Inverse Problems in Astrophysics

•Part 1: Introduction inverse problems and image deconvolution

•Part 2: Introduction to Sparsity and Compressed Sensing

•Part 3: Wavelets in Astronomy: from orthogonal wavelets and to the Starlet transform.

•Part 4: Beyond Wavelets

•Part 5: Inverse problems and their solution using sparsity: denoising, deconvolution, inpainting, blind source separation.

•Part 6: CMB & Sparsity

•Part 7: Perspective of Sparsity & Compressed Sensing in Astrophsyics

WMAP-Planck CMB Map

- Cosmic Microwave Background (CMB) and Planck

- Part 1: Joint WMAP-Planck CMB Map Reconstruction

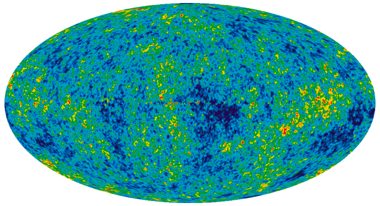
Joint Planck and WMAP CMB Map Reconstruction (arXiv:1401.6016), A&A,563, Id. A105, 2014.

- Part 3: Large Scale Anomalies Studies

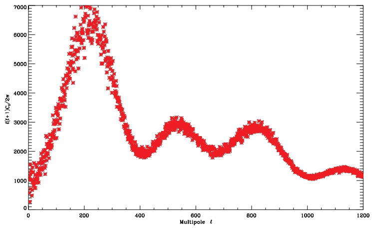
Planck CMB Anomalies: Astrophysical and Cosmological Foregrounds and the Curse of Masking (arXiv:1405.1844), JCAP, 08 id 006, 2014.

- Part 4: Primordial Pk Power Spectrum Reconstruction

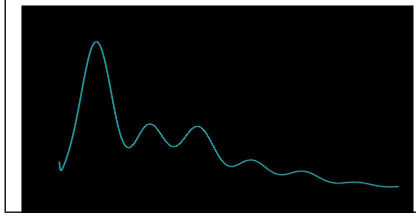
PRISM: Sparse Recovery of the Primordial Power Spectrum (arXiv:1406.7725), A&A, 566, id.A77, 2014.
PRISM: Sparse recovery of the primordial spectrum from WMAP9 and Planck datasets, arXiv:1406.7725, in press.



$$\Theta(\vec{p}) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m} Y_{\ell m}(\vec{p})$$



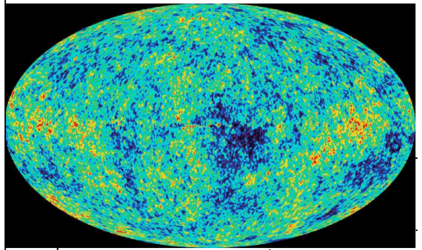
$$\widehat{C}_\ell^{\rm th} = \frac{1}{2\ell+1} \sum_{m=-\ell}^{\ell} |a_{\ell m}|^2$$

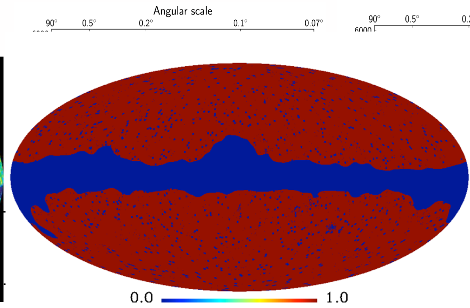


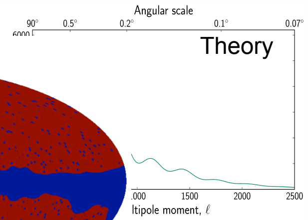
$$\widehat{C}_\ell^{\rm th} = C_\ell^{\rm th}\ Z_\ell$$

$$L Z_\ell \sim \chi^2_L \,, \qquad \text{where } L = 2\ell+1$$

Statistical Properties of the CMB fluctuation





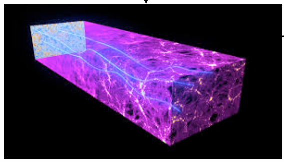


al Parameters

on inflation

inflation models

models (nl)



Statistical analysis of the weak lensing effect

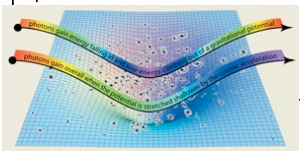
gravitationnal potential mapping

+

power spectrum

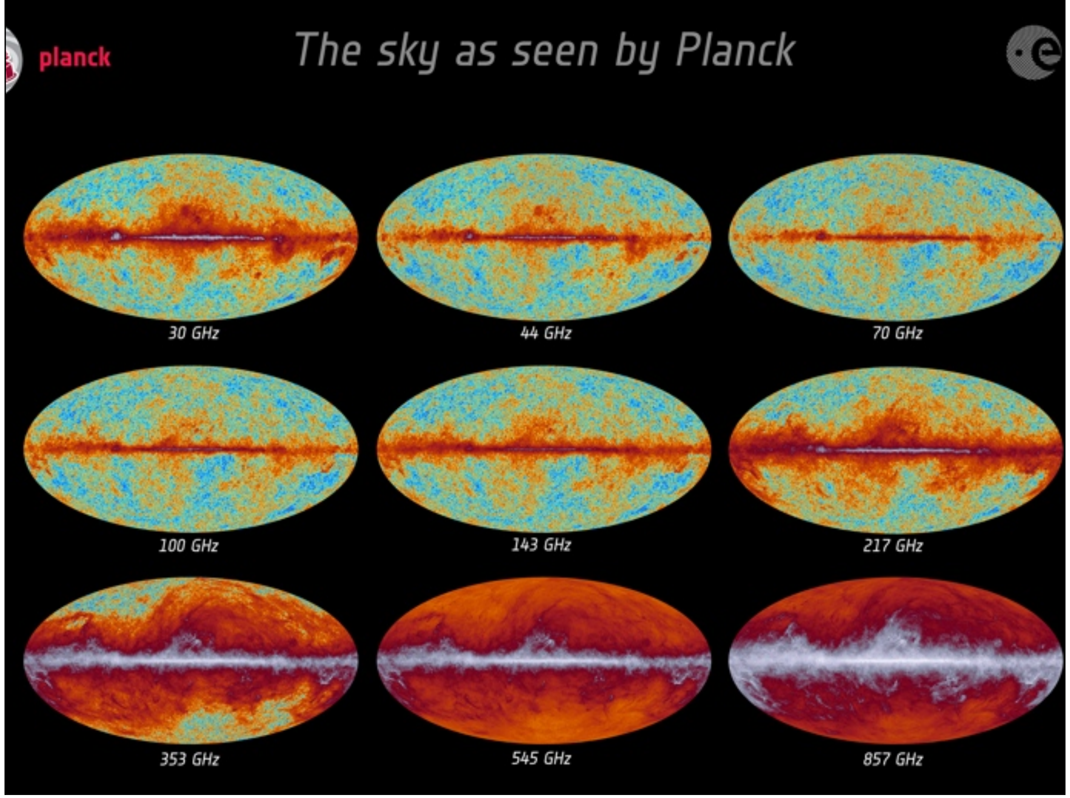
Large scale analysis

Topology of the univers, inflation, ISW, etc

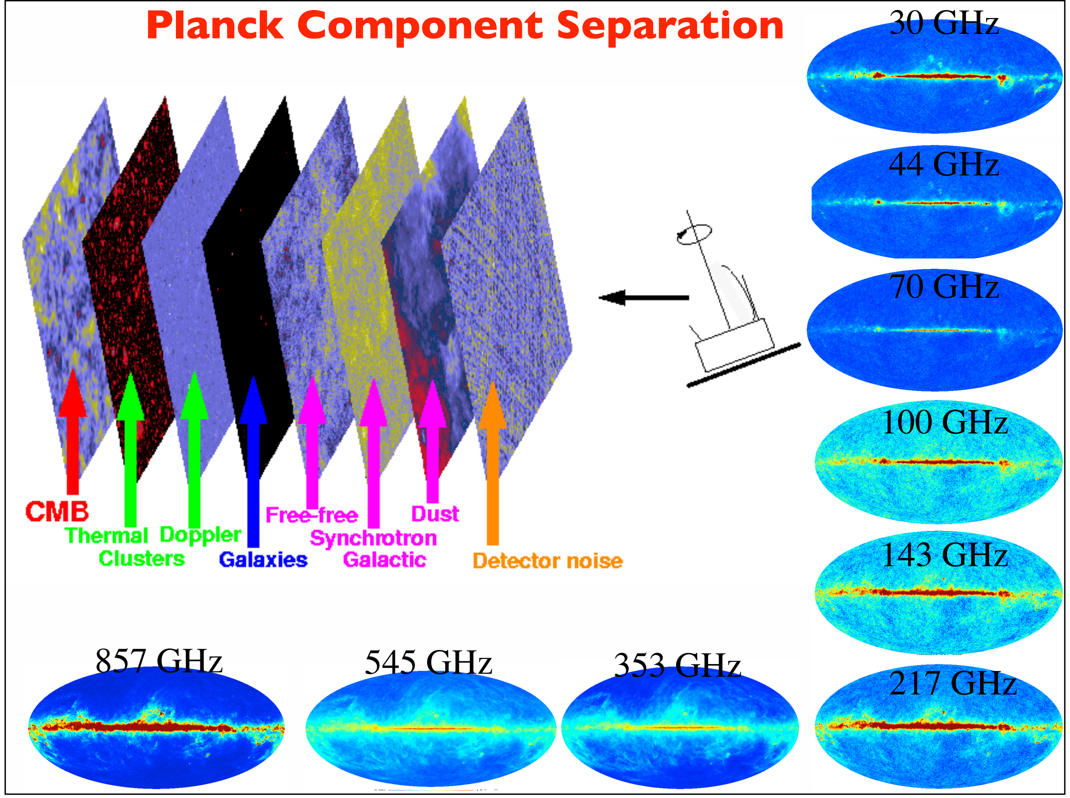


Integrated Sachs-Wolfe Effect (ISW)

Constraint on dark energy

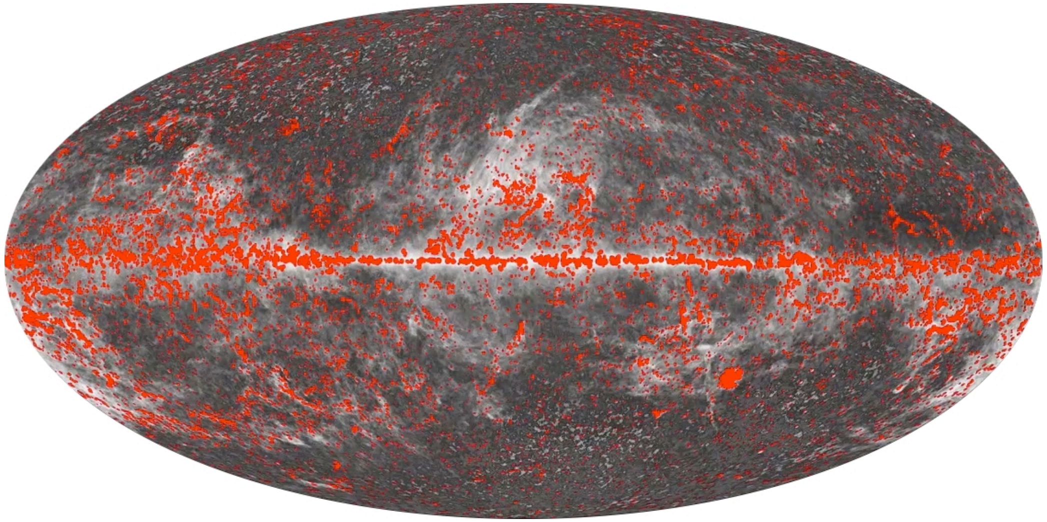


planck
The sky as seen by Planck
30 GHz
44 GHz
70 GHz
100 GHz
143 GHz
217 GHz
353 GHz
545 GHz
857 GHz

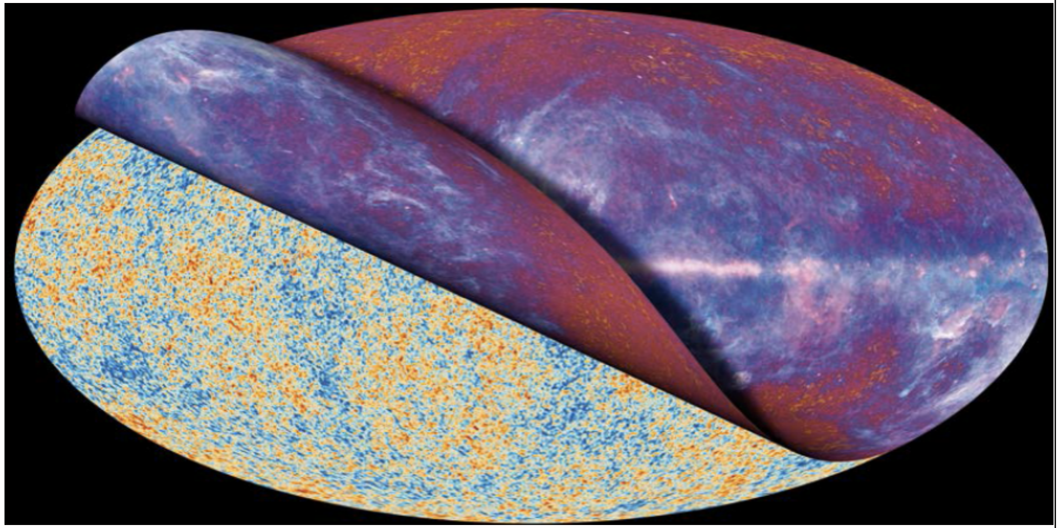


Planck Component Separation
30 GHz
44 GHz
70 GHz
100 GHz
143 GHz
217 GHz
857 GHz
545 GHz
353 GHz
CMB
Thermal Clusters
Doppler Clusters
Galaxies
Free-free
Synchrotron Galactic
Dust
Detector noise

Detected Compact Sources in Planck



Credit: ESA/Planck Collaboration



Caché dans les autres emissions du ciel

ILL POSED INVERSE PROBLEM

$$Y = A X + N$$

Need to add constraint

$$min_{A,X} = \parallel Y - AX \parallel^2 \quad s.t. \; \mathcal{C}(X, A)$$

Planck Component Separation Principle

- **Bayesian method:** MODEL at low resolution with 4 components:
  CMB,
  low-frequency emission,
  CO emission
  thermal dust emission
  + parameter interpolation to full resolution

- **Template fitting** in two regions: Clean the 100 and 143 Ghz map by:

$$T_c(\boldsymbol{x},\nu) = d(\boldsymbol{x},\nu) - \sum_{j=1}^{n_t} \alpha_j t_j(\boldsymbol{x}),$$

where templates are difference maps (30–44), (44–70), (545–353) and (857–545).

Planck Component Separation Principle

- **Internal Linear Combination** (**ILC**), used by WMAP :

  - CMB spectrum is assumed to be **known**: a
  - Modelling: $X = as + R$

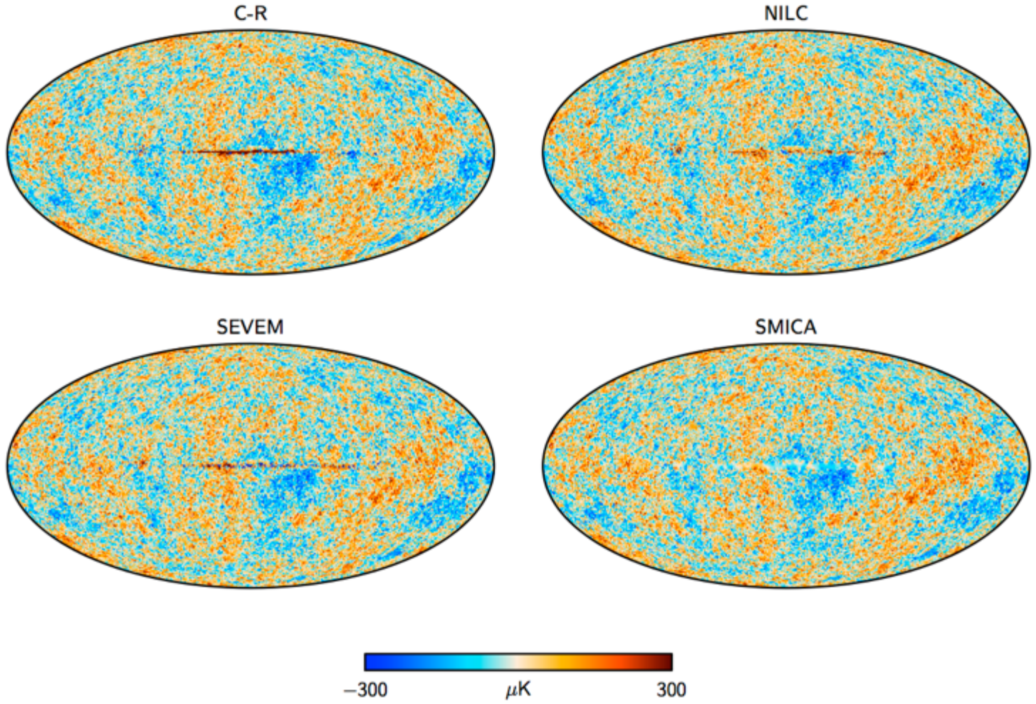
  Solution ILC : $$\hat{s} = \mathrm{Argmin}_s \left(X - as\right) R_X^{-1} \left(X - as\right)^T$$
  $$\hat{s} = \frac{1}{a^T R_X^{-1} a} a^T R_X^{-1} X$$

  Well known in statistics as the BLUE (Best Linear Unbiased Estimator) method.

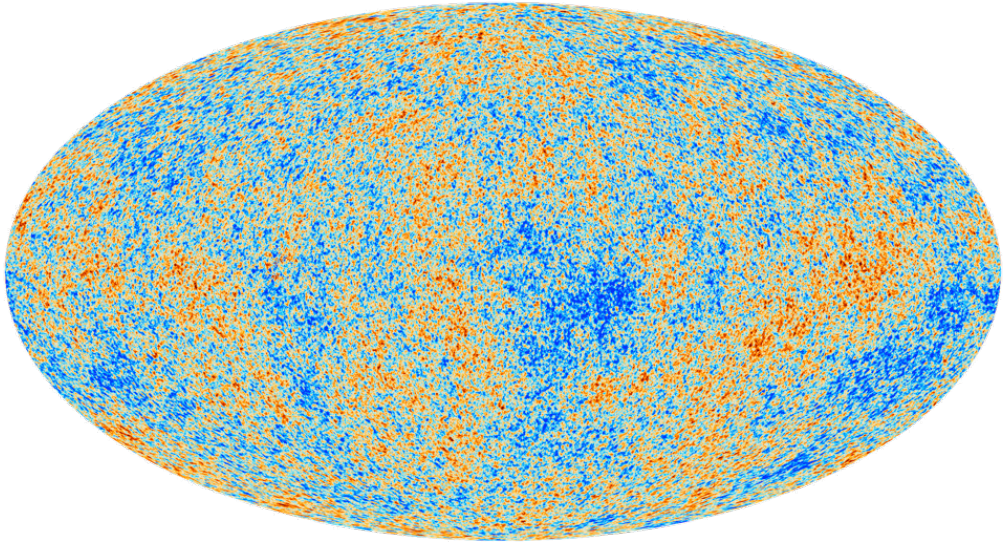
Nilc = ILC in the wavelet domain
one ILC per wavelet scale and per region. No localization at the coarsest scales and up to 20 regions at the finest scale.
Smica = ILC in spherical harmonic domain
\+ modeling of the covariance matrix at low l,(l < 1500)

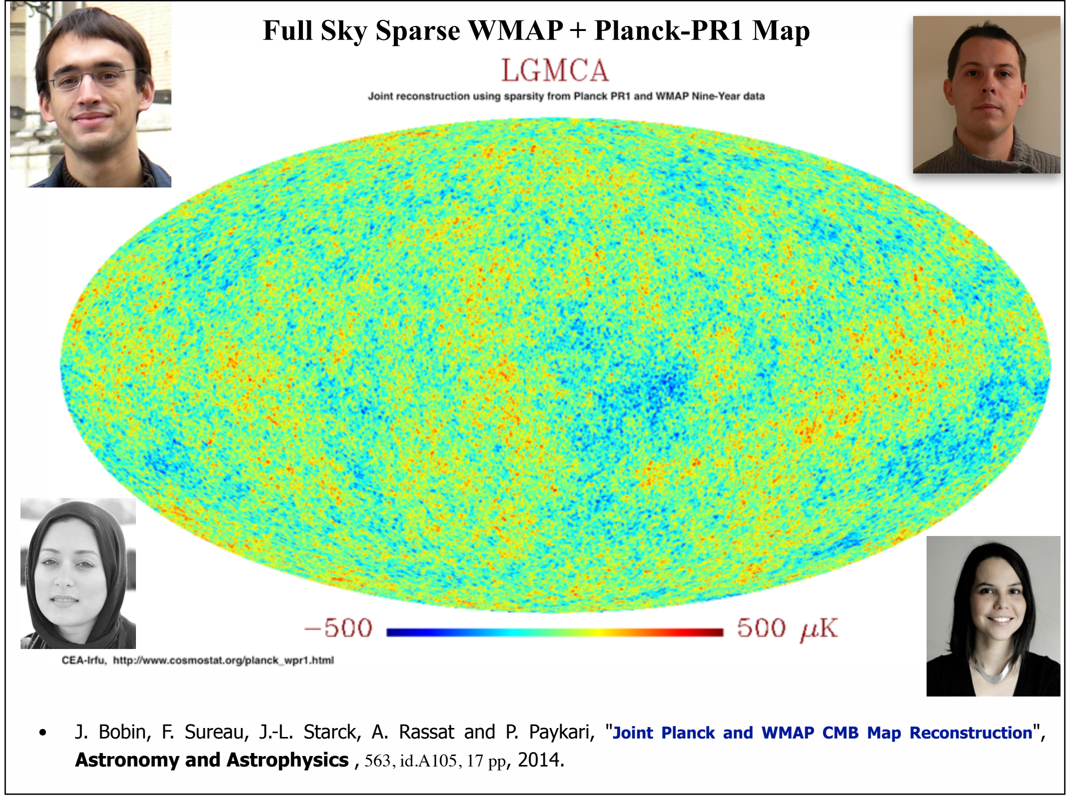
Commander-Ruler, Sevem, NILC, Smica

Planck Collaboration: *Planck* 2013 results. XII. Component separation



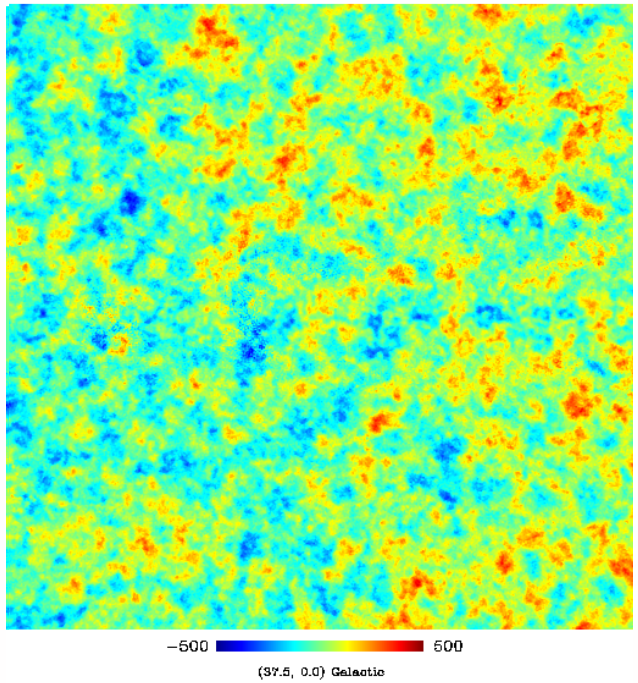
PLANCK PR1 CMB MAP

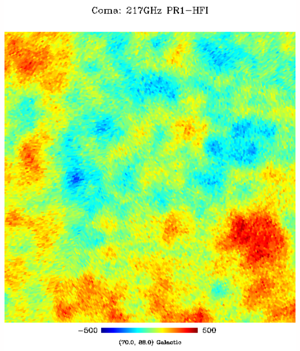


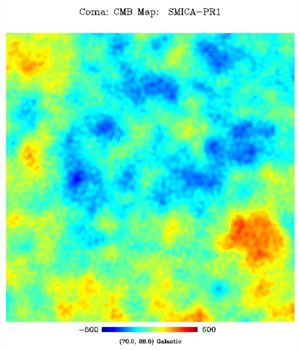


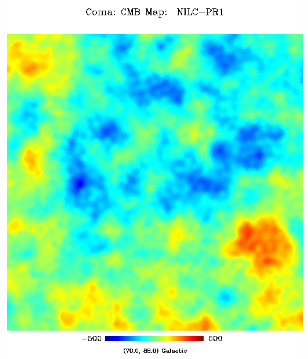
The anisotropies of the Cosmic microwave background (CMB) as observed by Planck. The CMB is a snapshot of the oldest light in our Universe, imprinted on the sky when the Universe was just 380 000 years old. It shows tiny temperature fluctuations that correspond to regions of slightly different densities, representing
Credits: ESA and the Planck Collaboration
the seeds of all future structure: the stars and galaxies of today.
La plus belle carte du fond diffus cosmologique

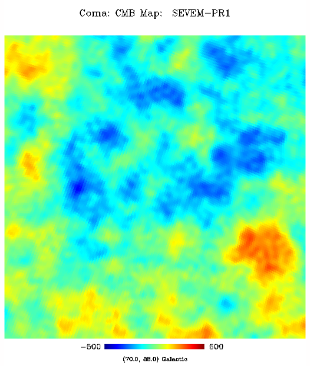
Galactic plane region:PR1-WMAP9yr-GMCA

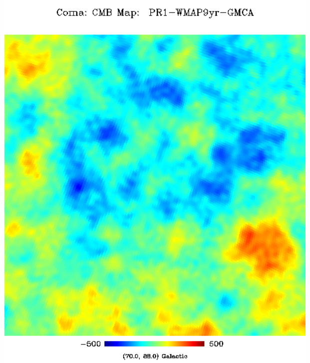


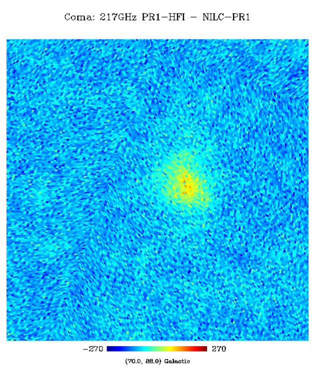

Coma: 217GHz PR1-HFI
−500
500
(70.0, 88.0) Galactic

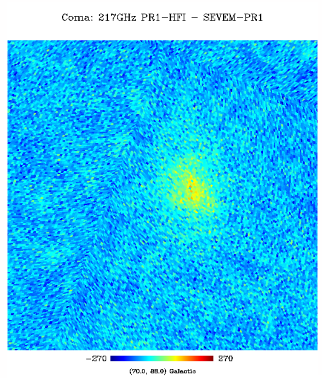

Coma: CMB Map: SMICA−PR1
−500
500
(70.0, 88.0) Galactic

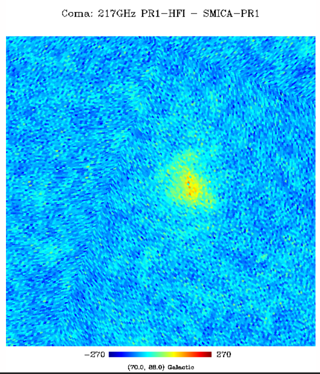

Coma: CMB Map: NILC−PR1
−500
500
(70.0, 88.0) Galactic

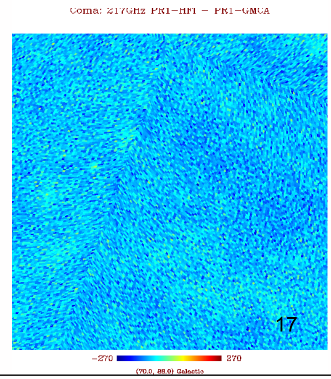

Coma: CMB Map: SEVEM−PR1
−500
500
(70.0, 88.0) Galactic


Coma: CMB Map: PR1−WMAP9yr−GMCA
−500
500
(70.0, 88.0) Galactic


Coma: 217GHz PR1−HFI − NILC−PR1
−270
270
(70.0, 88.0) Galactic


Coma: 217GHz PR1−HFI − SEVEM−PR1
−270
270
(70.0, 88.0) Galactic


Coma: 217GHz PR1−HFI − SMICA−PR1
−270
270
(70.0, 88.0) Galactic


Coma: 217GHz PR1−HFI − PR1−GMCA
−270
270
(70.0, 88.0) Galactic

QUALITY MAP

Expected power in a given wavelet band :

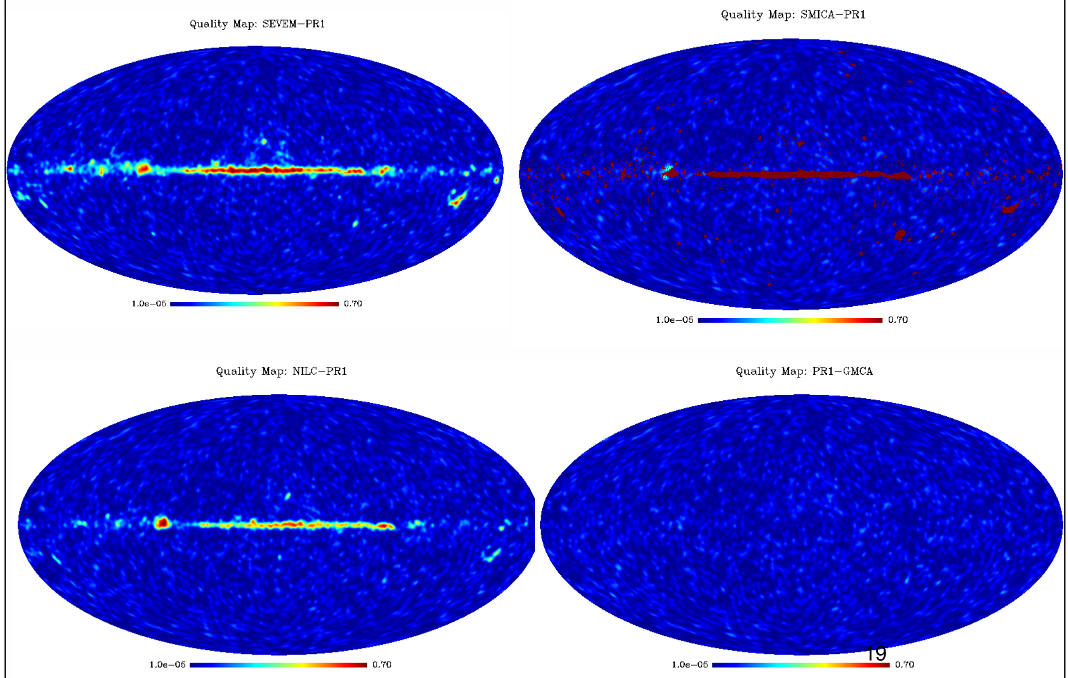
$$P_j = \frac{1}{4\pi} \sum_{\ell} \ell(\ell+1) \parallel a_{\ell,0}^{(\psi_j)} \parallel^2 C_\ell$$

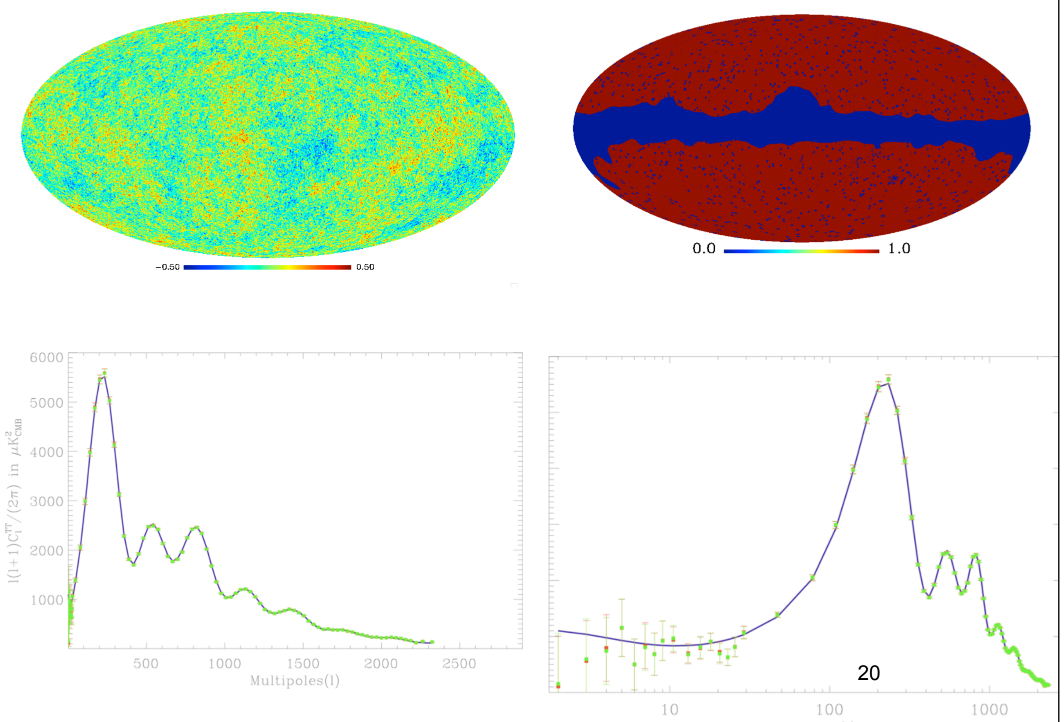
Quality coefficient :

$$q_{j,k} = P_j / \left(D_{j,k} - N_{j,k}\right)$$

$$Q_k = 1 - \max_j q_{j,k}$$

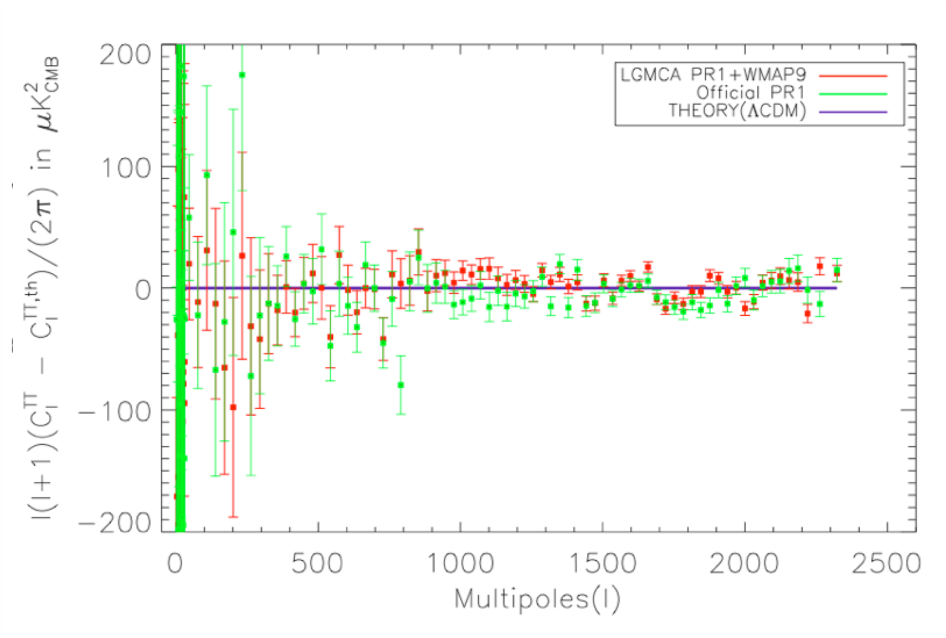
QUALITY MAPS





−0.50
0.50
0.0
1.0
6000
5000
4000
3000
2000
1000
500
1000
1500
2000
2500
l(l+1)C$_l^{TT}$/(2π) in μK$^2_{CMB}$
Multipoles(l)
10
100
1000

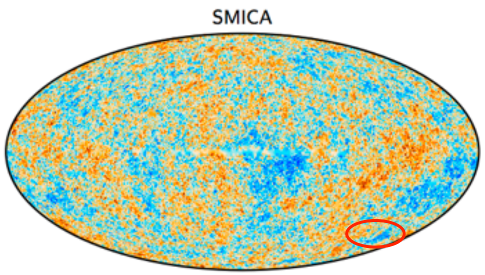
Power Spectrum



CMB & ANOMALIES

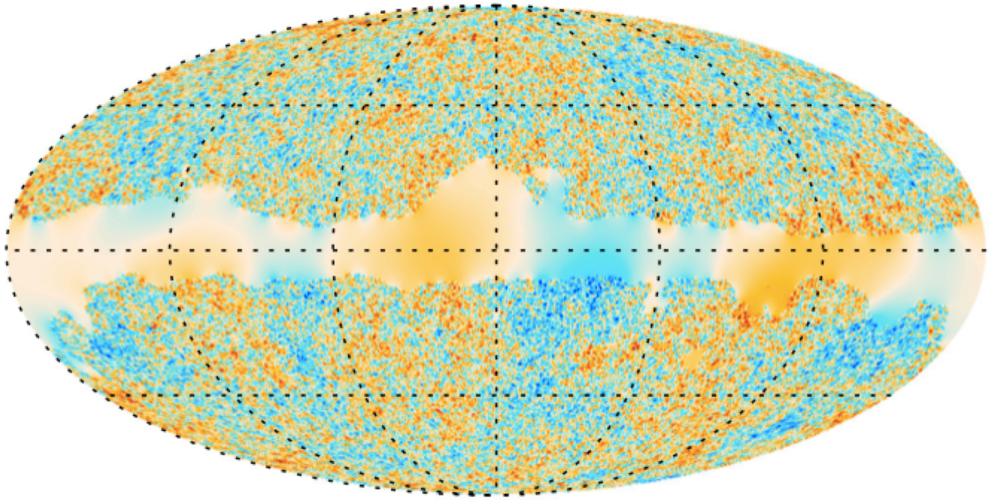
- Anomalies in WMAP CMB maps:
  - Low Power in CMB Quadrupople (Hinshaw 96, Spergel 03).
  - North /South Asymmetry (Erikson 04).
  - Planarity of low multipoles, 'Axis of Evil' (Tegmark 03, de Oliveira-Costa 04, Land & Maguiejo 05).
  - Small scale **cold spot** in southern hemisphere (Vielva 2004).
  - Few hot spots.

Anomalies confirmed by Planck



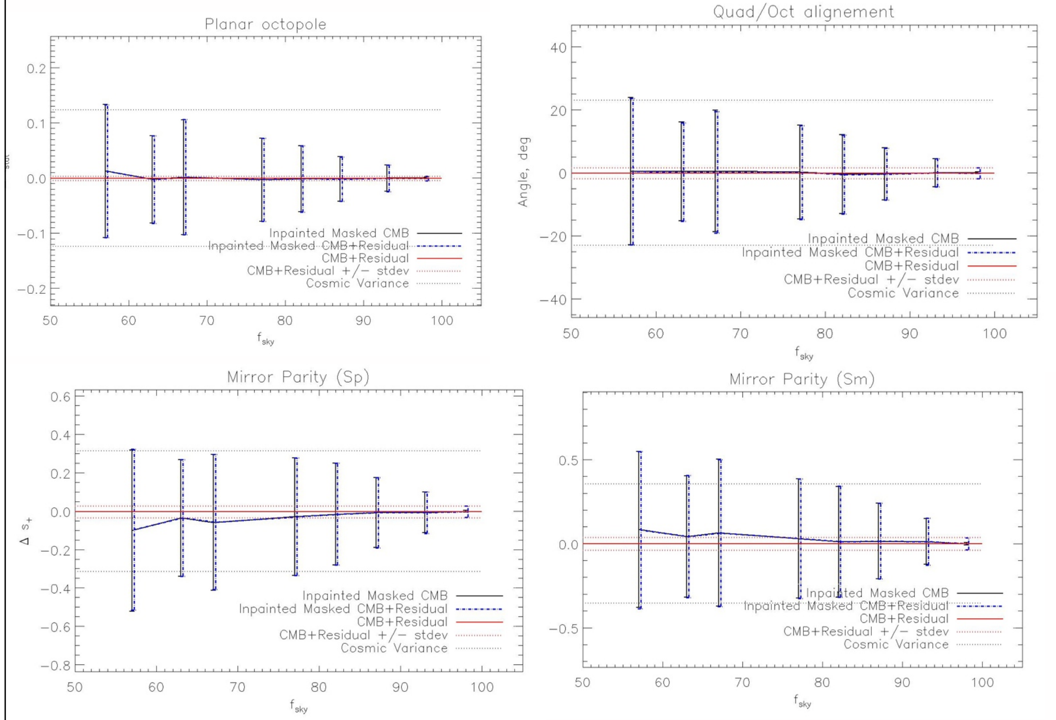
INPAINTING for large scale studies

Mask 30% of the sky and use Wiener Inpainting

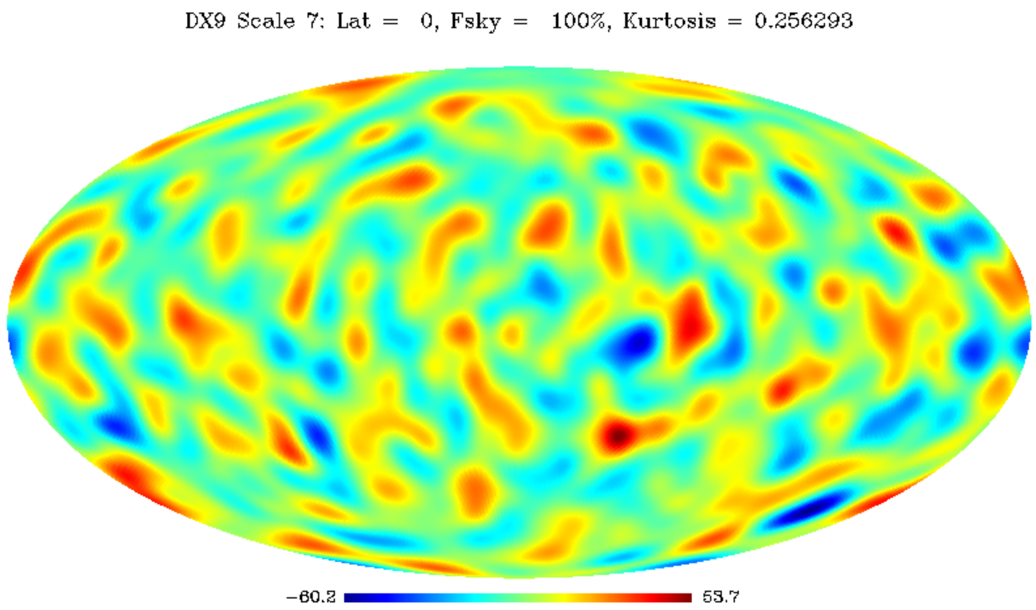


How aggressive should be the mask ?

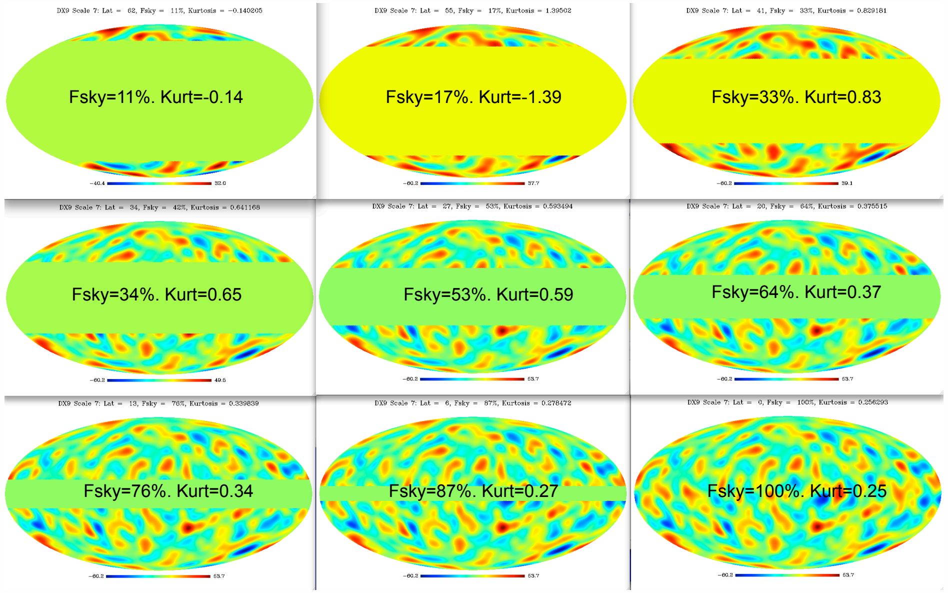
Errors on Anamolies Measurements versus Mask Size



Kurtosis & Masking

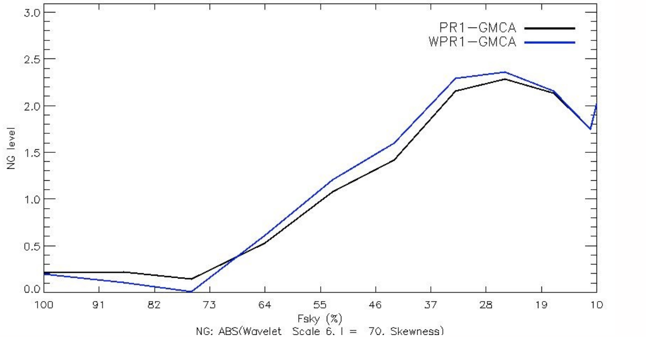


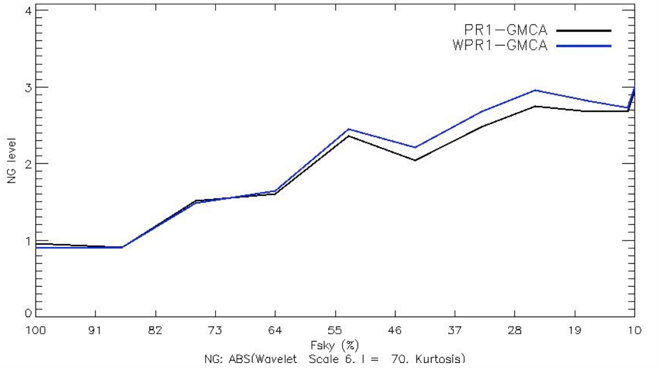
Kurtosis versus Mask Size (Scale 7, l=[17,70])



A. Rassat, J.-L. Starck, P. Paykari, F. Sureau J. Bobin, “Planck CMB Anomalies: Astrophysical and Cosmological Foregrounds and the Curse of Masking”, arXiv:1405.1844, JCAP, in press, 2014.

Normalized Skewness & Kurtosis





Sparsity & CMB

✦Sparsity is very efficient for Component Separation

- High quality and full sky CMB map, from WMAP and Planck data.
- Masking is even not necessary anymore for large scale studies.

Joint Planck and WMAP CMB Map Reconstruction (arXiv:1401.6016), A&A,563, Id. A105, 2014.

✦CMB full sky analysis at large scales:

- After kDq ISW and kSZ subtraction, octopole planarity, AoE, mirror parity the quadrupole/octopole alignment and cold spot are not anomalous.

Planck CMB Anomalies: Astrophysical and Cosmological Foregrounds and the Curse of Masking (arXiv: 1405.1844), JCAP, 2014.

✦PRISM method for Pk reconstruction: No detection of any strong deviation from WMAP9 or Planck PR1 near-scale invariant fiducial primordial power spectra.

PRISM: Sparse Recovery of the Primordial Power Spectrum (arXiv:1406.7725), A&A, 566, id.A77, 2014.
PRISM: Sparse recovery of the primordial spectrum from WMAP9 and Planck datasets, arXiv:1406.7725, A&A,2014.

http://www.cosmostat.org/gmca_mainpage.html
http://www.cosmostat.org/planck_wpr1.html
http://www.cosmostat.org/prism.html

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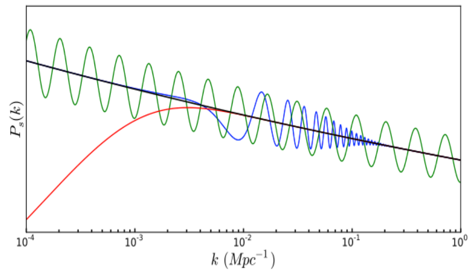
Importance of P(k)

Inflation is currently the most favoured model

Simplest models predict a near scale-invariant power spectrum

$$P(k) = A_s \left(\frac{k}{k_p}\right)^{n_s - 1 + \frac{1}{2}\alpha_s \ln(k/k_p)}$$

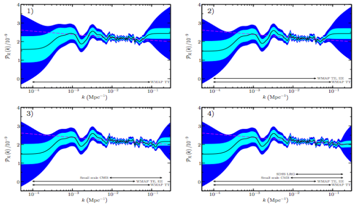


Why is P(k) important?

- Distinguish between different models of inflation
- Cosmological parameters is sensitive to shape of P(k)

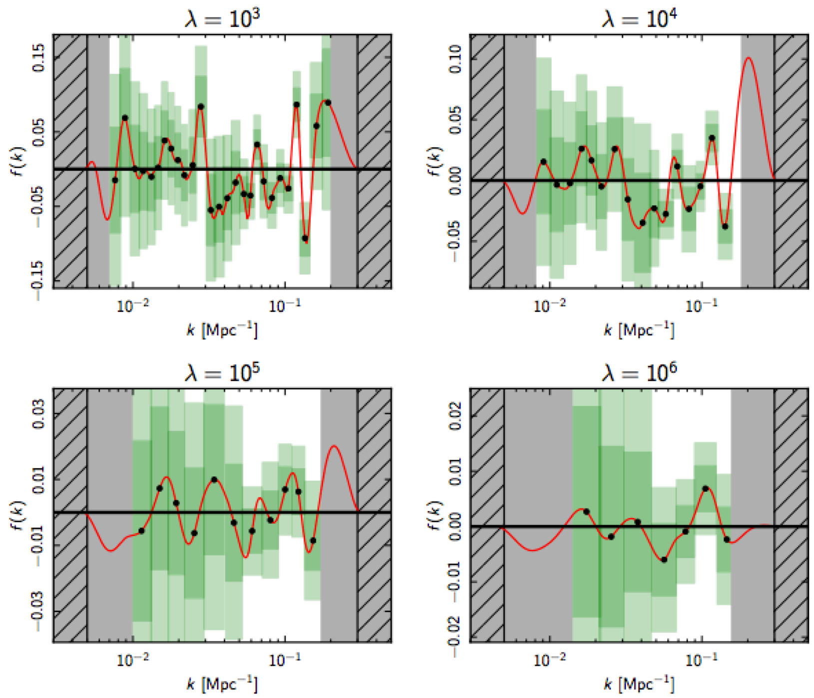
Estimation of P(k)?

- Reconstruction
- Parametric methods



Hunt & Sarkar 2014

Planck PR1 Official P(k)



P(k) reconstruction : An Inverse Problem

Formally, the observed pseudo-power spectrum computed from masked CMB maps can be linked to the underlying primordial power spectrum through a linear relation of the form:

$$\widetilde{C}_\ell = \left(\sum_{\ell' k} M_{\ell \ell'} T_{\ell' k} P_k + N_\ell \right) Z_\ell$$

where T and M are a linear operators encoding respectively the angular transfer function of CMB anisotropies and the effects of masks and beams, and Z is a multiplicative noise term.

The recovery of the primordial power spectrum is performed by solving an optimization problem of the form:

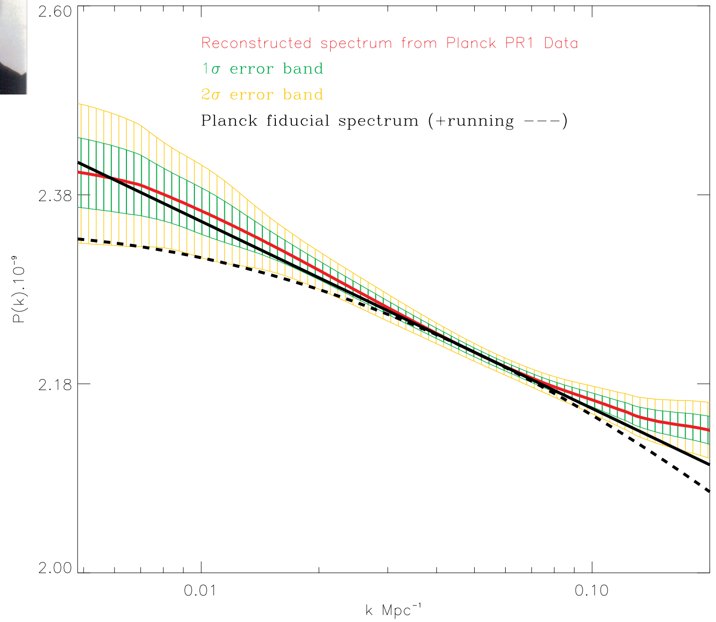
Sparse recovery:

$$\min_X \frac{1}{2} \parallel C_\ell - (\mathbf{MT}X + N_\ell) \parallel_2^2 + \lambda \parallel \mathbf{\Phi}^t X \parallel_0 \,,$$

P. Paykari, F. Lanusse, J.-L. Starck, F. Sureau, J. Bobin, "PRISM: Sparse Recovery of the Primordial Power Spectrum", A&A, 566, id A77, 2014, **arXiv:1402.1983**.

http://www.cosmostat.org/prism.html

PRISM & Planck PR1 Data



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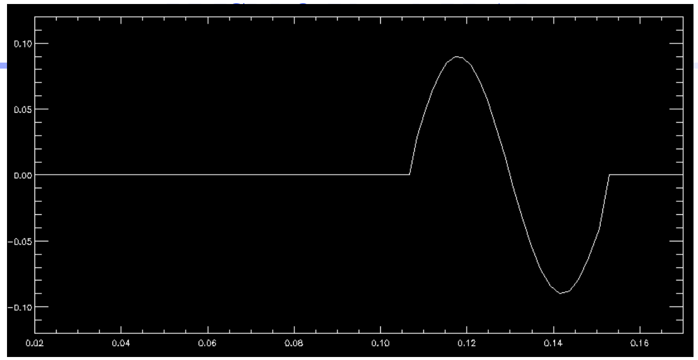
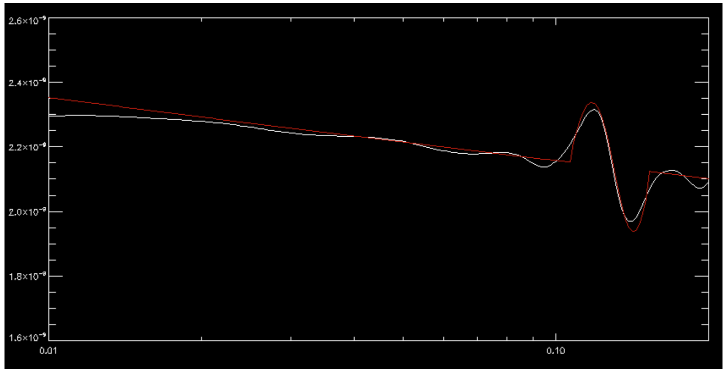




http://www.cosmostat.org/prism.html

Sparsity and the Bayesian Contreversy

$$Y = MX = M\Phi\alpha \text{ with } \|\alpha\|_1 \quad \text{minimum}$$

Prior on the solution: $P(\alpha) = e^{-\lambda\|\alpha\|_1}$

Gaussian noise prior: $P(Y/\alpha) = e^{-\|Y - M\Phi\alpha\|_2^2}$

Bayes: $P(\alpha|Y) = P(Y|\alpha)P(\alpha)$

$$\Big\downarrow \text{ Maximum a Posteriori (MAP)}$$

$$\min_{\alpha} -log\left(P(\alpha|Y)\right) = \| Y - M\Phi\alpha \|_2^2 + \lambda \| \alpha \|_1,$$

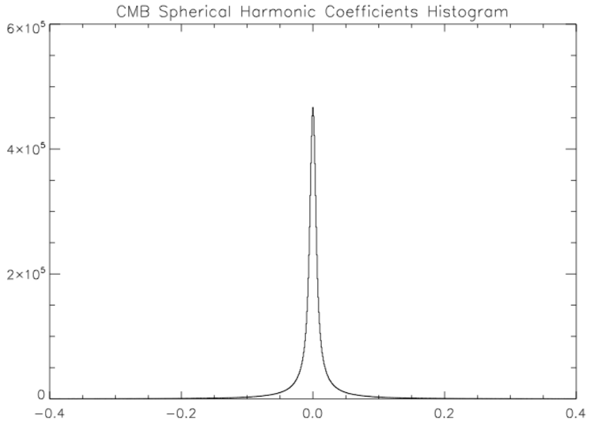
Severe Critics from Bayesian Cosmologists against CMB Sparse Inpainting

Sparsity consists in assuming an anisotropy and a non Gaussian prior, which **does not make sense** for the CMB, which is Gaussian and isotropic.

But what is exactly the prior in the sparse analysis ?

Bayesian: each (spherical harmonic) coefficient is a realization of a stochastic process.

Sparsity: we see the data as a function, and the coefficients follows a given distribution. Even if each spherical harmonic coefficient is a realization of Gaussian variable, the distribution of all coefficients is not necessary Gaussian.



There is no assumption that the CMB is Gaussian or isotropic, but there is also no assumption that it is non Gaussian or anisotropic. In this sense, using the l1-regularized inpainting to test if the CMB is indeed Gaussian and isotropic may be better than other methods, including Wiener filtering, which in the Bayesian framework assumes Gaussianity and isotropy.

Compressed sensing and the Bayesian interpretation failure

The critic is that the l1 regularization is equivalent to assume that the solution is Laplacian and not Gaussian, which does not make sense in case of CMB analysis.

==> ~~The MAP solution verifies the distribution of the prior.~~

(Nikolova, 2007; Gribonval, 2011, Gribonval, 2012, Unser, 2012)

The beautiful Compressed Sensing counter-example

$$\min_x \|x\|_1 \quad \text{s.t.} \quad y = \Theta x$$

but x does NOT follow a Laplacian distribution

What Bayesian Perspective Cannot See !!!

For most Bayesian cosmologists, if a prior derives an algorithm, therefore to use this algorithm, we must have the coefficients distributed according to this prior.

But this is simply a false logical chain.

What compressed sensing shows is that:

we can have prior A be completely true, but impossible to use for computation time or any other reason, and can use prior B instead, and get the correct results!

We need to take into account **the operator involved** in the inverse problem, and this requires much **deeper** mathematical developments than a simple and naive Bayesian interpretation.

Compressed sensing theory shows that for some operators, **beautiful geometrical phenomena** allows us to recover perfectly the solution of an underdetermined inverse problem. Similar results were derived for a **random sampling on the sphere**.

Starck, Donoho, Fadili, Rassat, "Sparsity and the Bayesian Perspective", Astronomy and Astrophysics, 552, A133, 2013 [arXiv:1302.2758].