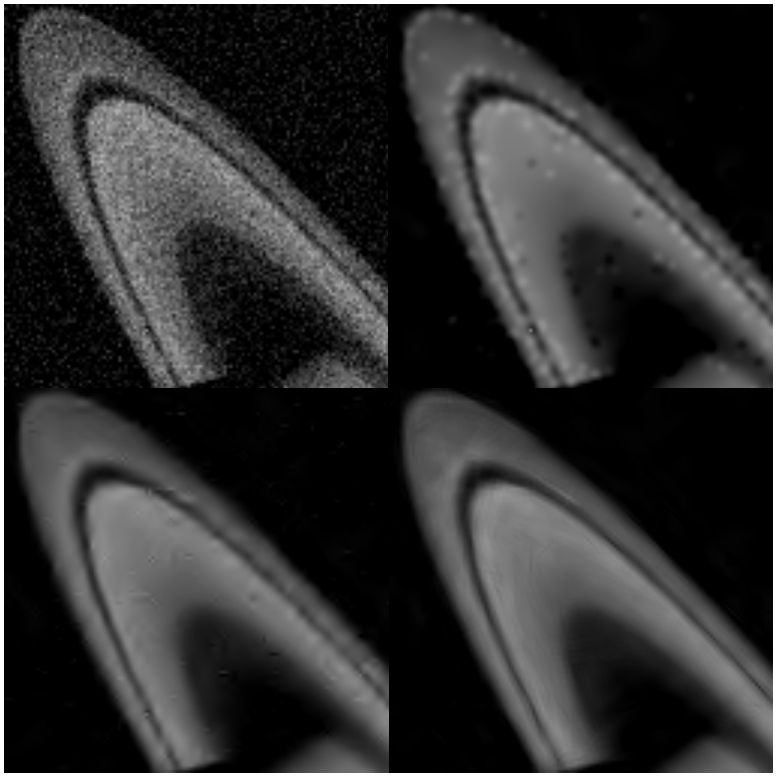




(a)





CONTRAST ENHANCEMENT USING THE CURVELET TRANSFORM

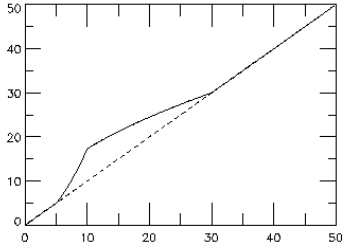
J.-L Starck, F. Murtagh, E. Candes and D.L. Donoho, ***"Gray and Color Image Contrast Enhancement by the Curvelet Transform"***,

IEEE Transaction on Image Processing, 12, 6, 2003.

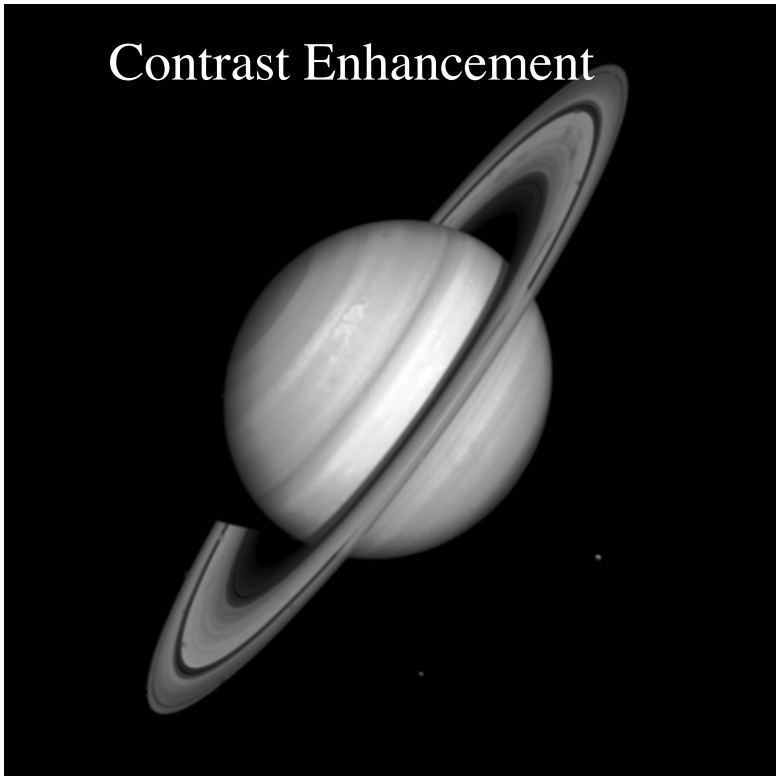
$$\tilde{I} = C_R\left(y_c\left(C_T I\right)\right)$$

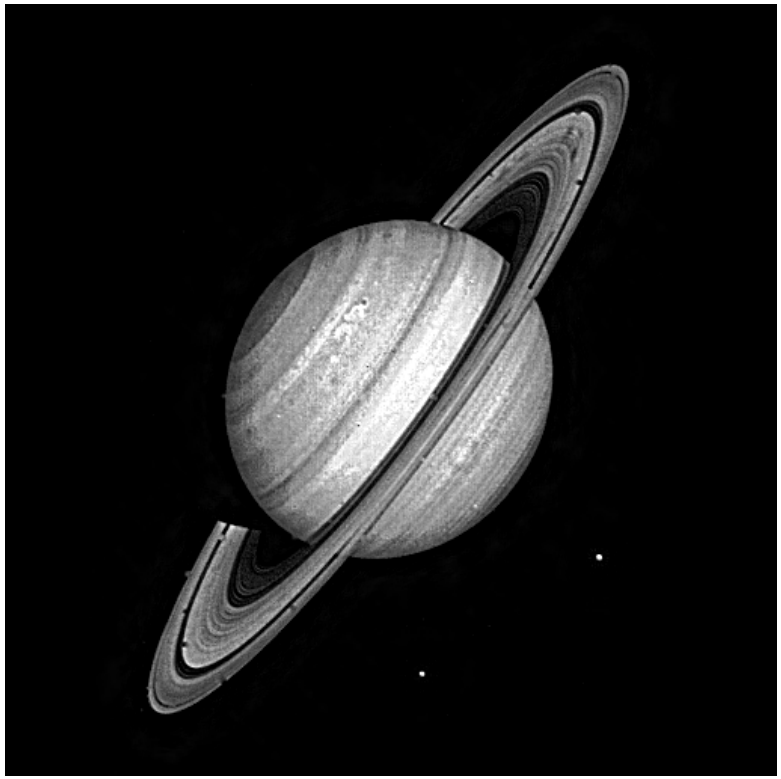
$$\begin{cases} y_c(x,\sigma) = 1 & \text{if} \quad x < c\sigma \\ y_c(x,\sigma) = \dfrac{x - c\sigma}{c\sigma}\left(\dfrac{m}{c\sigma}\right)^p + \dfrac{2c\sigma - x}{c\sigma} & \text{if} \quad x < 2c\sigma \\ y_c(x,\sigma) = \left(\dfrac{m}{x}\right)^p & \text{if} \quad 2c\sigma \le x < m \\ y_c(x,\sigma) = \left(\dfrac{m}{x}\right)^s & \text{if} \quad x > m \end{cases}$$

Modified curvelet coefficient

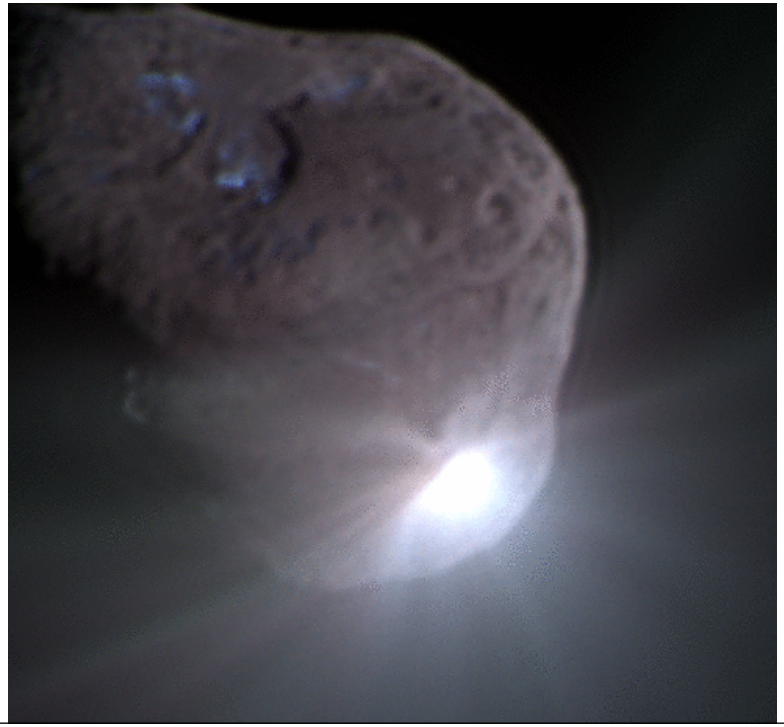


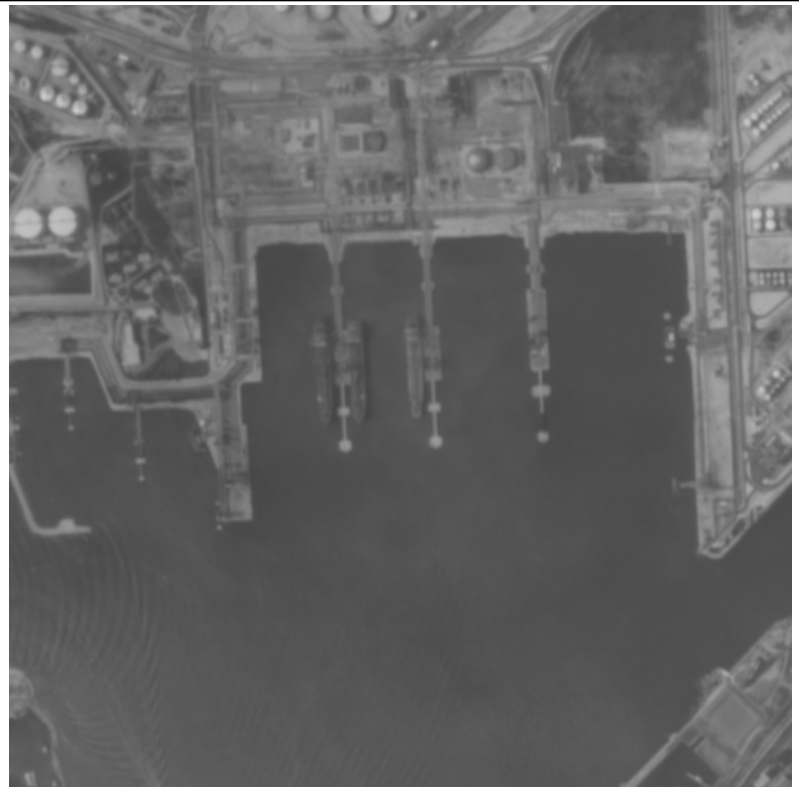
Curvelet coefficient


Contrast Enhancement



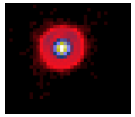
Comet 9P/Tempel-1: Impact on July 4, 2005

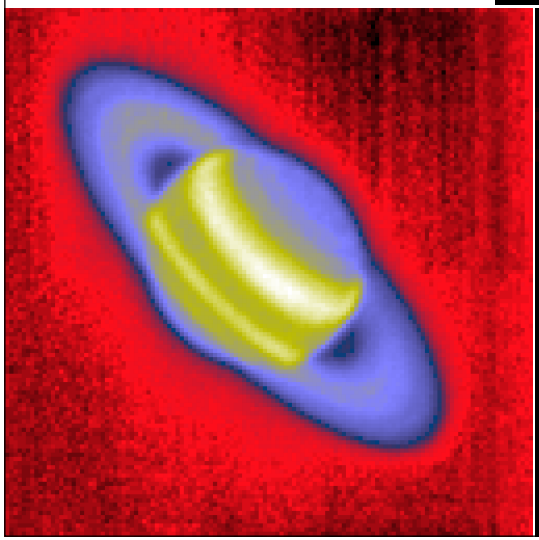


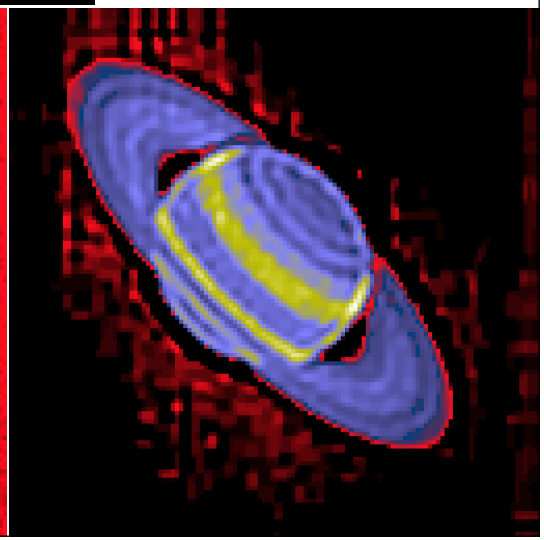


DECONVOLUTION

- E. Pantin, J.-L. Starck, and F. Murtagh, "Deconvolution and Blind Deconvolution in Astronomy", in Blind image deconvolution: theory and applications, pp 277--317, 2007.
- J.-L. Starck, F. Murtagh, and M. Bertero, "The Starlet Transform in Astronomical Data Processing: Application to Source Detection and Image Deconvolution", Springer, Handbook of Mathematical Methods in Imaging, in press, 2011.

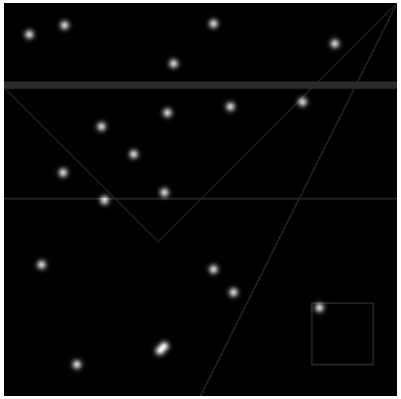




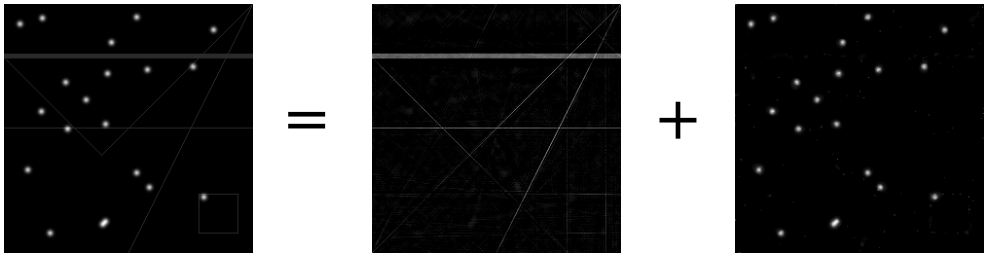


A difficult issue

Is there any representation that well represents the following image ?



Going further



Curvelets Wavelets

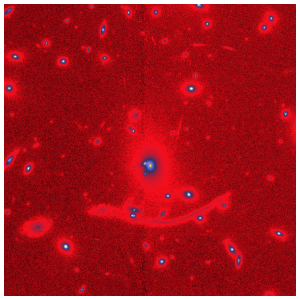
REDUNDANT REPRESENTATIONS

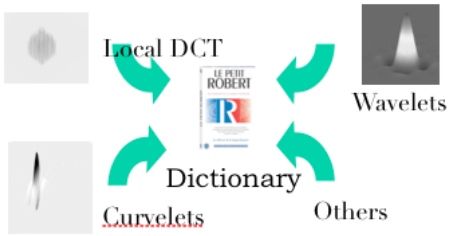
Morphological Diversity

•J.-L. Starck, M. Elad, and D.L. Donoho, ***Redundant Multiscale Transforms and their Application for Morphological Component Analysis***, *Advances in Imaging and Electron Physics, 132, 2004.*

•J.-L. Starck, M. Elad, and D.L. Donoho, ***Image Decomposition Via the Combination of Sparse Representation and a Variational Approach***, *IEEE Trans. on Image Proces., 14, 10, pp 1570--1582, 2005.*

•J.Bobin et al, ***Morphological Component Analysis: an adaptive thresholding strategy***, IEEE Trans. on Image Processing, Vol 16, No 11, pp 2675--2681, 2007.





$$\phi = [\phi_1,\ldots,\phi_L], \quad \alpha = \{\alpha_1,\ldots,\alpha_L\}, \quad s = \phi\alpha = \sum_{k=1}^{L} \phi_k \alpha_k$$

Sparsity Model 2: we consider a signal as a sum of K components s_k, $s = \sum_{k=1}^{K} s_k$ each of them being sparse in a given dictionary :

$$s_k = \Phi_k \alpha_k$$

$$s = \sum_{k=1}^{K} s_k = \sum_{k=1}^{K} \Phi_k \alpha_k = \Phi\alpha$$

New Perspectives

Morphological Component Analysis (MCA)

•***Redundant Multiscale Transforms and their Application for Morphological Component Analysis***, *Advances in Imaging and Electron Physics, 132, 2004.*
•***Image Decomposition Via the Combination of Sparse Representation and a Variational Approach***, *IEEE Trans. on Image Proces., 14, 10, pp 1570--1582, 2005*
• ***Morphological Component Analysis: an adaptive thresholding strategy***, IEEE Trans. on Image Processing, Vol 16, No 11, pp 2675--2681, 2007.

$$J(s_1,\ldots,s_L) = \left\| s - \sum_{k=1}^{L} s_k \right\|_2^2 + \lambda \sum_{k=1}^{L} \left\| T_k s_k \right\|_p$$

Morphological Component Analysis (MCA)

$$J(s_1,\ldots,s_L) = \left\| s - \sum_{k=1}^{L} s_k \right\|_2^2 + \lambda \sum_{k=1}^{L} \left\| T_k s_k \right\|_p$$

- Initialize all s_k to zero
- Iterate j=1,...,Niter
  - Iterate k=1,..,L

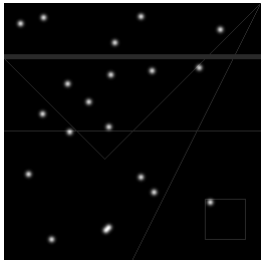
    Update the kth part of the current solution by fixing all other parts and minimizing:

$$J(s_k) = \left\| s - \sum_{i=1, i\neq k}^{L} s_i - s_k \right\|_2^2 + \lambda^{(j)} \left\| T_k s_k \right\|_p$$

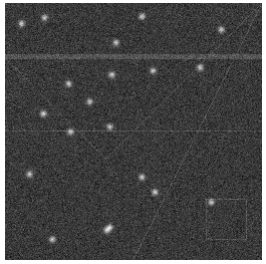
Which is obtained by a simple **hard**/soft thresholding of : $s_r = s - \sum_{i=1, i\neq k}^{L} s_i$

- Decrease the threshold $\lambda^{(j)}$

$$MIN_{s_1,s_2}\left(\left\|Ws_1\right\|_p + \left\|Cs_2\right\|_p\right) \text{ subject to } \left\|s-(s_1+s_2)\right\|_2^2 < \varepsilon$$

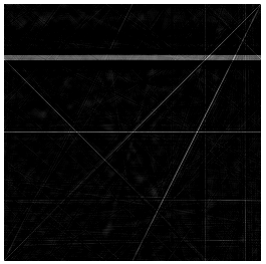


a) Simulated image (gaussians+lines)

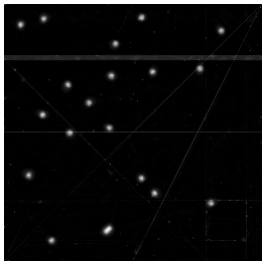


b) Simulated image + noise

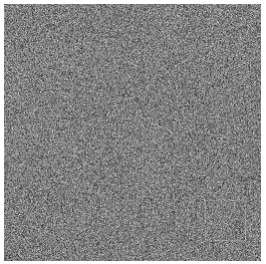
c) A trous algorithm



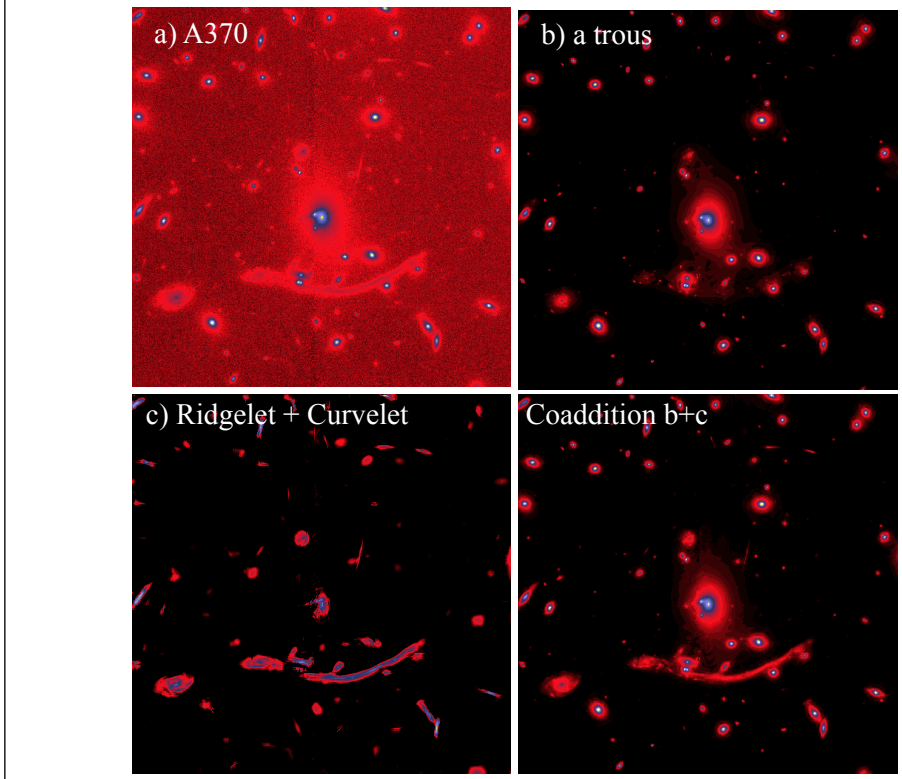
d) Curvelet transform

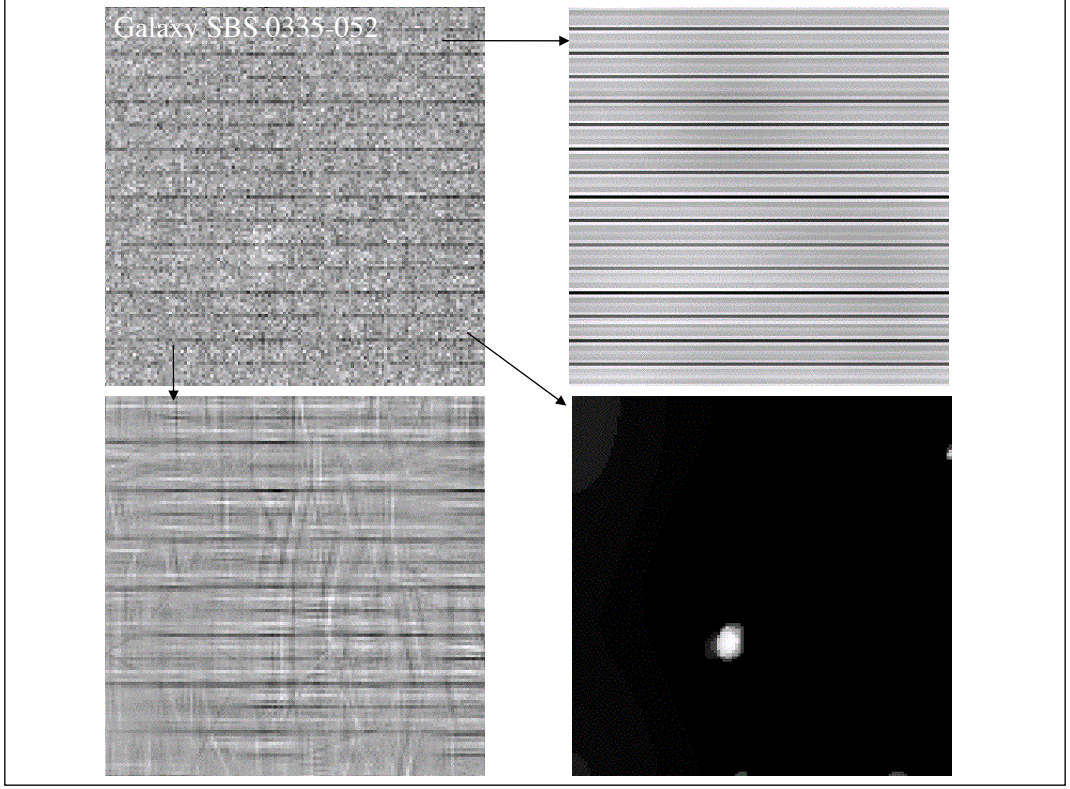


e) coaddition c+d

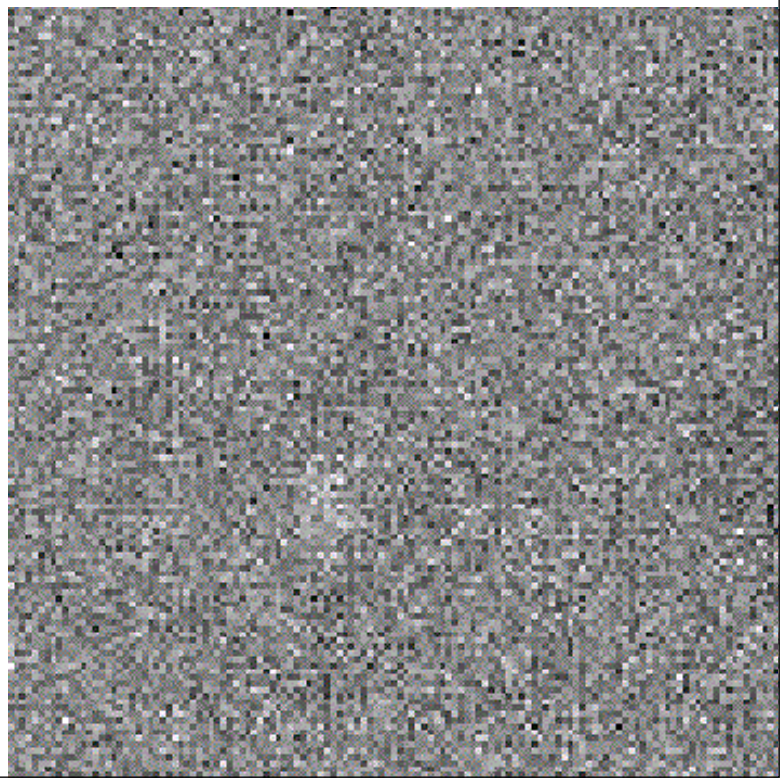


f) residual = e-b

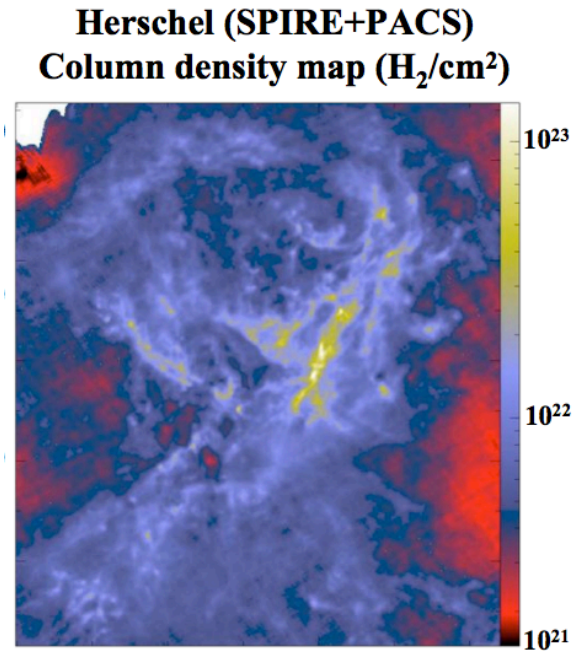

a) A370
b) a trous
c) Ridgelet + Curvelet
Coaddition b+c


Galaxy SBS 0335-052

Galaxy SBS 0335-052
10 micron
GEMINI-OSCIR



Revealing the structure of one of the nearest infrared dark clouds (Aquila Main: d ~ 260 pc)



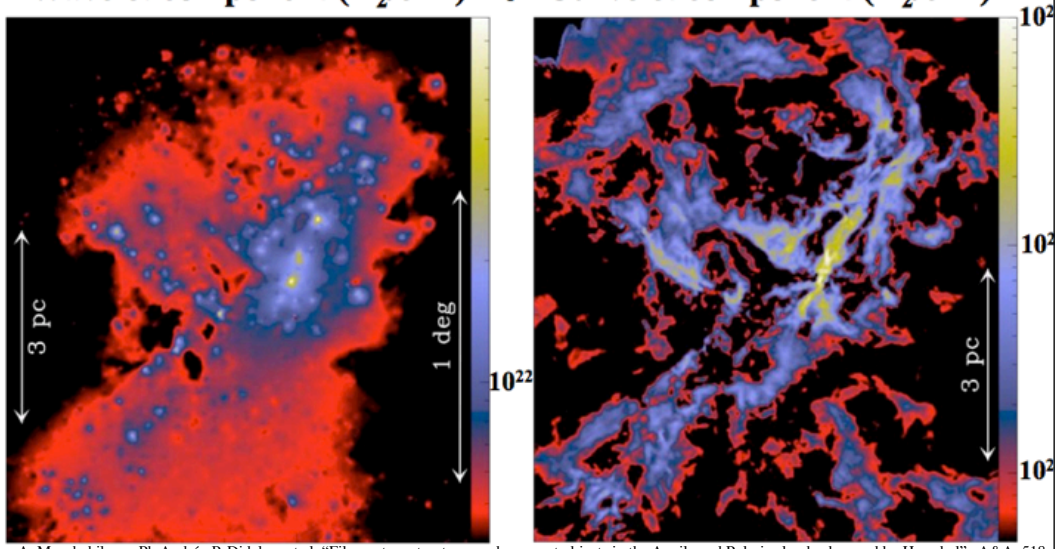
Dense cores form primarily in filaments

Morphological Component Analysis:

***Herschel* Column density map** (P. Didelon based on Starck et al. 2003)

Cores = Filaments

Wavelet component (H_2/cm^2) + Curvelet component (H_2/cm^2)

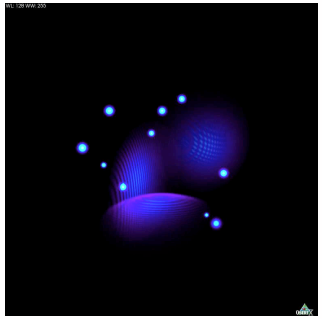


A. Menshchikov, Ph.André. P. Didelon, et al, "Filamentary structures and compact objects in the Aquila and Polaris clouds observed by Herschel", A&A, 518, id.L103, 2010.

3D Morphological Component Analysis

A. Woiselle

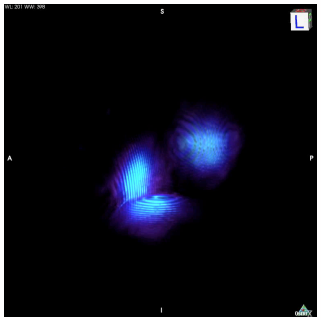
Original (3D shells + Gaussians)



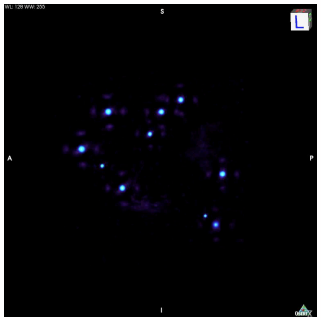
Dictionary

RidCurvelets + 3D UDWT.

Shells

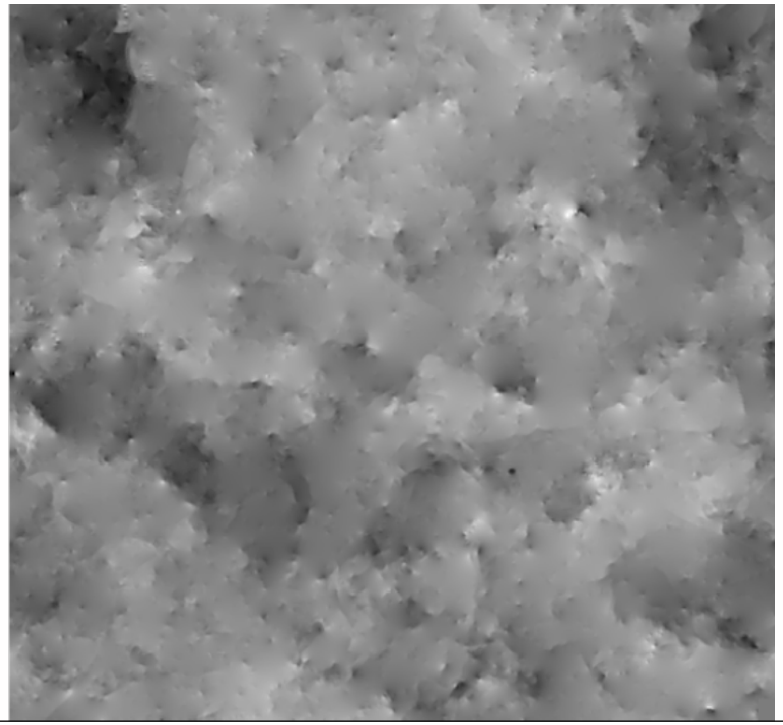


Gaussians

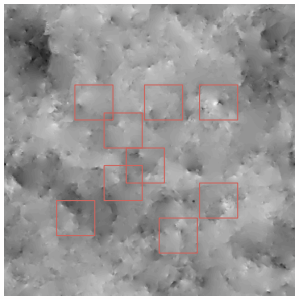


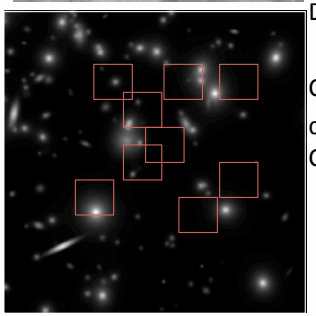
- A . Woiselle, J.L. Starck, M.J. Fadili, "3D Data Denoising and Inpainting with the Fast Curvelet transform", **JMIV**, 39, 2, pp 121-139, 2011.
- A. Woiselle, J.L. Starck, M.J. Fadili, "3D curvelet transforms and astronomical data restoration", **Applied and Computational Harmonic Analysis**, Vol. 28, No. 2, pp. 171-188, 2010.

Simulated Cosmic String Map



Dictionary Learning





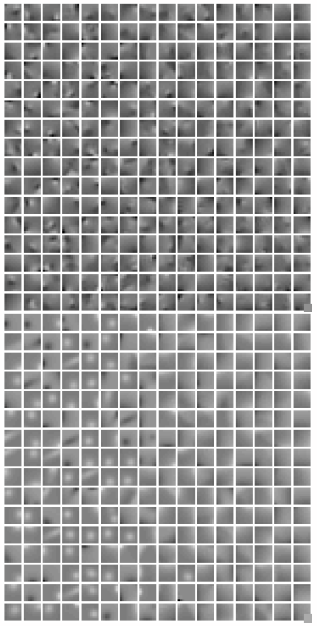
Training basis.

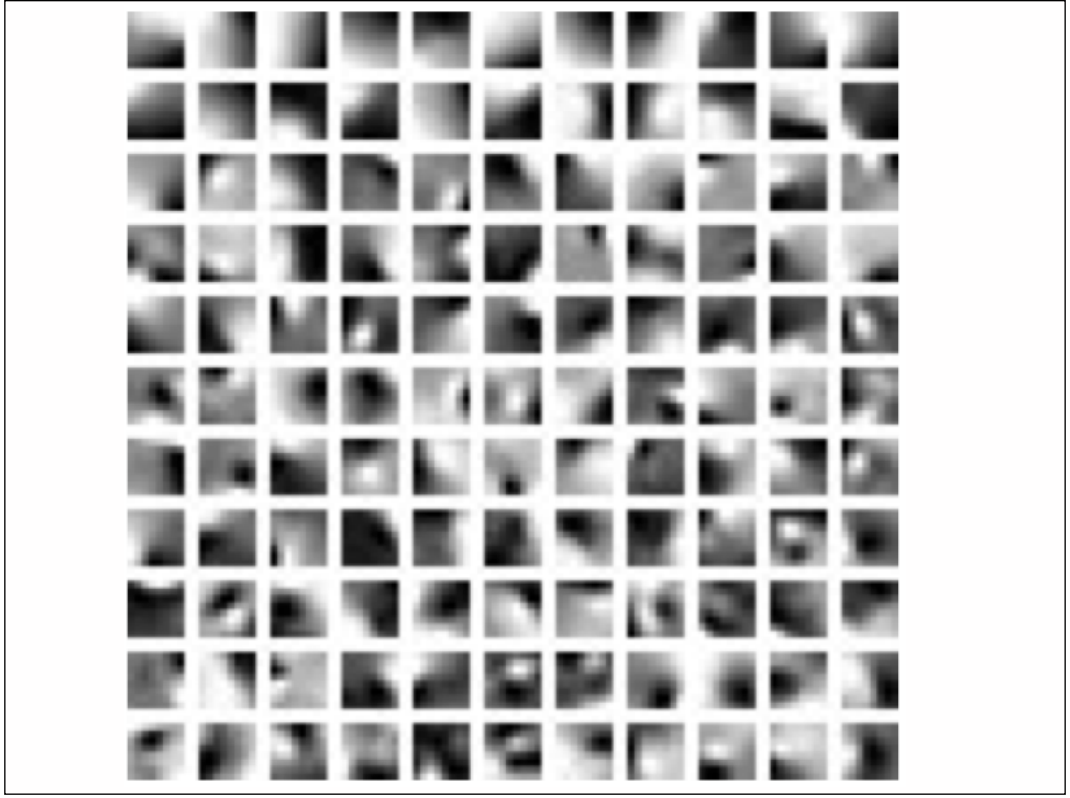
$$\left(\hat{D},\hat{\mathrm{A}}\right) = \underset{\substack{D\in \mathrm{C}_1 \\ A\in \mathrm{C}_2}}{\arg\min}\left(Y = DA\right)$$

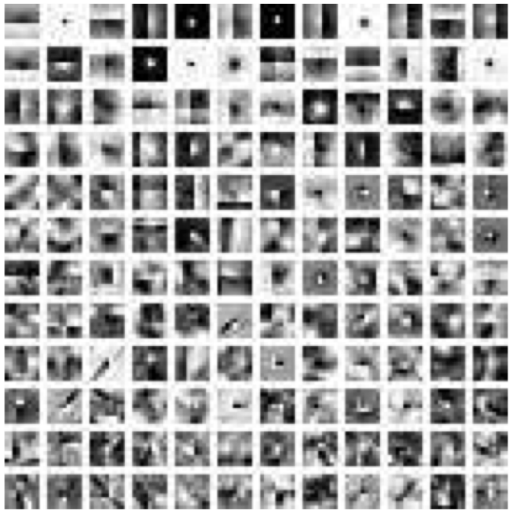
DL: Matrix Factorization problem

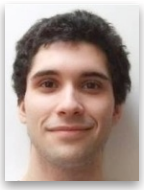
C_1: Constraints on the Sparsifying dictionary D

C_2: Constraints on the Sparse codes



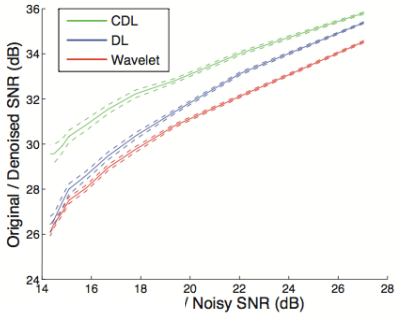


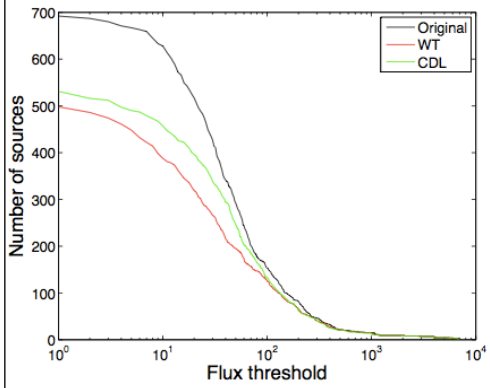


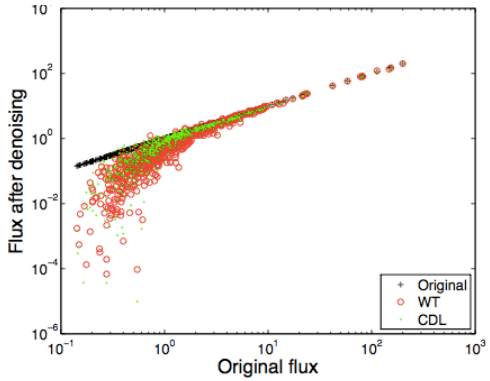


S. Beckouche

Astronomical Image Denoising Using Dictionary Learning, S. Beckouche, J.L. Starck, and J. Fadili, A&A, submitted.







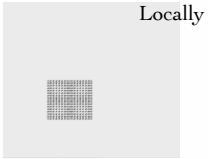
Sparsity Model 1: we consider a dictionary which has a fast transform/reconstruction operator:

$$\Phi = \{\phi_1, \ldots, \phi_K\}$$

$$s = \sum_{k=1}^{K} \alpha_k \phi_k = \Phi\alpha$$

Local DCT — Stationary textures

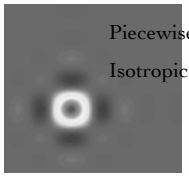
Locally oscillatory



Wavelet transform

Piecewise smooth

Isotropic structures



Curvelet transform

Sparsity Model 2: Morphological Diversity:

$$\phi = [\phi_1, \ldots, \phi_L], \quad \alpha = \{\alpha_1, \ldots, \alpha_L\}, \quad s = \phi\alpha = \sum_{k=1}^{L} \phi_k \alpha_k$$

Sparsity Model 3: we adapt/learn the dictionary directly from the data

Model 3 can be also combined with model 2:

G. Peyre, M.J. Fadili and J.L. Starck, , "Learning the Morphological Diversity", **SIAM Journal of Imaging Science**, 3 (3) , pp.646-669, 2010.

Advantages of model 1 (fixed dictionary) : extremely fast.

Advantages of model 2 (union of fixed dictionaries):
- more flexible to model 1.
- The coupling of local DCT+curvelet is well adapted to a relatively large class of images.

Advantages of model 3 (dictionary learning):
atoms can be obtained which are well adapted to the data, and which could never be obtained with a fixed dictionary.
Drawback of model 3 versus model 1,2:
We pay the price of dictionary learning by being less sensitive to detect very faint features.
Complexity: Computation time, parameters, etc