Cosmology and
Multiscale Geometric Analysis

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1. MGA: Wavelets + New Multiscale Methods
    - Ridgelet transform
    - Curvelet transform

J.L. Starck, E. Candes, D.L. Donoho, "The Curvelet Transform for Image Denoising"
IEEE Transaction on Image Processing, 11, 6, 2002.

1. MGA and Non Gaussianity signature detection in the CMB

2. MGA in 3D: Analysis of the spatial distribution of the galaxies

Status of the Wavelet Transform in Astronomy

The wavelet transform is used in many astronomical domains:

galaxies counting: large scale structures analysis, void detection, …
Gamma: Gamma ray burst detection
X-ray images (ROSAT, XMM, Chandra): extended sources and filaments detection
Optics: asteroid and planetary ring detection, deconvolution
Infrared (ISO): calibration, source detection, deconvolution, …
Cosmic Microwave Background: Gaussianity test
Radio: aperture synthesis deconvolution

ADS Abstract Service

Keyword “wavelet in title”: 469 papers
Keyword “wavelet in abstract: 1115 papers

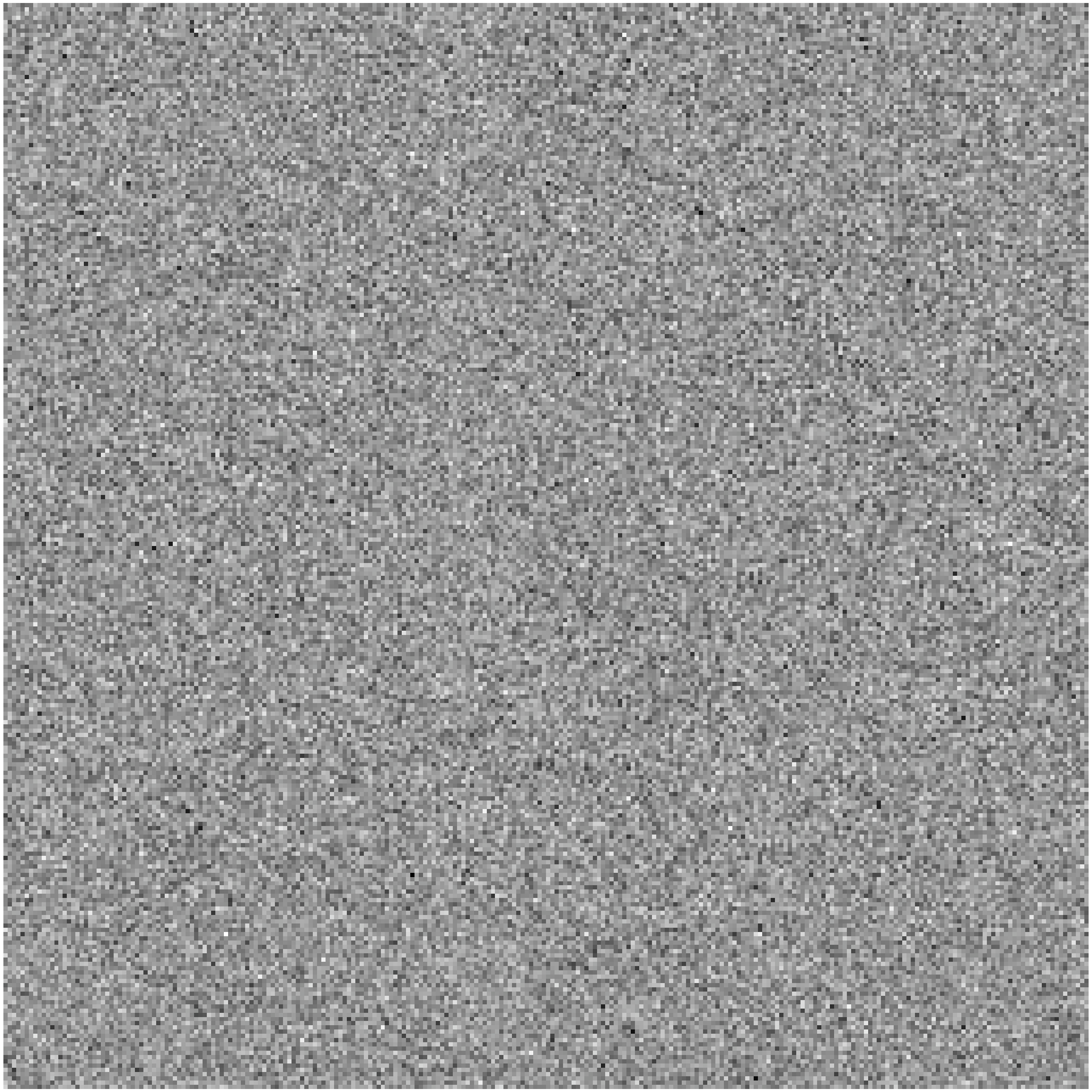
Problems related to the WT

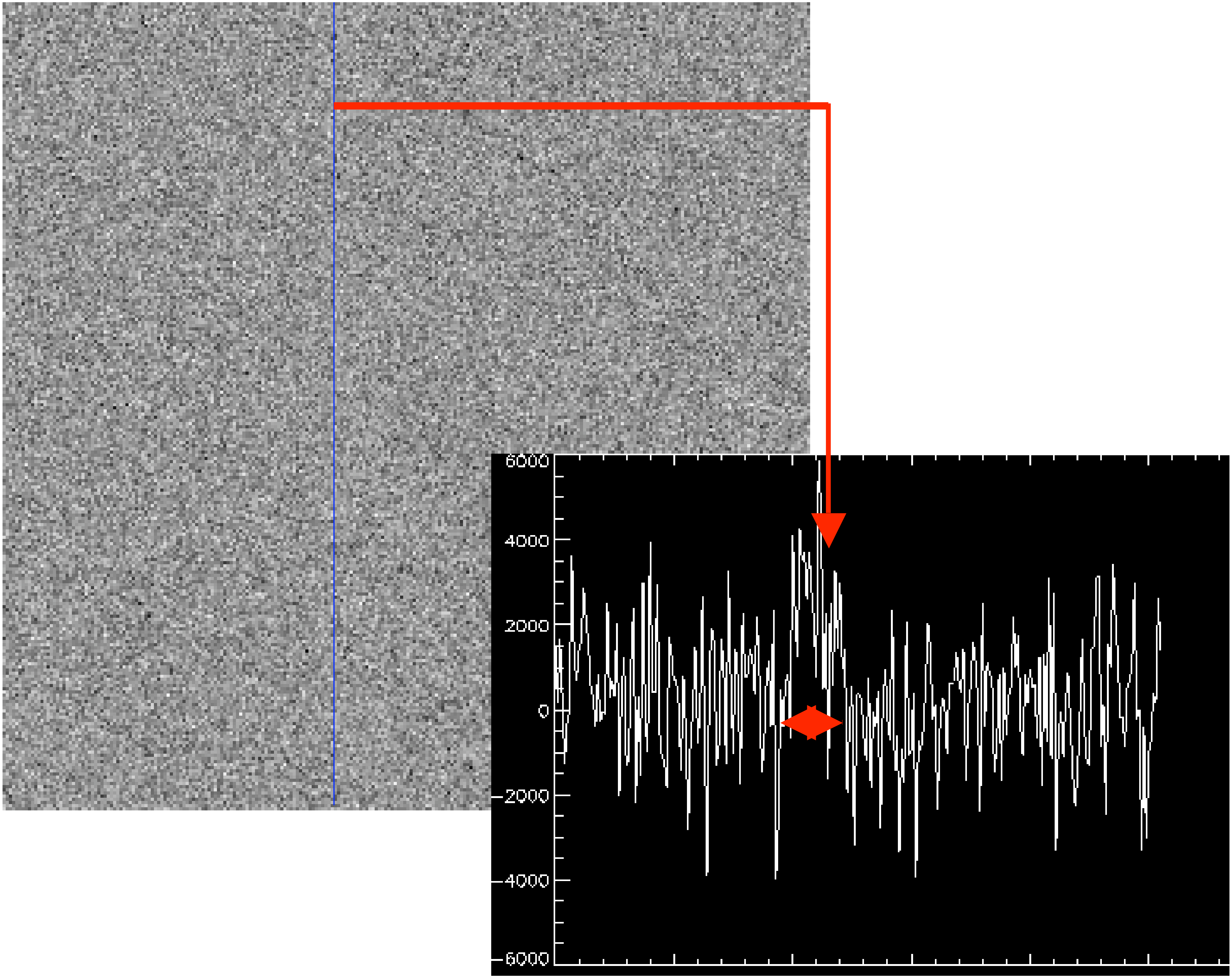
1) Edges representation:
if the WT performs better than the FFT to represent edges in an image, it is still not optimal.

2) There is only a fixed number of directional elements independent of scales.

3) Limitation of existing scale concepts:
there is no highly anisotropic elements.

SNR = 0.1




6000
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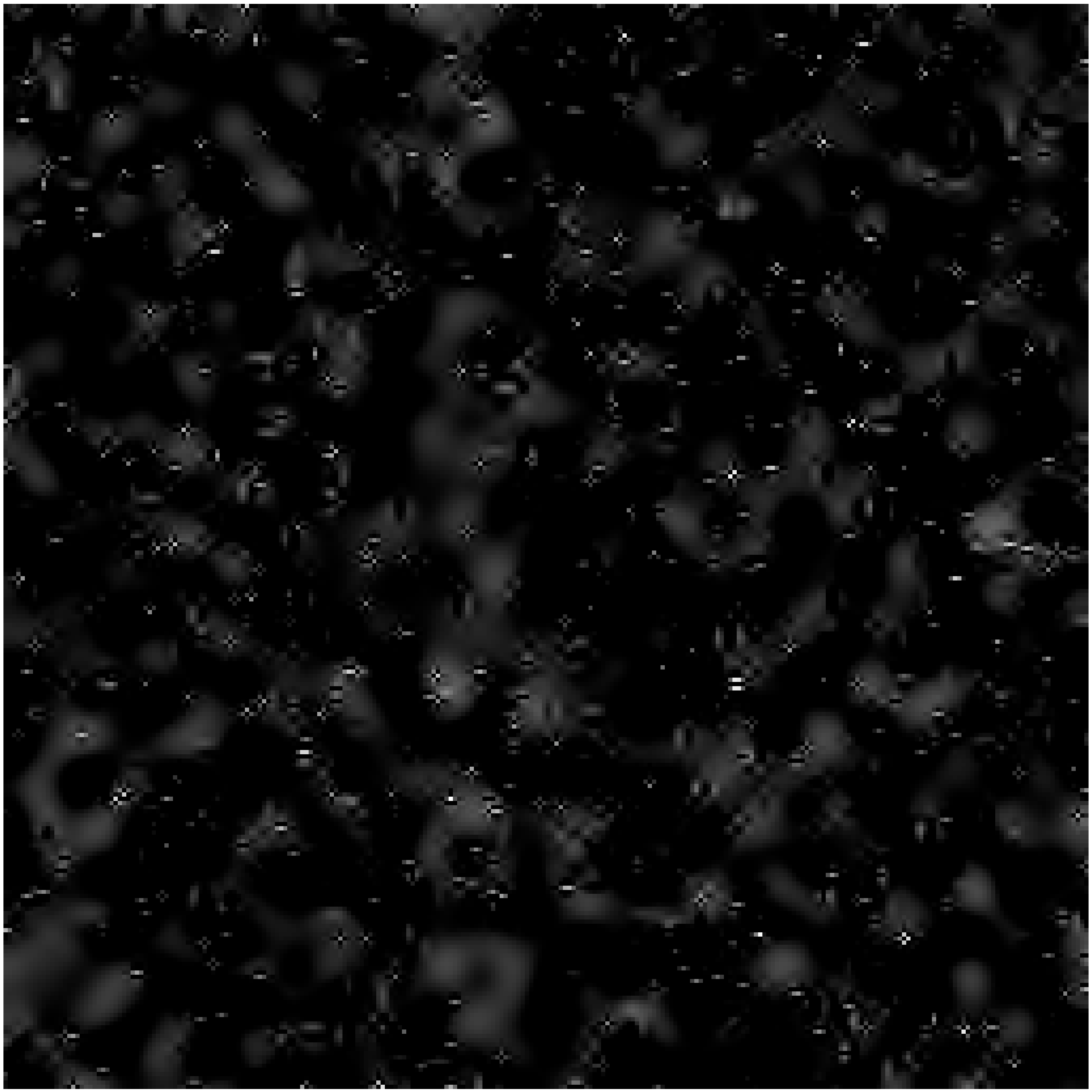
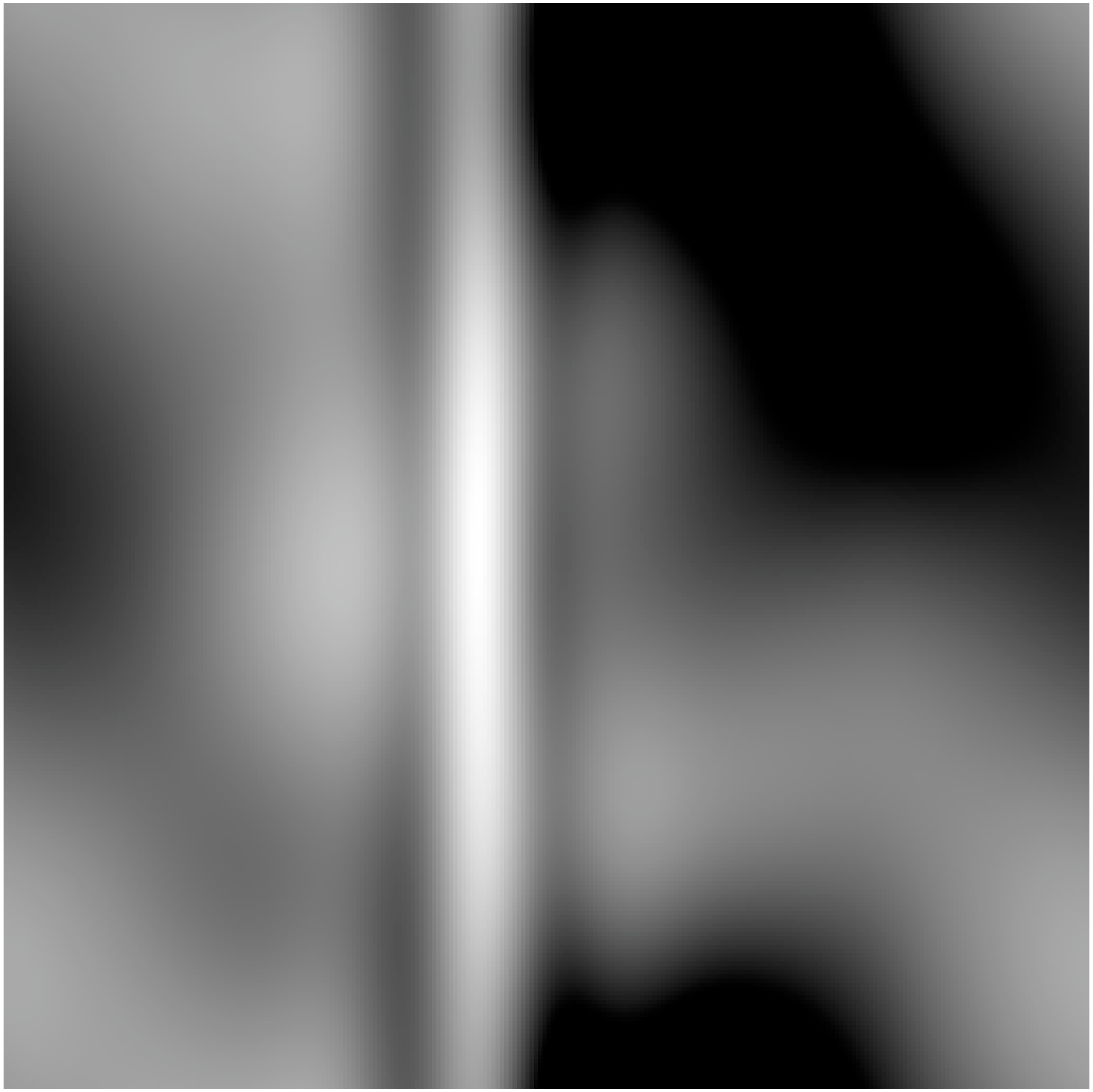
-2000
-4000
-6000

Undecimated Wavelet Filtering (3 sigma)



Ridgelet Filtering (5sigma)

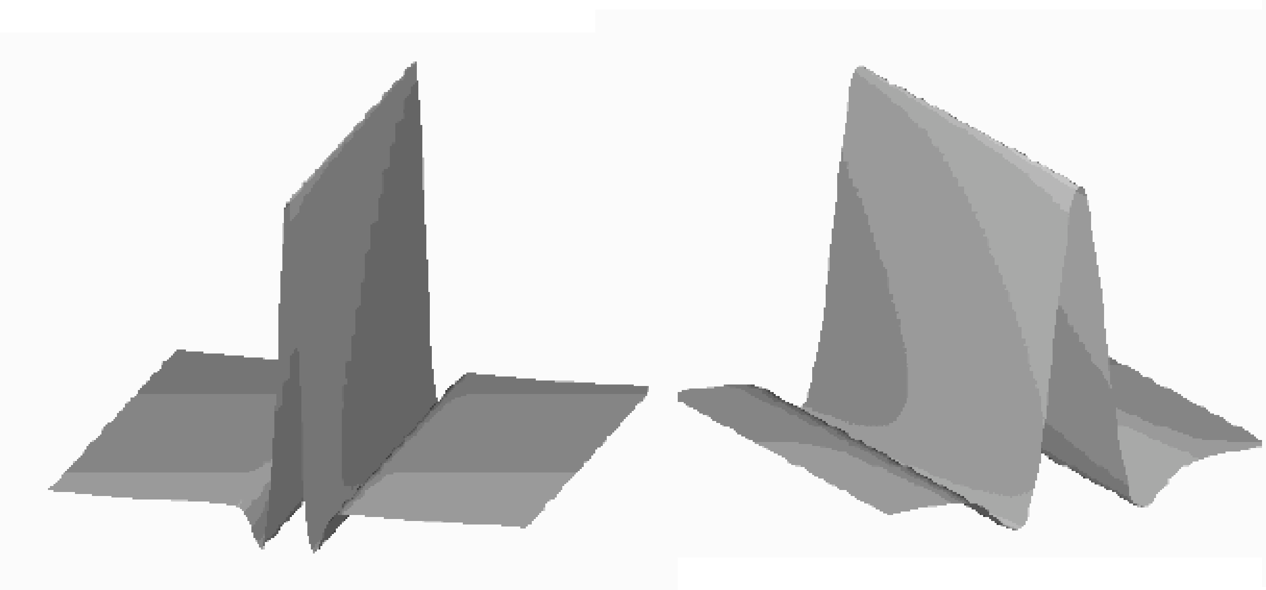


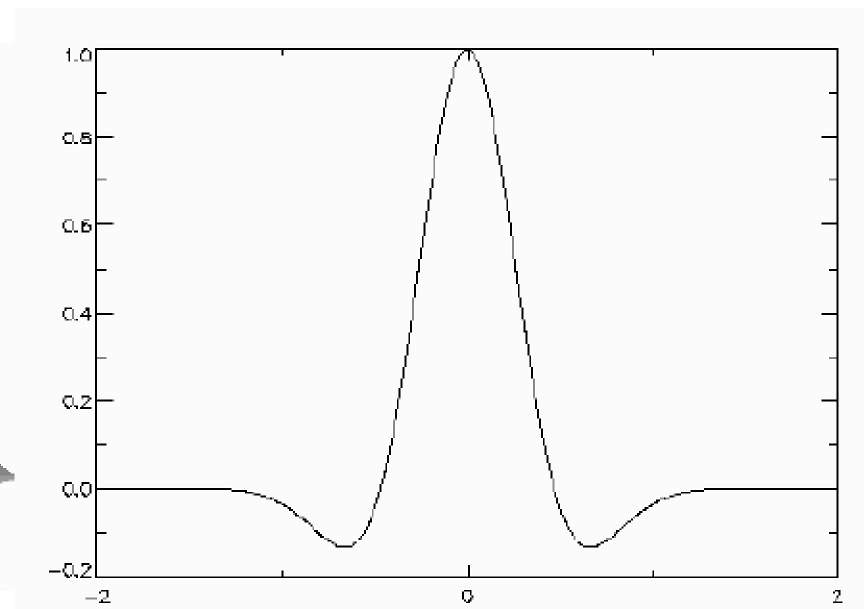
Continuous Ridgelet Transform

Ridgelet Transform (Candes, 1998): $R_f(a,b,\theta) = \int \psi_{a,b,\theta}(x) f(x) dx$

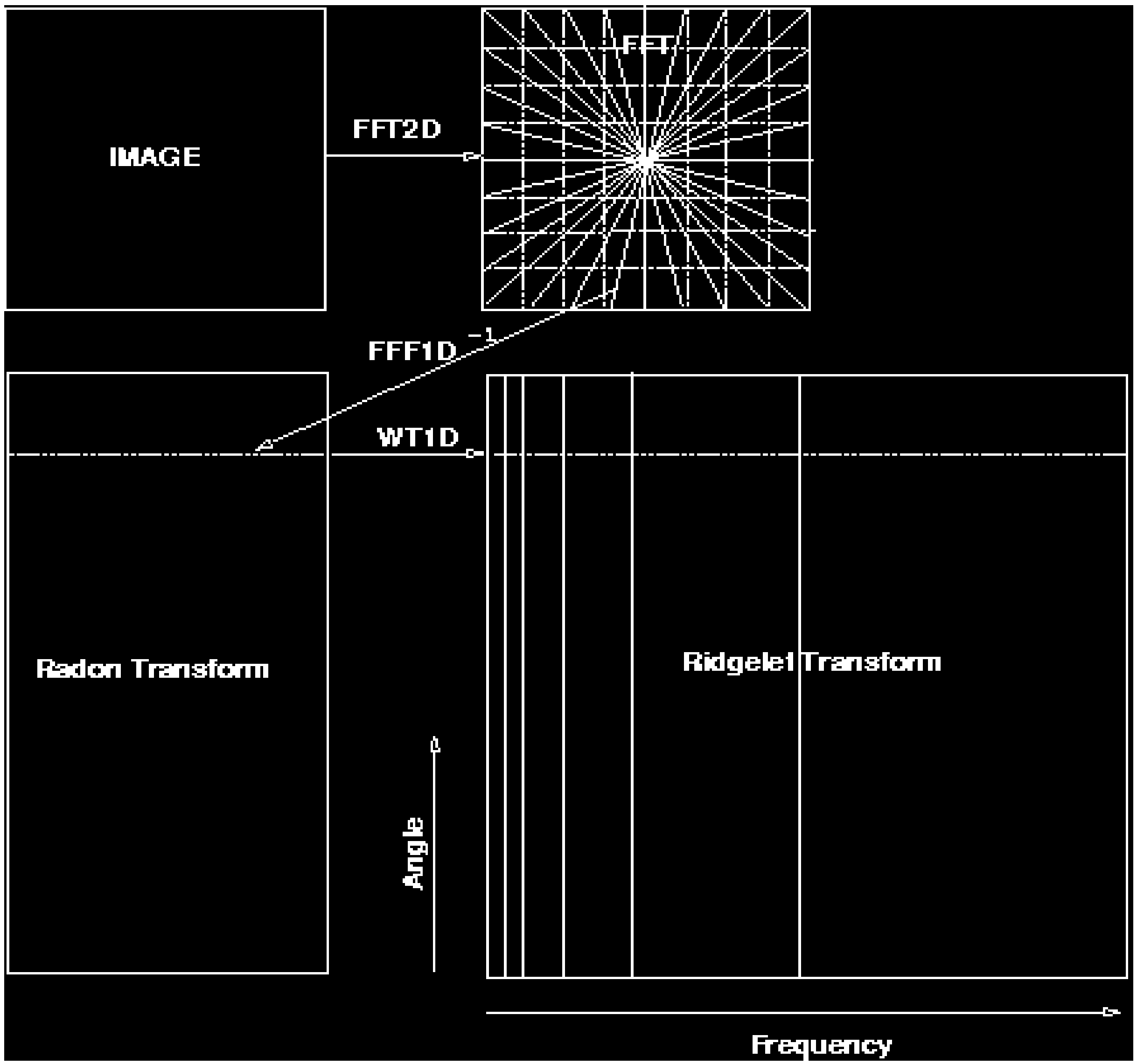
Ridgelet function: $\psi_{a,b,\theta}(x) = a^{\frac{1}{2}} \psi\left(\dfrac{x_1 \cos(\theta) + x_2 \sin(\theta) - b}{a} \right)$

The function is constant along lines. Transverse to these ridges, it is a wavelet.



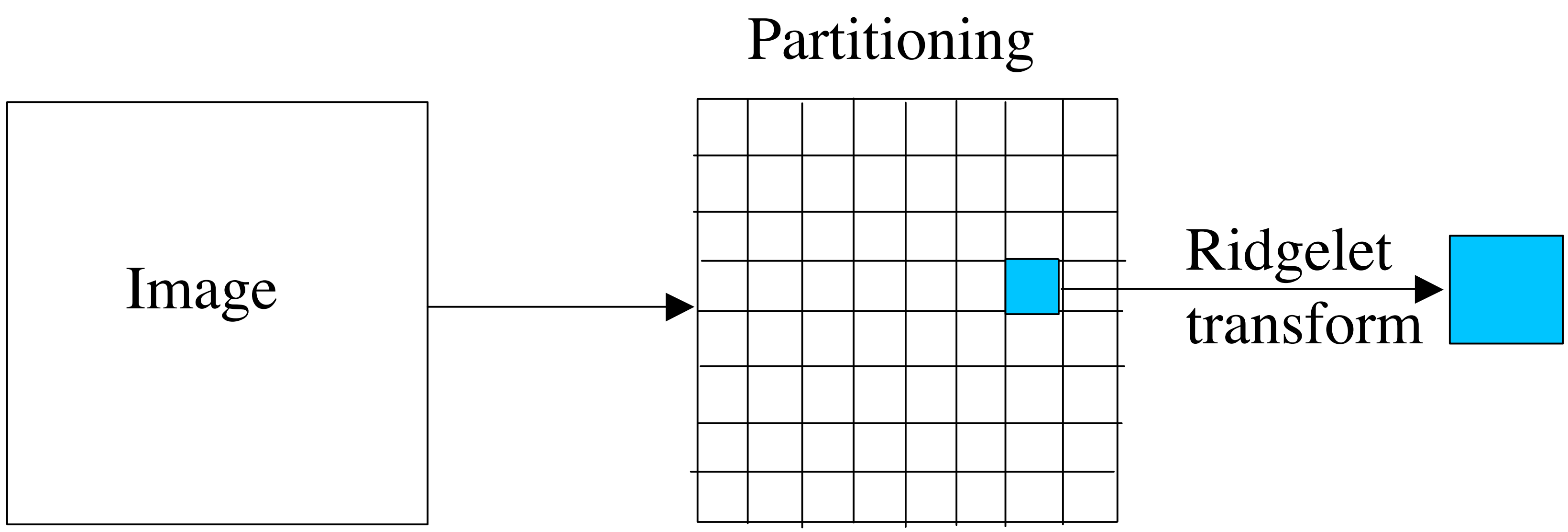


The ridgelet coefficients of an object f are given by analysis of the Radon transform via: $R_f(a,b,\theta) = \int Rf(\theta,t)\psi(\frac{t-b}{a})dt$



Local Ridgelet Transform

The ridgelet transform is optimal to find only lines of the size of the image. To detect line segments, a partitioning must be introduced. The image is decomposed into blocks, and the ridgelet transform is applied on each block.

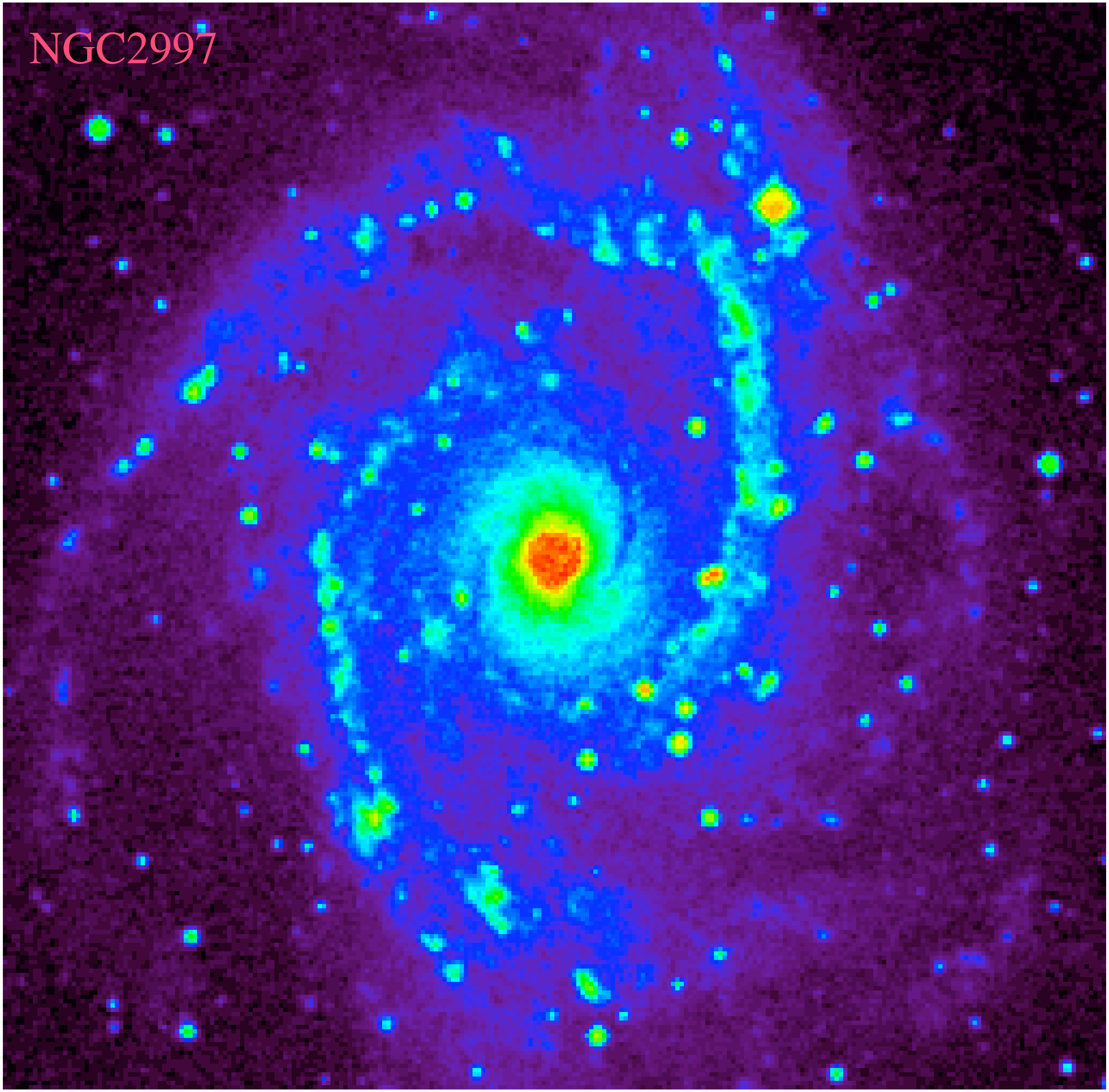


The Curvelet Transform

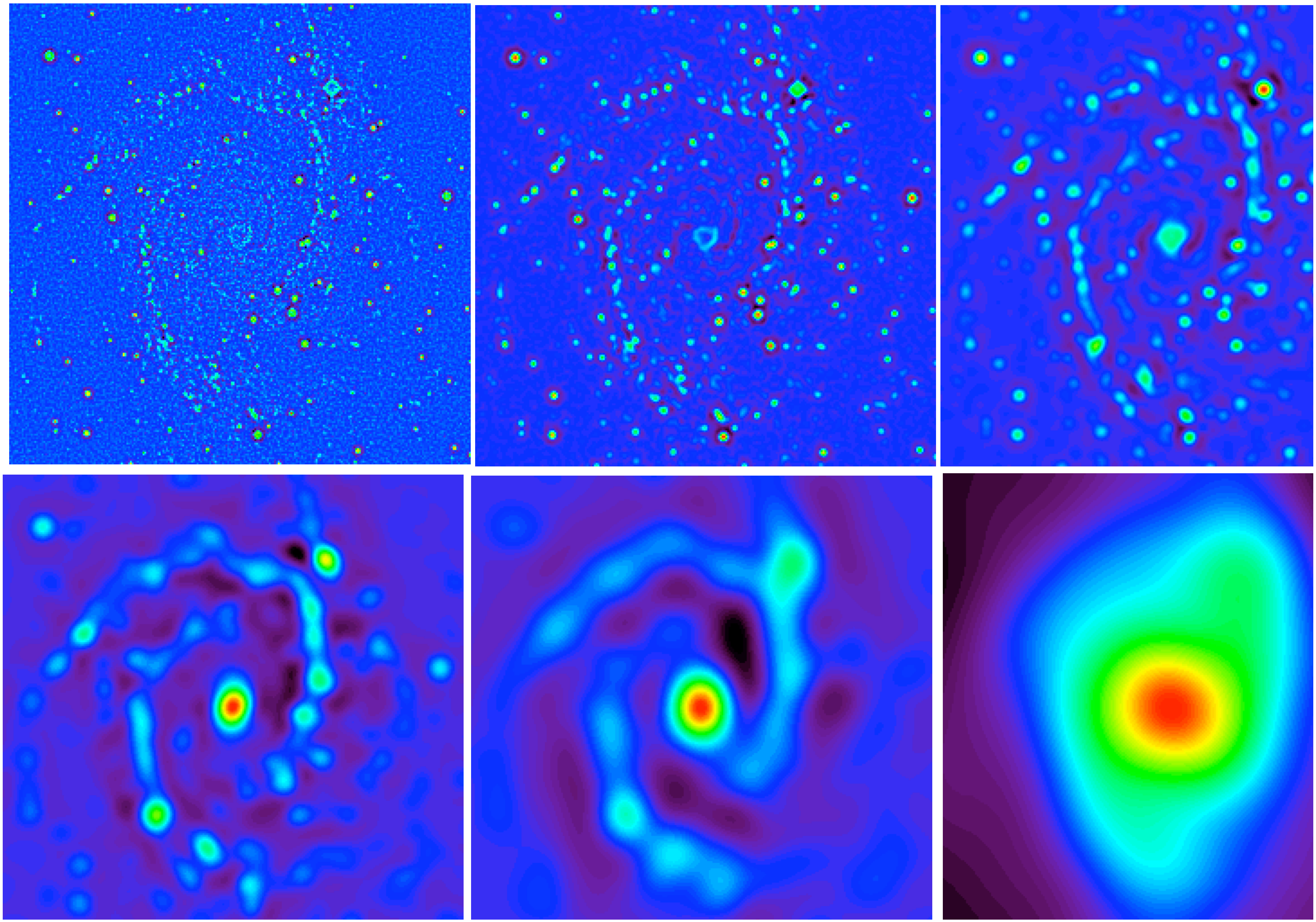
The curvelet transform opens us the possibility to analyse an image with different block sizes, but with a single transform.

The idea is to first decompose the image into a set of wavelet bands, and to analyze each band by a ridgelet transform. The block size can be changed at each scale level.

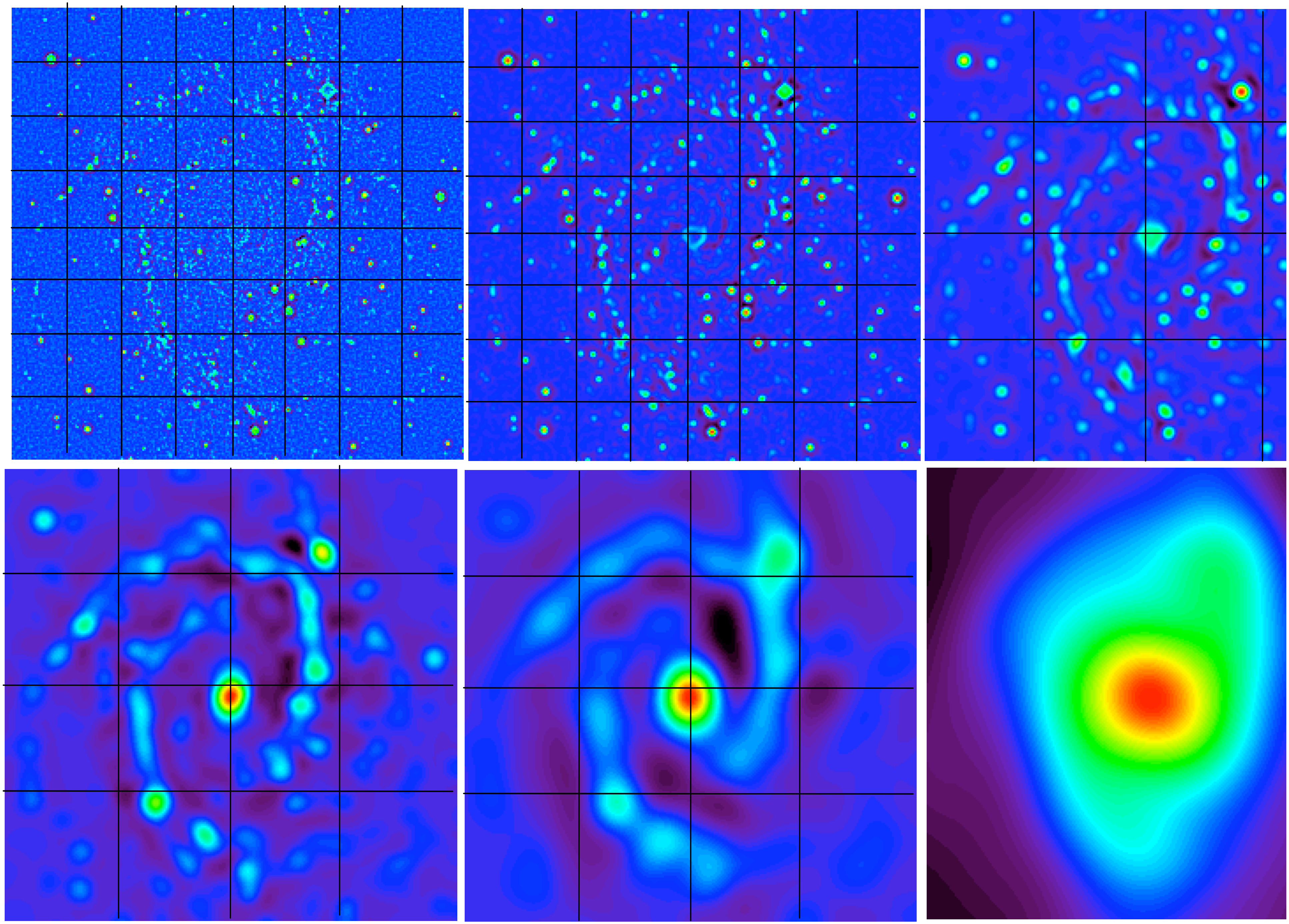
- à trous wavelet transform
- partitionning
- ridgelet transform
    . Radon Transform
    . 1D Wavelet transform


NGC2997

Undecimated Isotropic WT: $I(k,l) = c_{J,k,l} + \sum_{j=1}^{J} w_{j,k,l}$

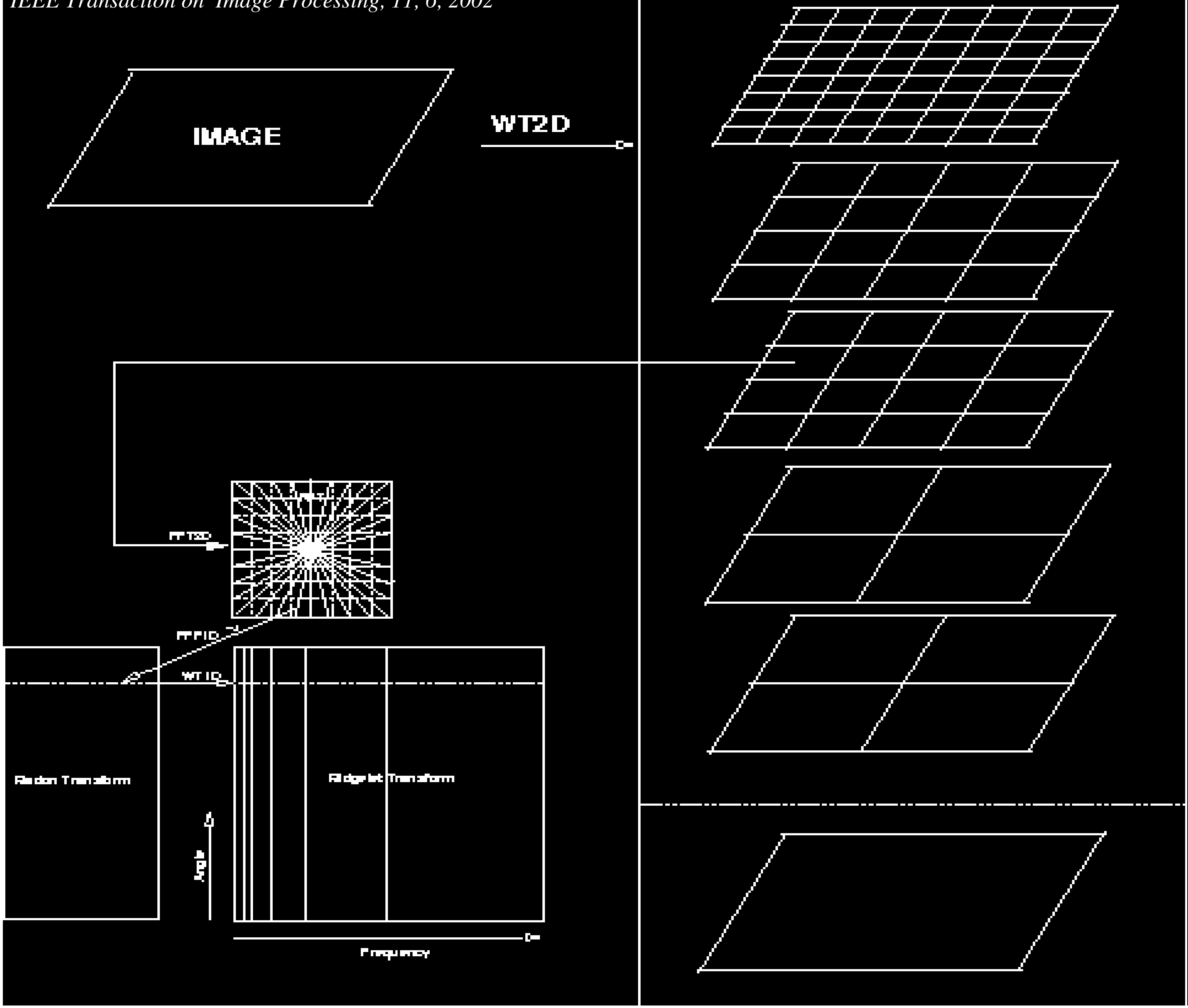


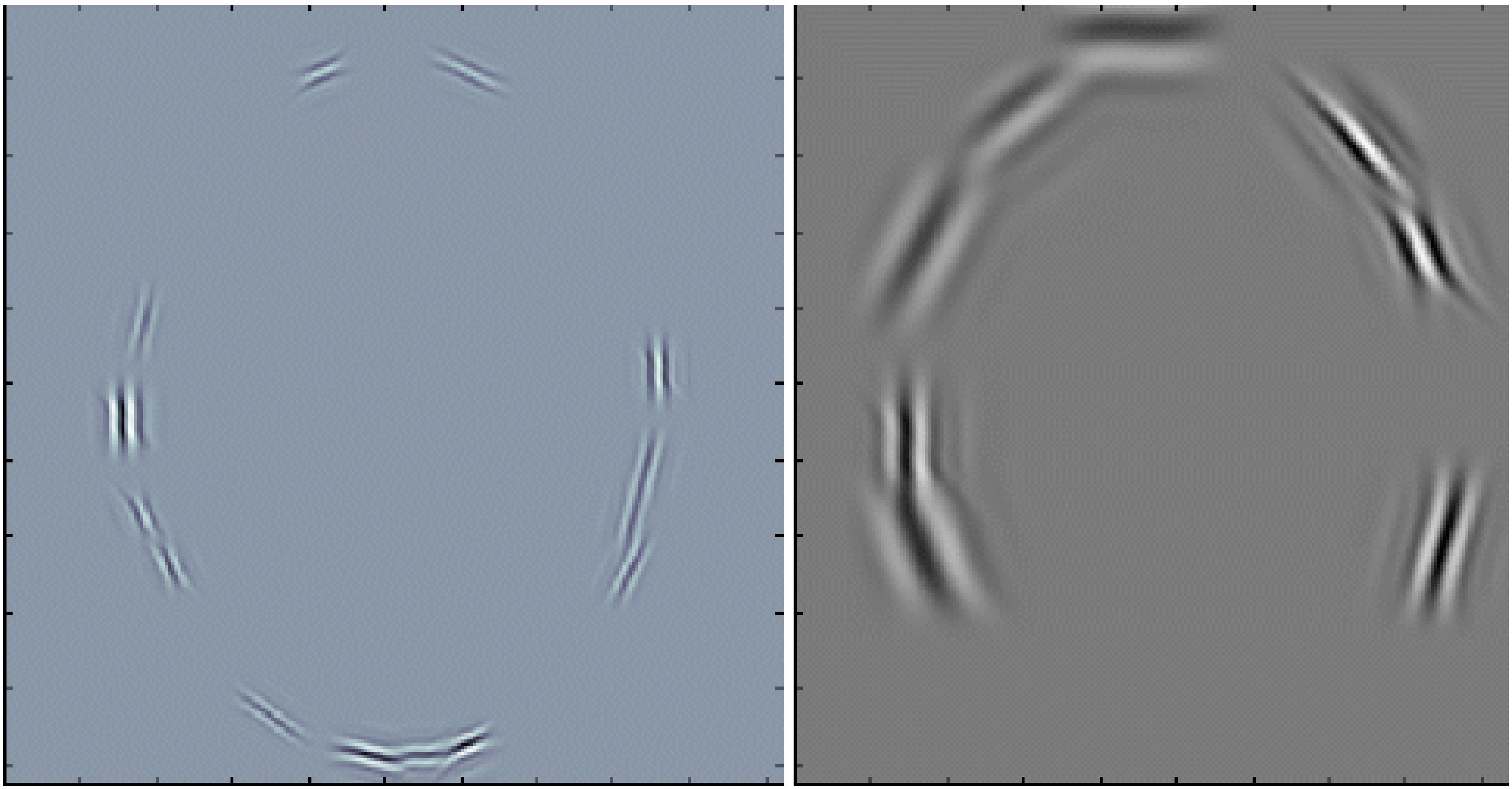
PARTITIONING



Starck, Candes, Donoho, “The Curvelet Transform for Image Denoising”

IEEE Transaction on Image Processing, 11, 6, 2002





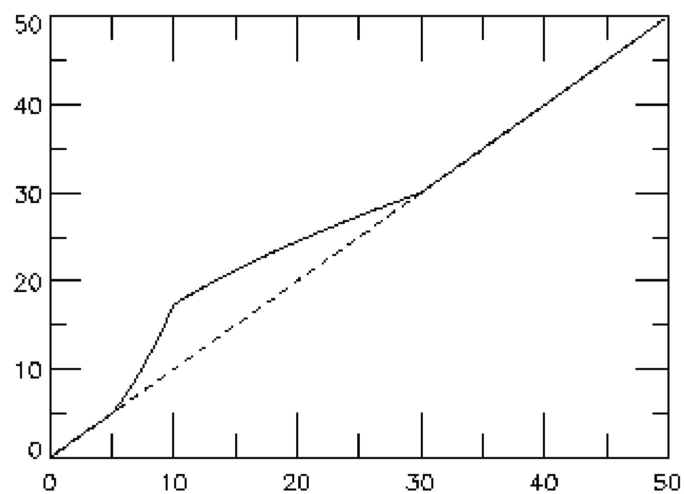
CONTRAST ENHANCEMENT

J.-L. Starck, F. Murtagh, E. Candes, and D.L. Donoho, "Gray and Color Image Contrast Enhancement by the Curvelet Transform", IEEE Transaction on Image Processing, 12, 6, pp 706--717, 2003.

$$\tilde{I} = C_R\left(y_c\left(C_T I\right)\right)$$

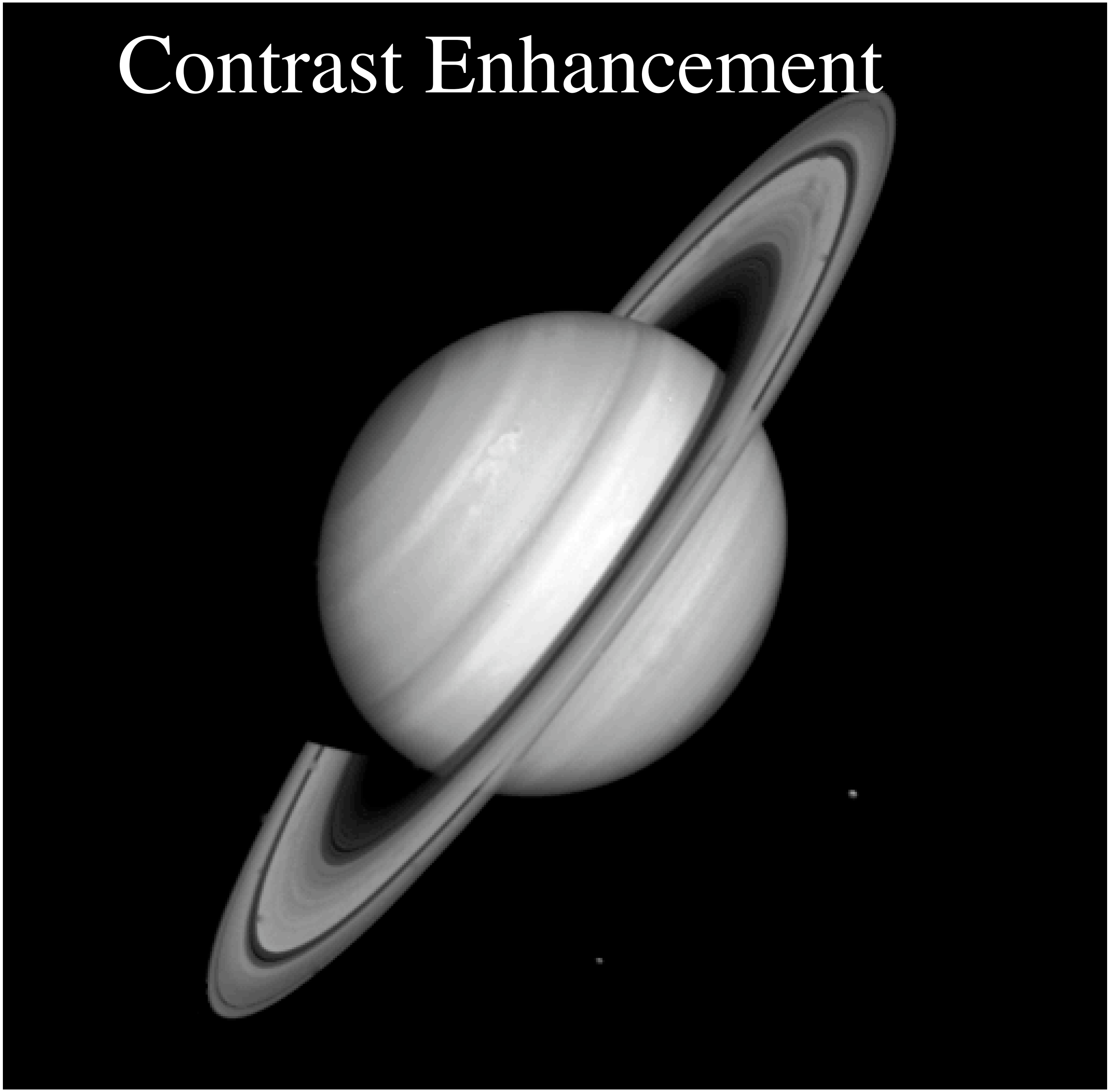
$$\begin{cases} y_c(x,\sigma) = 1 & \text{if} \quad x < c\sigma \\ y_c(x,\sigma) = \dfrac{x - c\sigma}{c\sigma}\left(\dfrac{m}{c\sigma}\right)^p + \dfrac{2c\sigma - x}{c\sigma} & \text{if} \quad x < 2c\sigma \\ y_c(x,\sigma) = \left(\dfrac{m}{x}\right)^p & \text{if} \quad 2c\sigma \le x < m \\ y_c(x,\sigma) = \left(\dfrac{m}{x}\right)^s & \text{if} \quad x > m \end{cases}$$

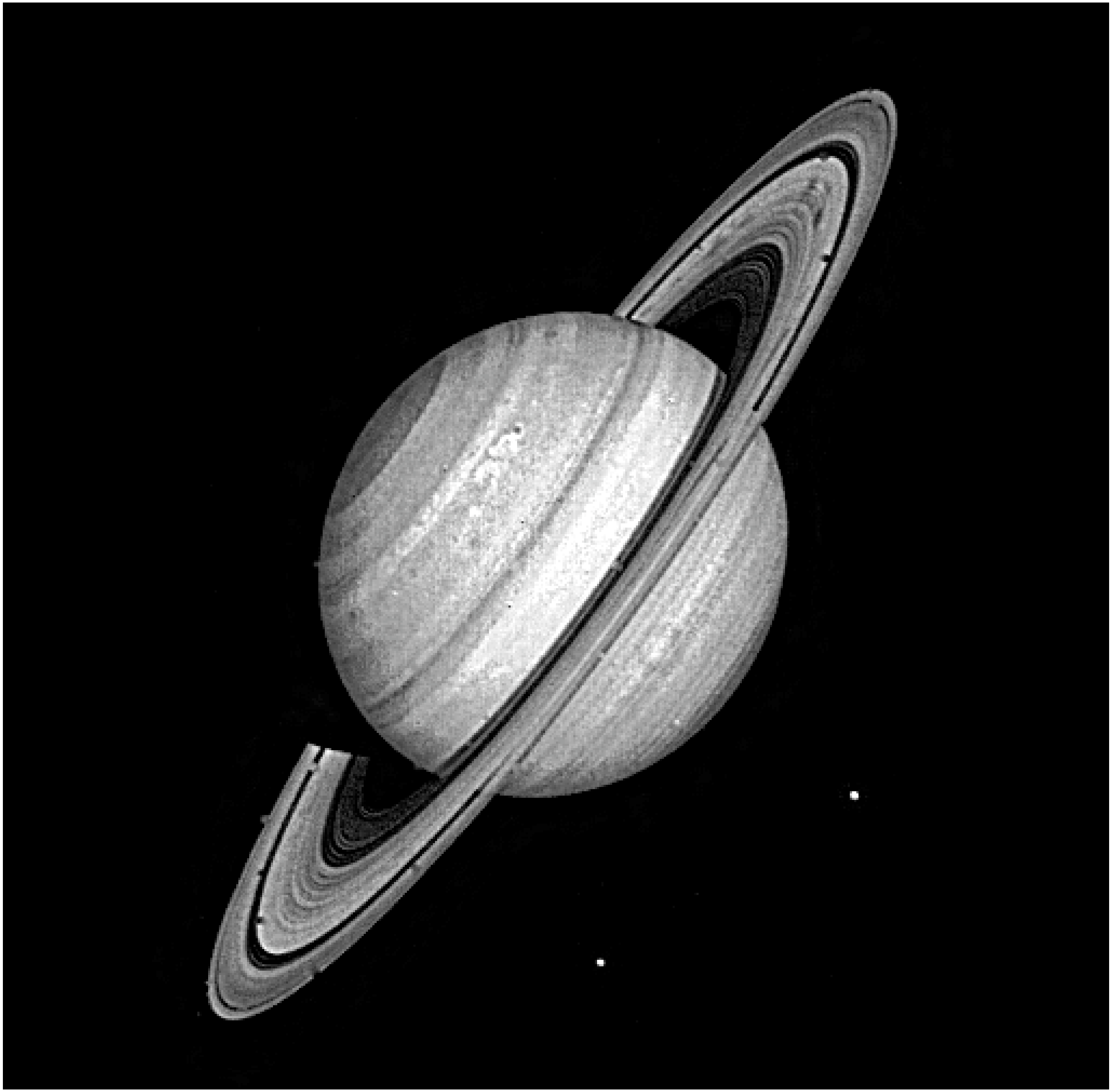
Modified curvelet coefficient



Curvelet coefficient

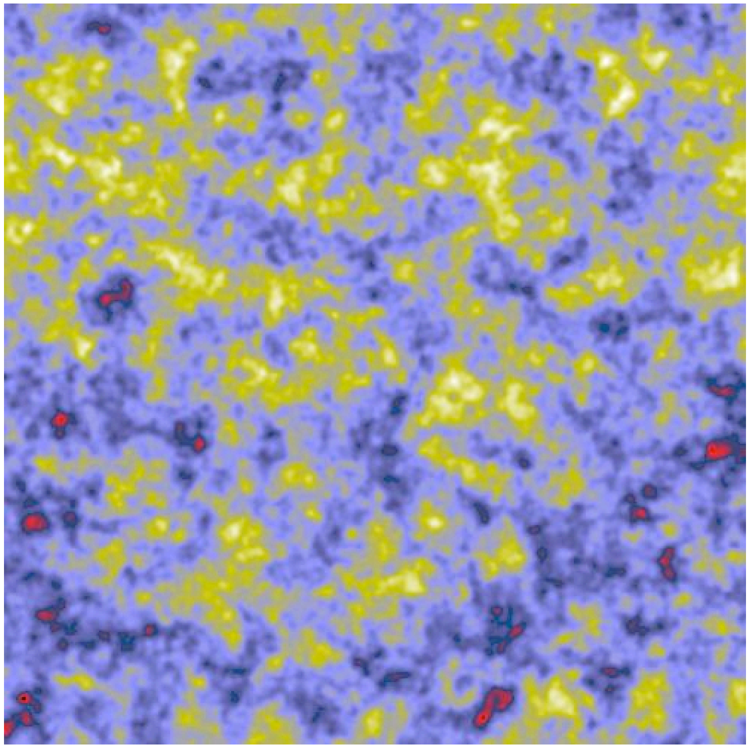
Contrast Enhancement



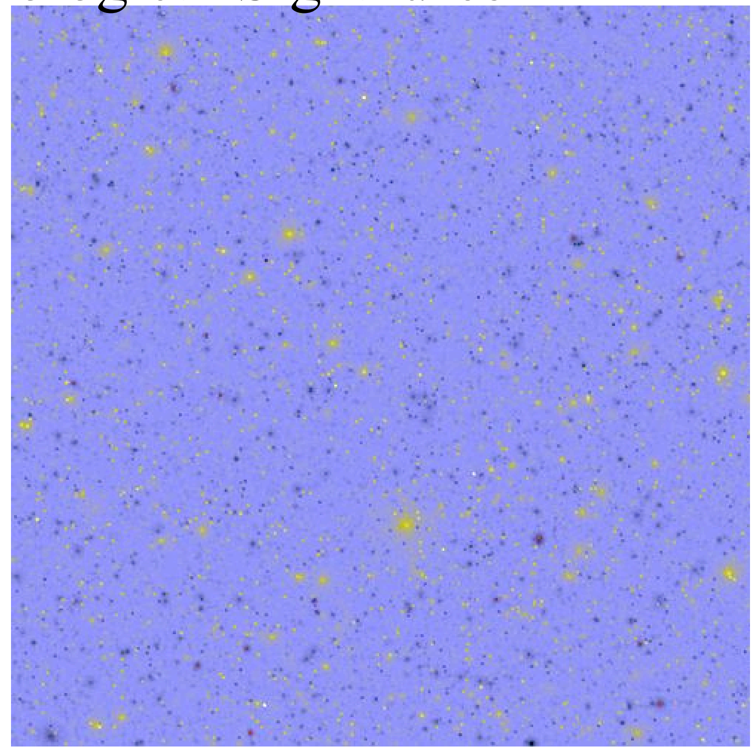


Detection of non-Gaussian Cosmological Signatures

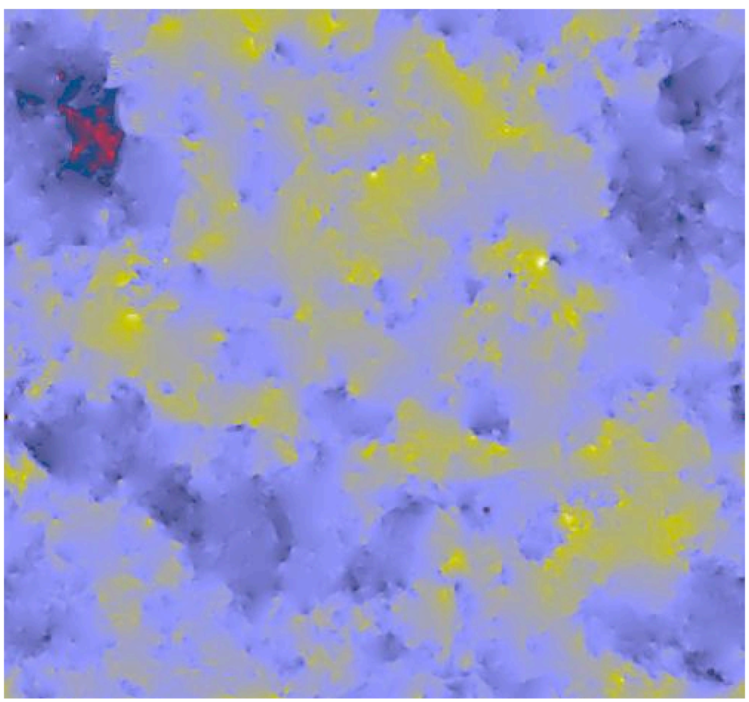
CMB



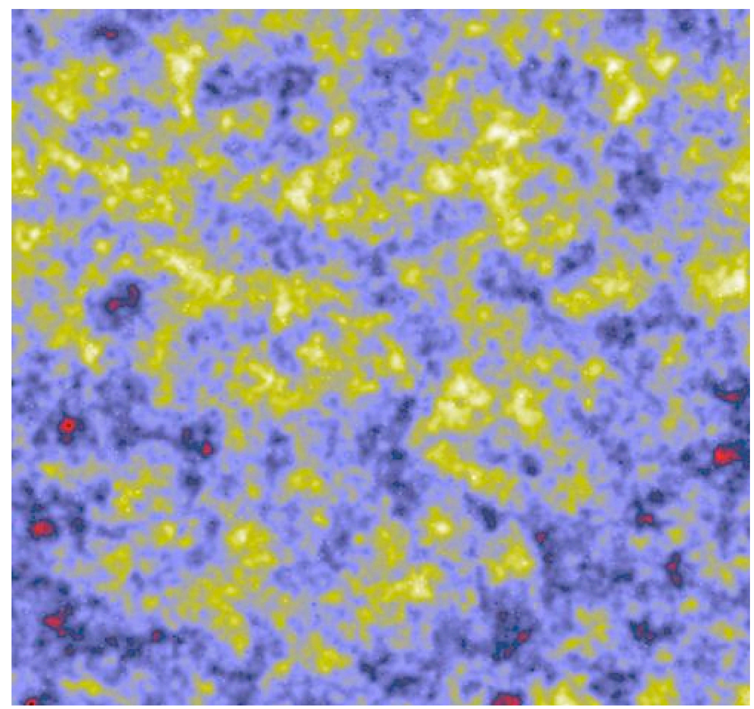
SZ

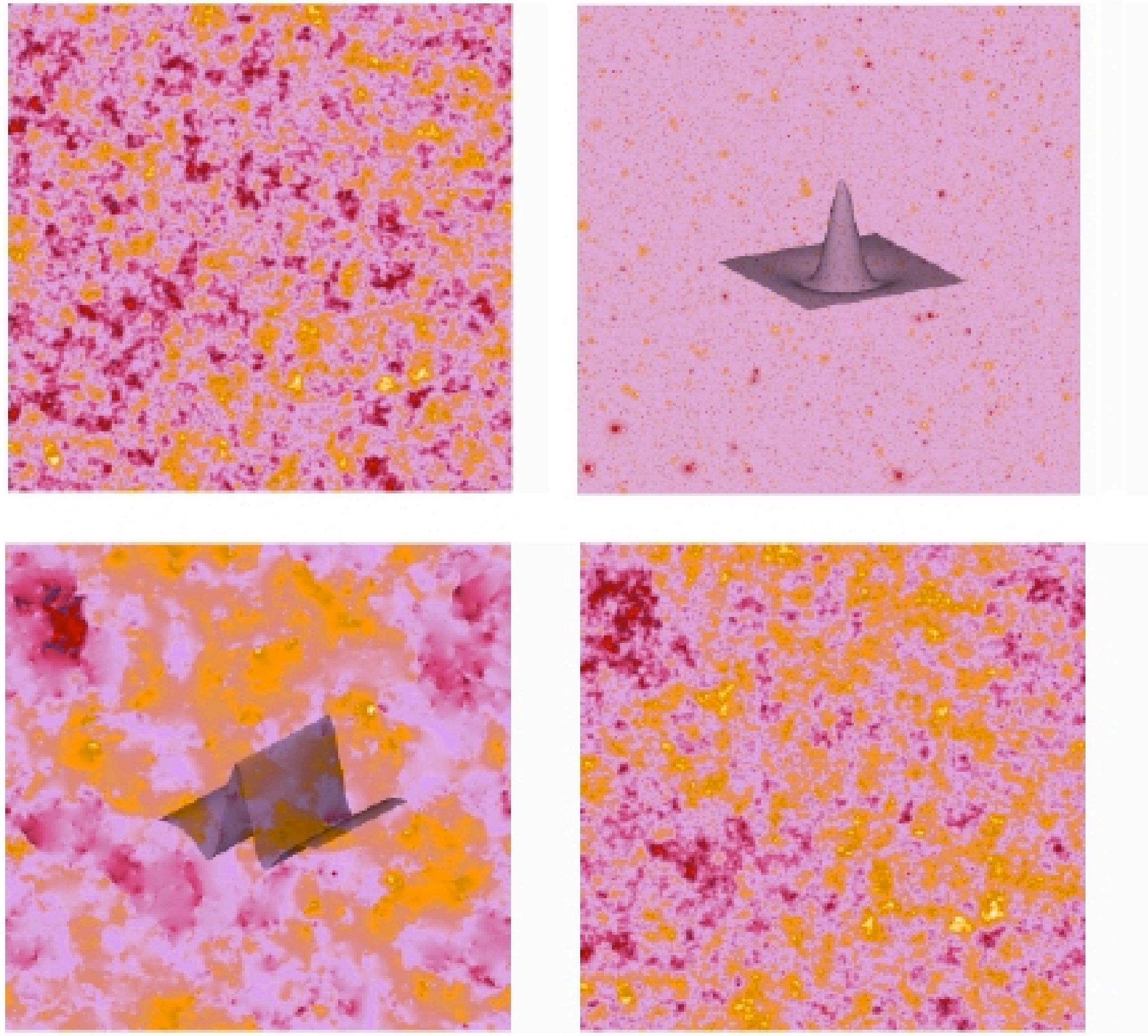


CS

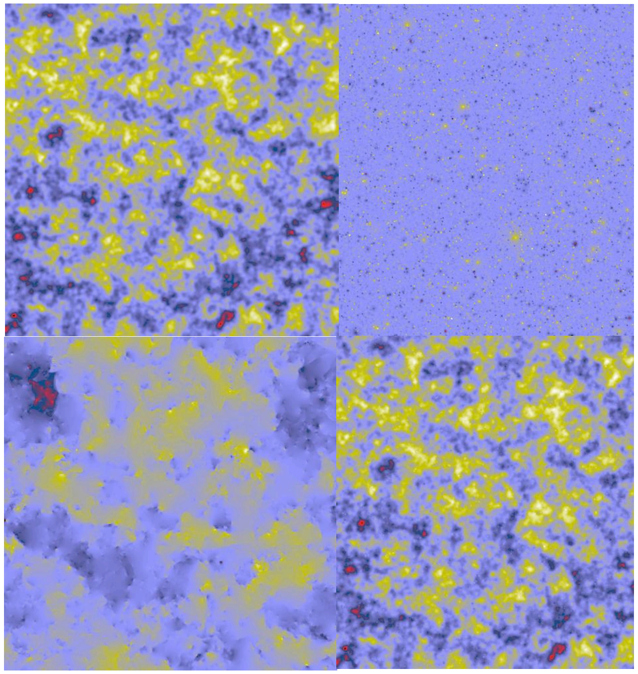


Total





Multiscale Analysis of the CMB



We have applied the following multiscale transforms

- Isotropic wavelet transform
- Bi-orthogonal wavelet transform
- Ridgelets (block size of 16 pixels)
- Ridgelets (block size of 32 pixels)
- Curvelets

On

1) 100 **CMB + KSZ** + 100 Gaussian realizations with the same power spectrum.

$$K_{CMB-SZ}(i,b) \Rightarrow K_{CMB-SZ}(b) = mean(K_{CMB-SZ}(1..100,b)), \bar{K}_{CMB-SZ}(b) = \frac{K_{CMB-SZ}(b)}{K_{CMB}(b)}$$

2) 100 **CMB + CS** + 100 Gaussian realizations with the same power spectrum

3) 100 **CMB + KSZ + CS** + 100 Gaussian realizations with the same power spectrum

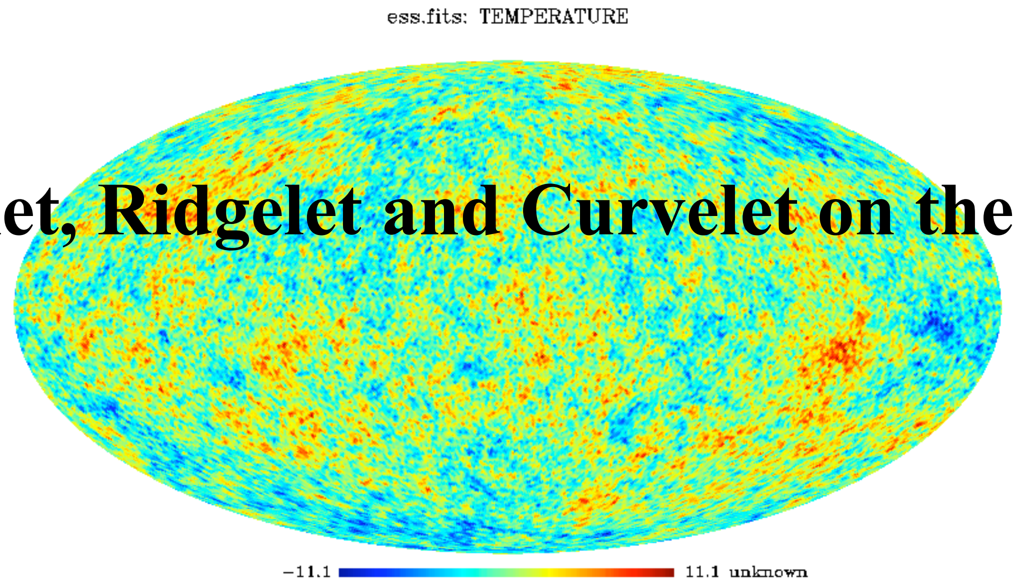
We compare the normalized kurtosis for the three data set.

Results

• **Curvelets are NOT sensitive to KSZ but *are* sensitive to cosmic strings**

	Bi-orthogonal WT	Ridgelet	Curvelet
CMB+KSZ	1106.	0.1	10.12
CMB+CS	1813.	5.7	198.
CMB+CS+KSZ	1040.	5.9	165.

Detecting cosmological non-Gaussian signatures by multi-scale methods, *Astron. and Astrophys., 416, 9--17, 2004* .



Wavelet, Ridgelet and Curvelet on the Sphere :

J.-L. Starck, P. Abrial, Y. Moudden and M. Nguyen,
"Wavelets, Ridgelets and Curvelets on the Sphere", A&A, in press.

Experiments available at: http://jstarck.free.fr/mrs.html

Denoising using Wavelets and Curvelets

Astrophysical Component Separation (ICA on the Sphere)

Analysis of the spatial distribution of galaxies

To map out the universe:

1) Measure redshifts of lots of galaxies: $\mathbf{z = \Delta\lambda / \lambda}$

2) Calculate speed from redshift: $\mathbf{V = c\,z}$

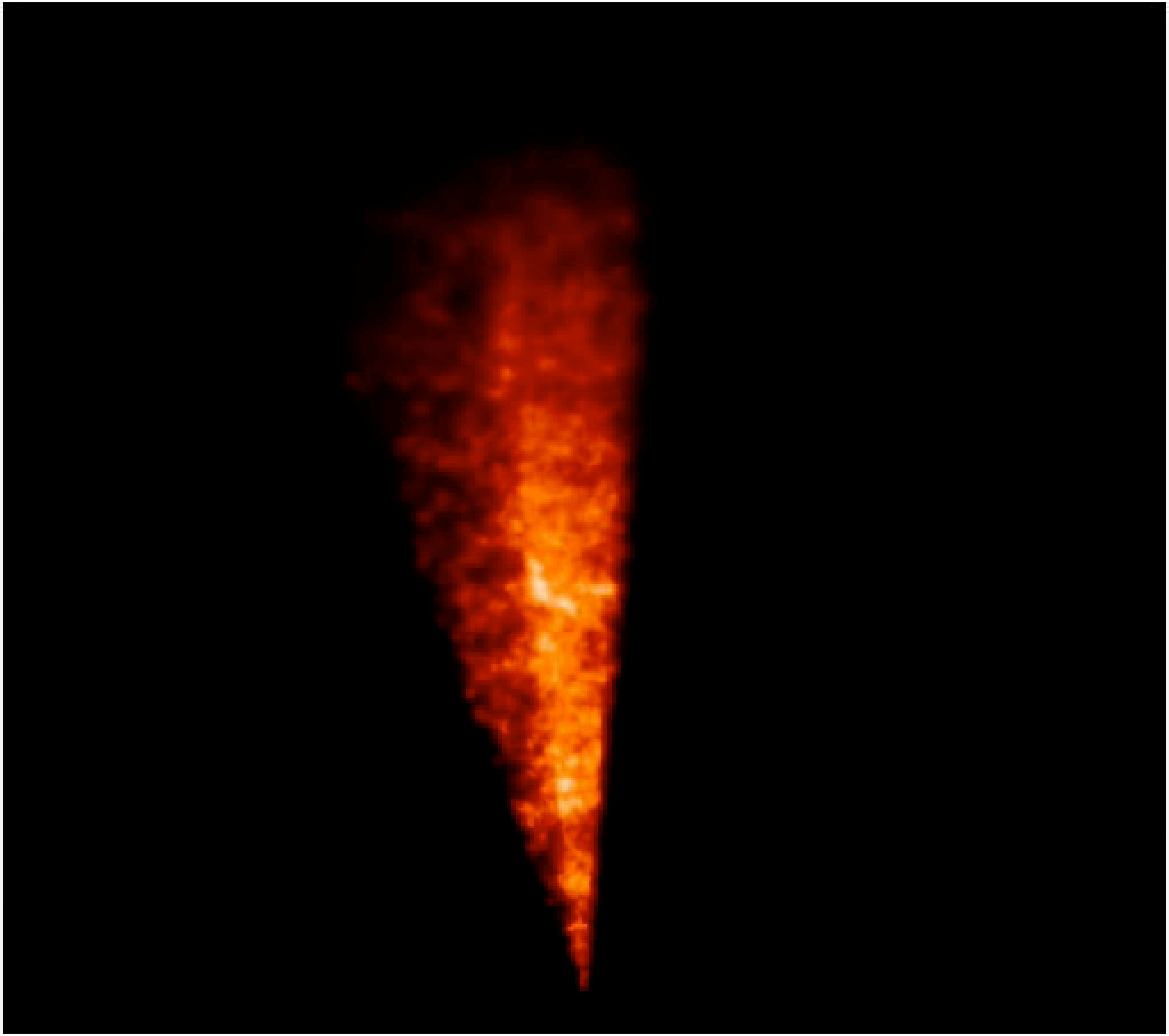
3) Calculate distance from Hubble Law: $\mathbf{V = H\,d}$

V = velocity (in km/s)
d = distance (in megaparsec = 3.08 10^19 km)
H = Hubble constant (around 70 km/s per Mpc)

4) make a map of direction vs. distance

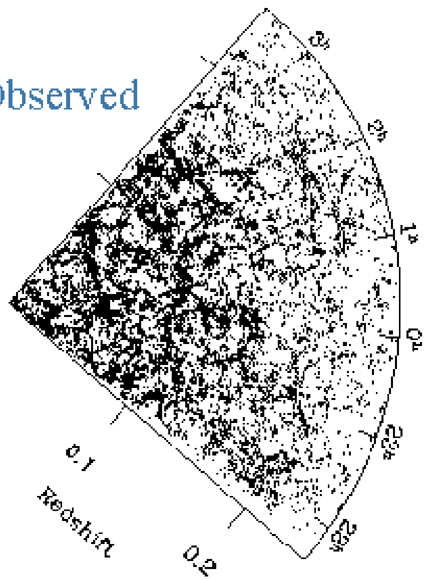
lots of structures
- bubbles and voids
- some structures more than 10 Mpc long
- voids at least that wide across

How do you form such huge things?

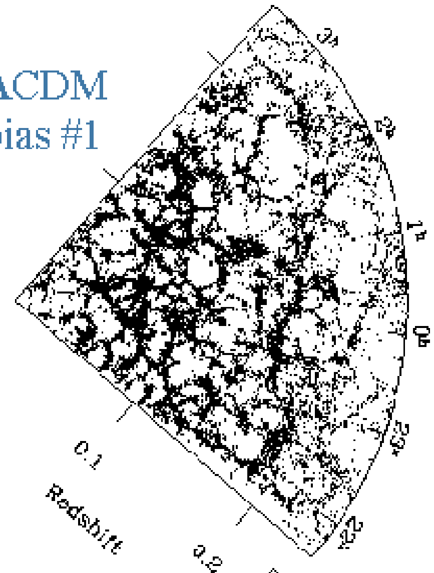


Models vs observations

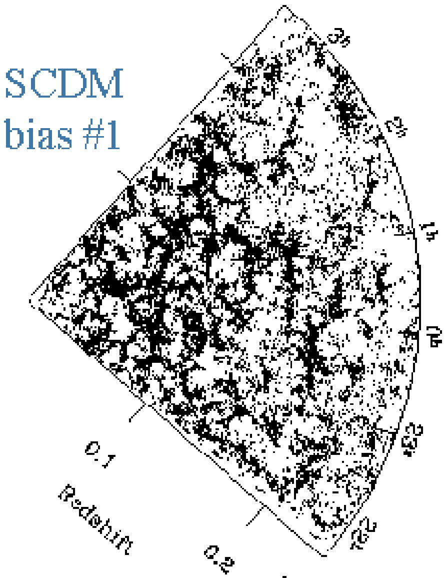
Observed



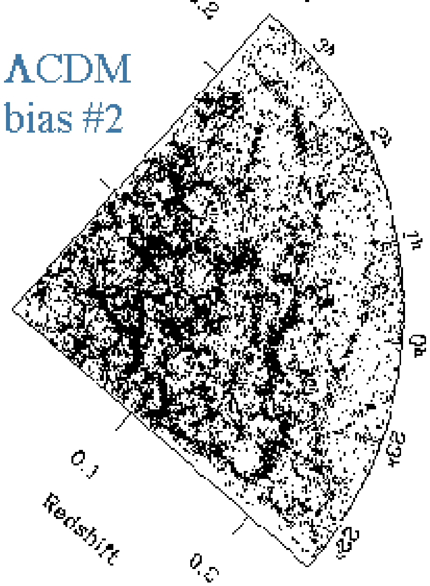
ΛCDM
bias #1



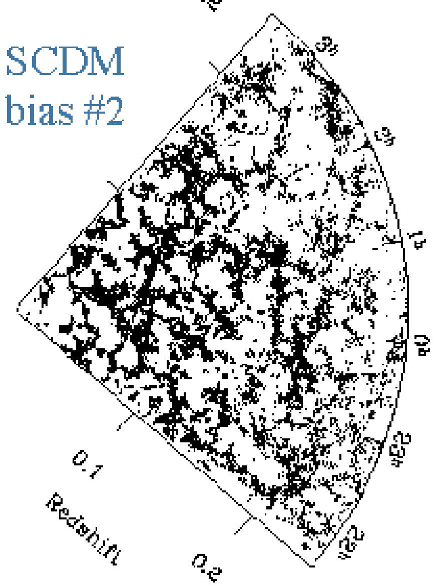
SCDM
bias #1



ΛCDM
bias #2



SCDM
bias #2



Methods

.Two or three point correlation function

. Genus curve

. Voronoi Tessellation

. Minimal spanning trees

. Power spectrum

. Fractals

GENUS FUNCTION

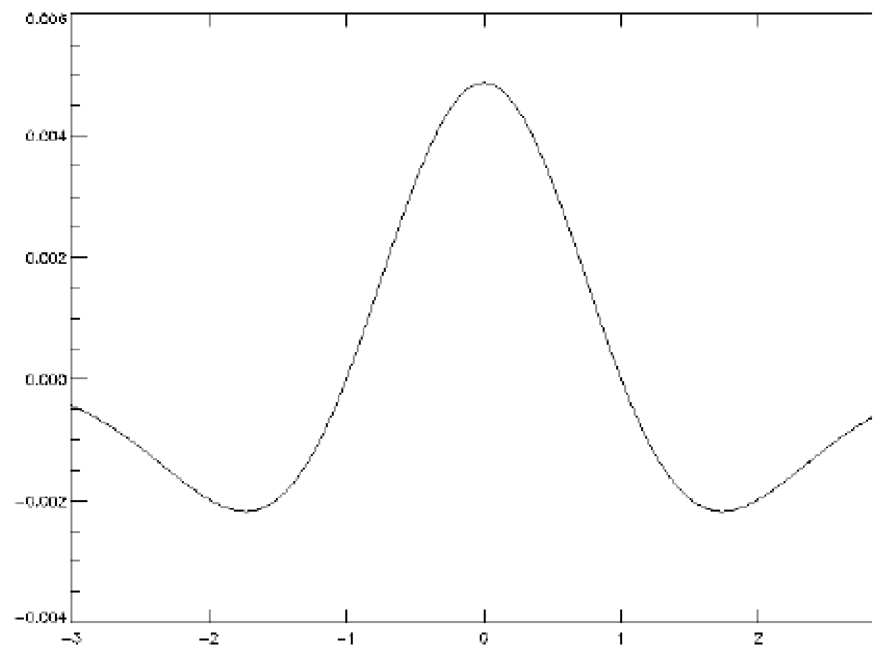
The genus of a surface G is

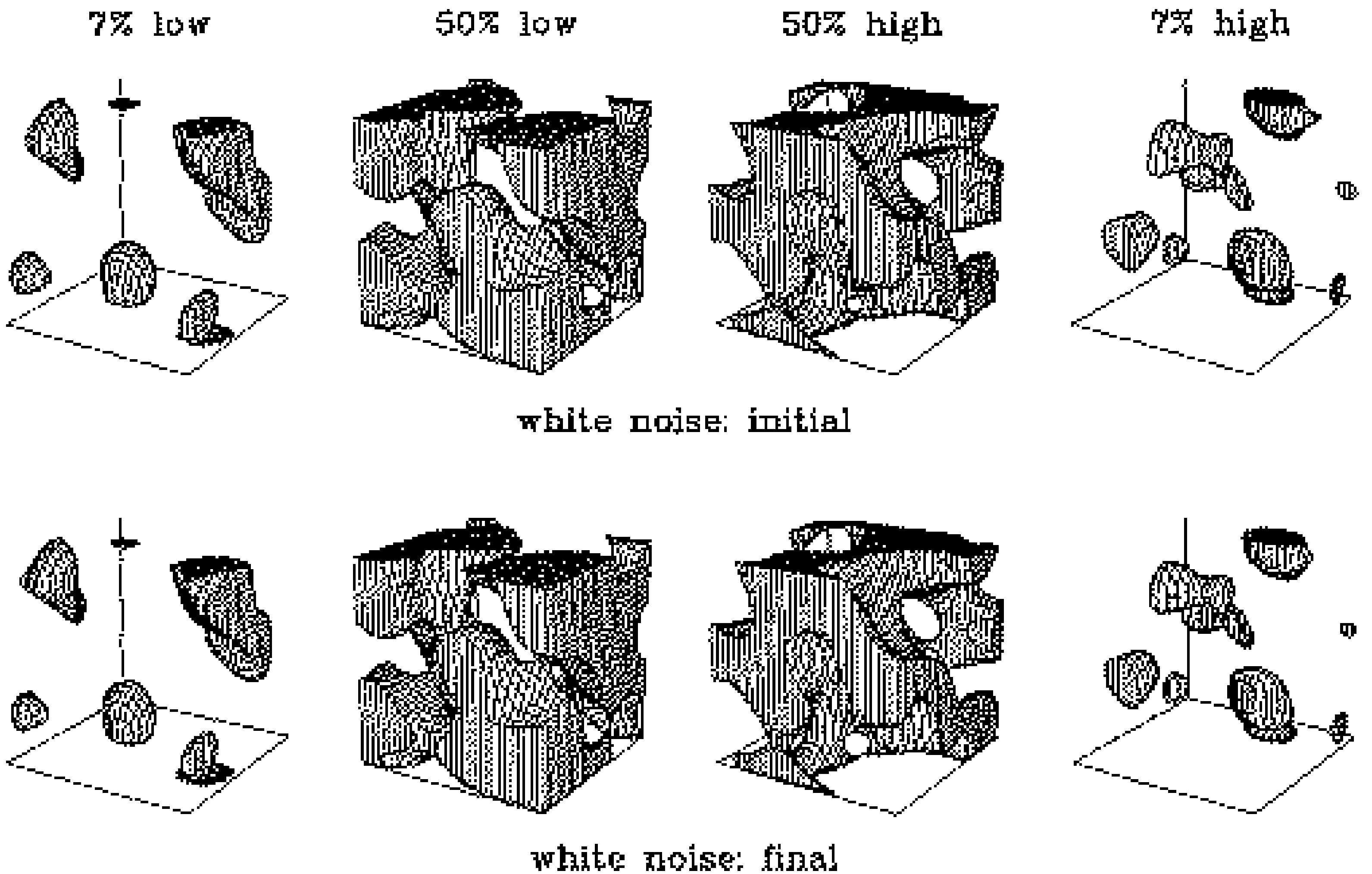
G(T) = (number of holes) - (number of isolated regions) + 1

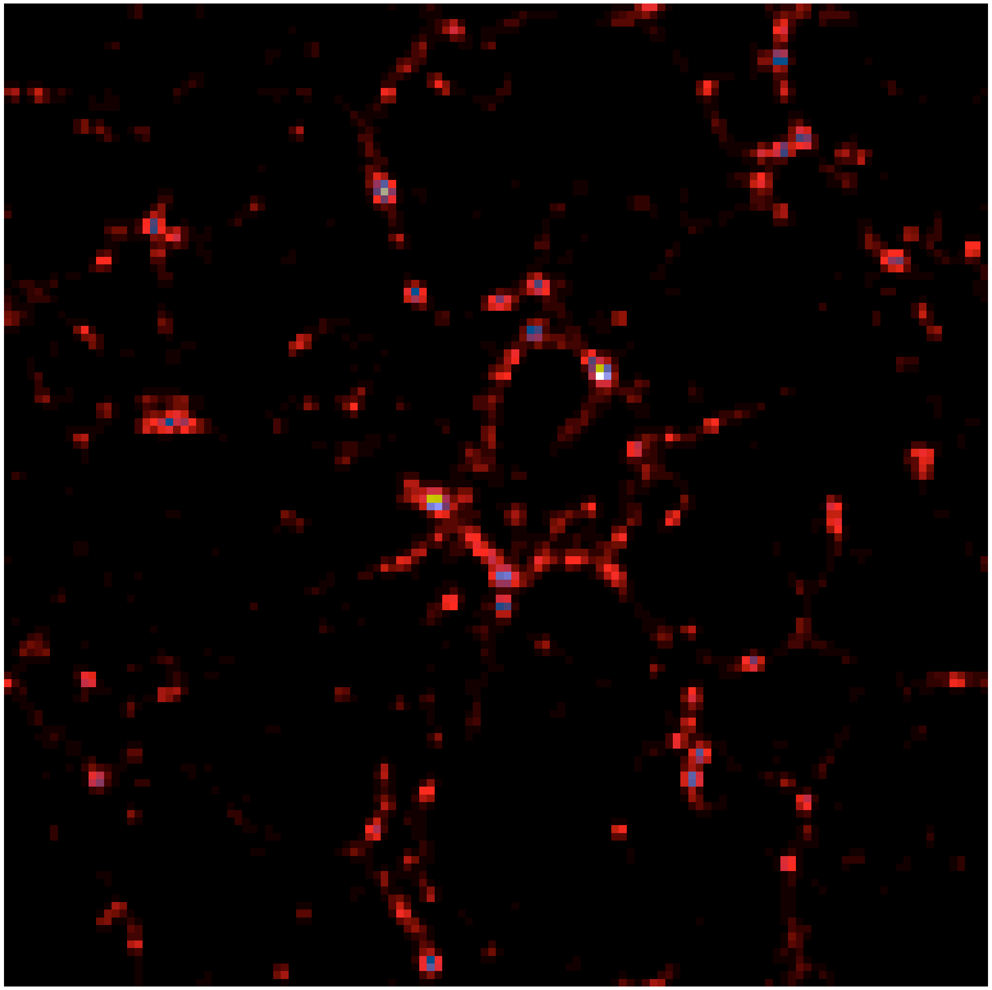
- Convolve the data by a Gaussian
    - Threshold all values under a threshold level T
    - G(T) = (number of holes) - (number of isolated regions) + 1

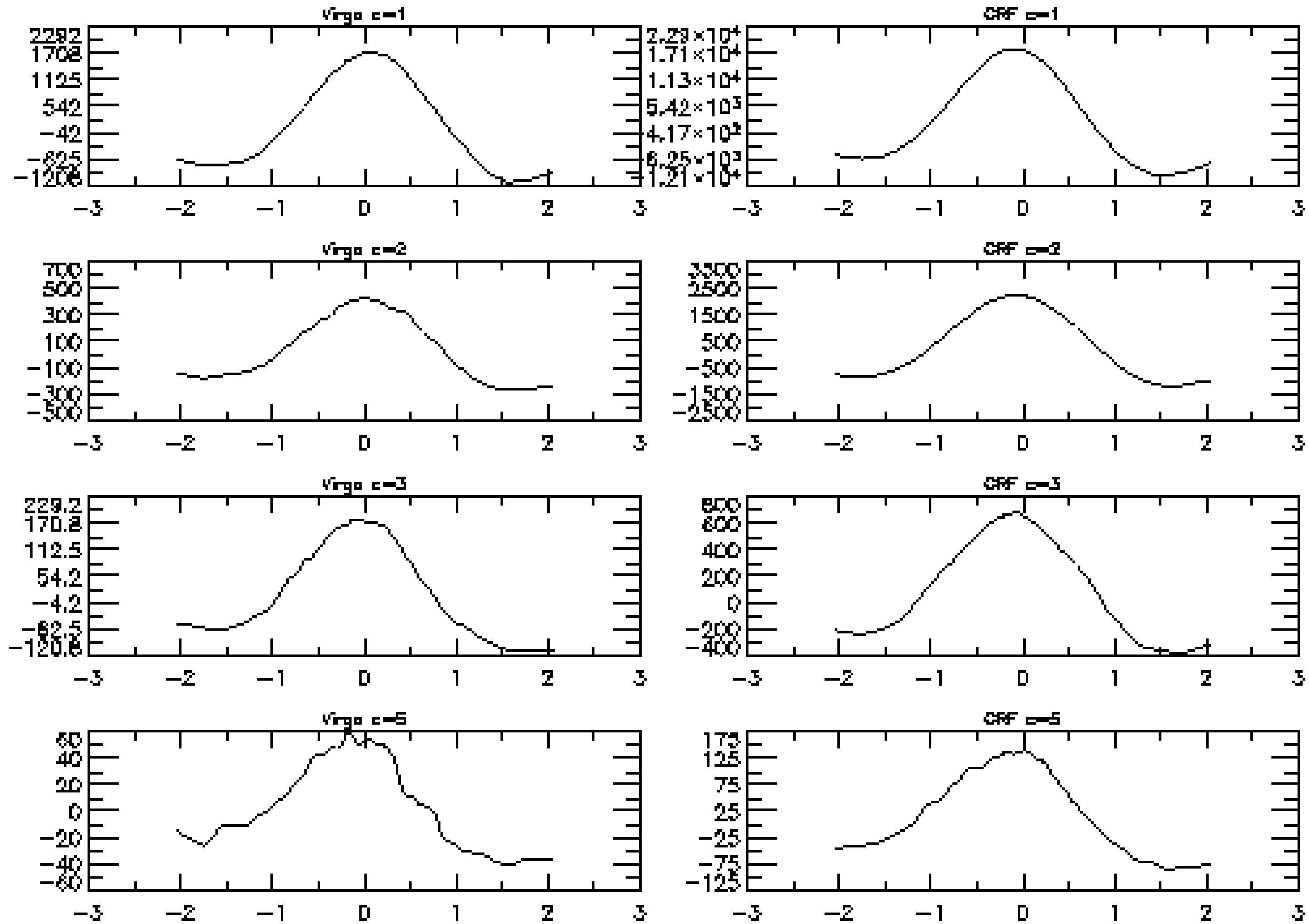
For a Gaussian field, the genus curve is:

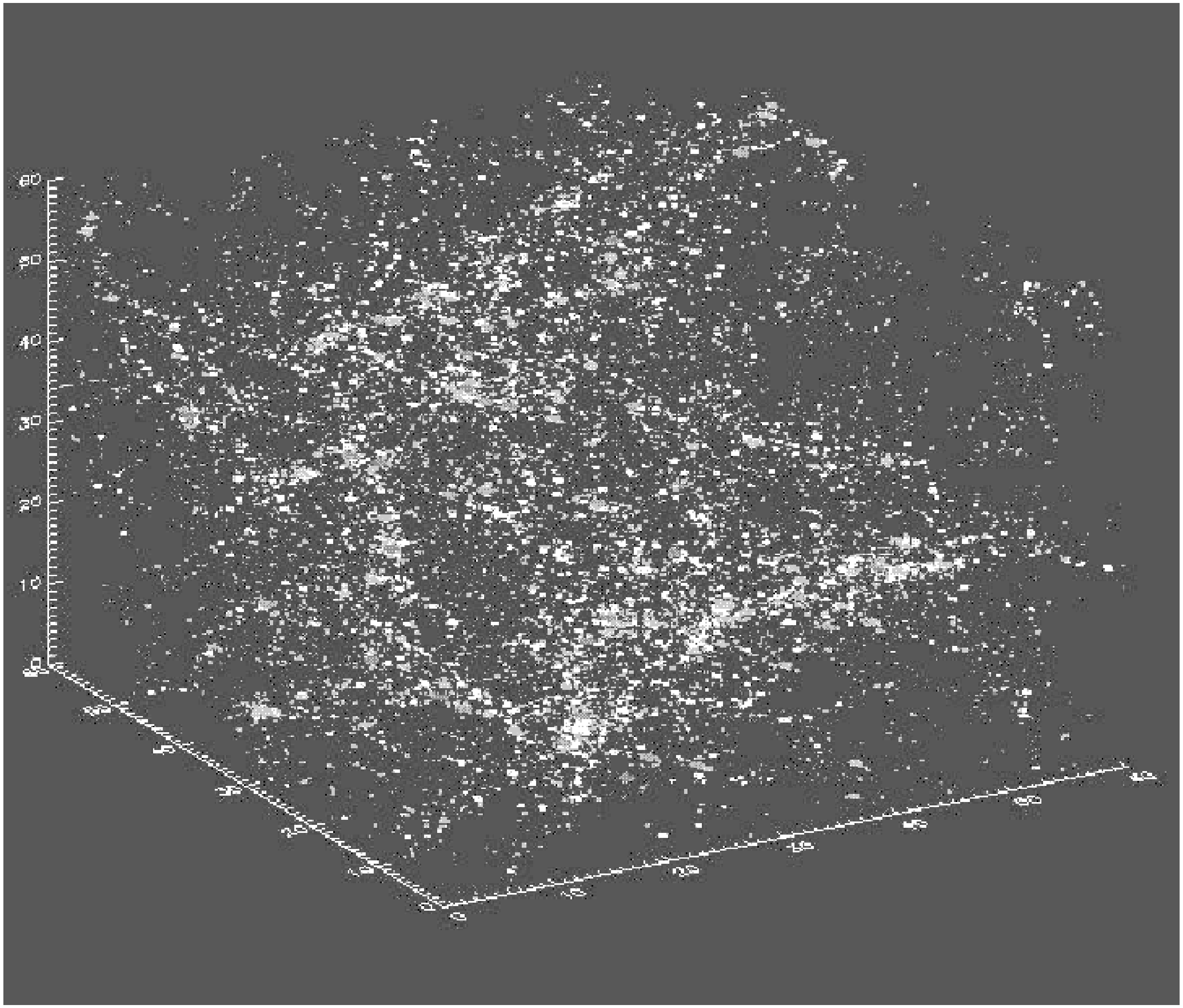
$$g(\nu) = N(1-\nu^2)\exp\left(-\frac{\nu^2}{2}\right)$$




7% low
50% low
50% high
7% high
white noise: initial
white noise: final




Virgo c=1
CRF c=1
Virgo c=2
CRF c=2
Virgo c=3
CRF c=3
Virgo c=5
CRF c=5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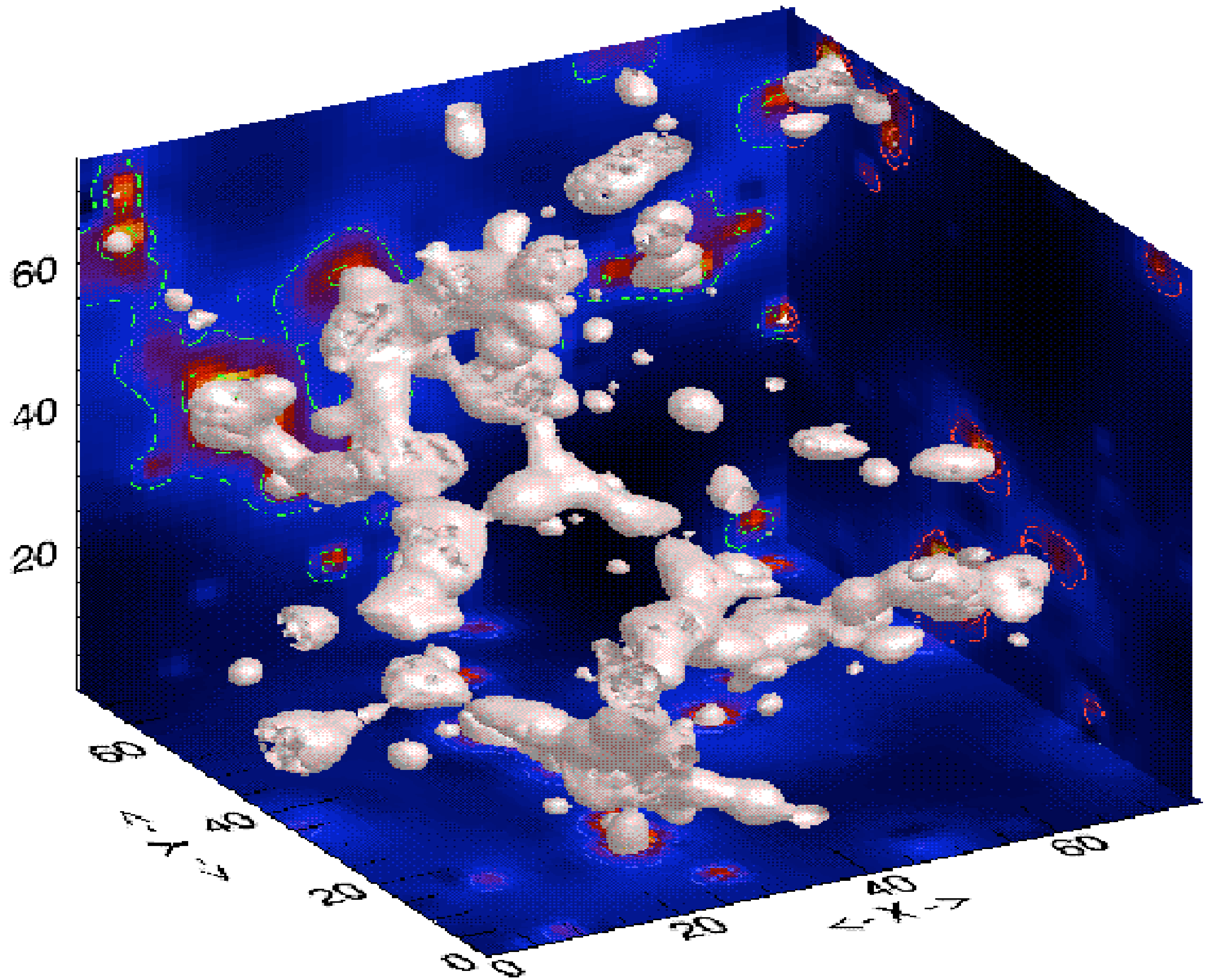

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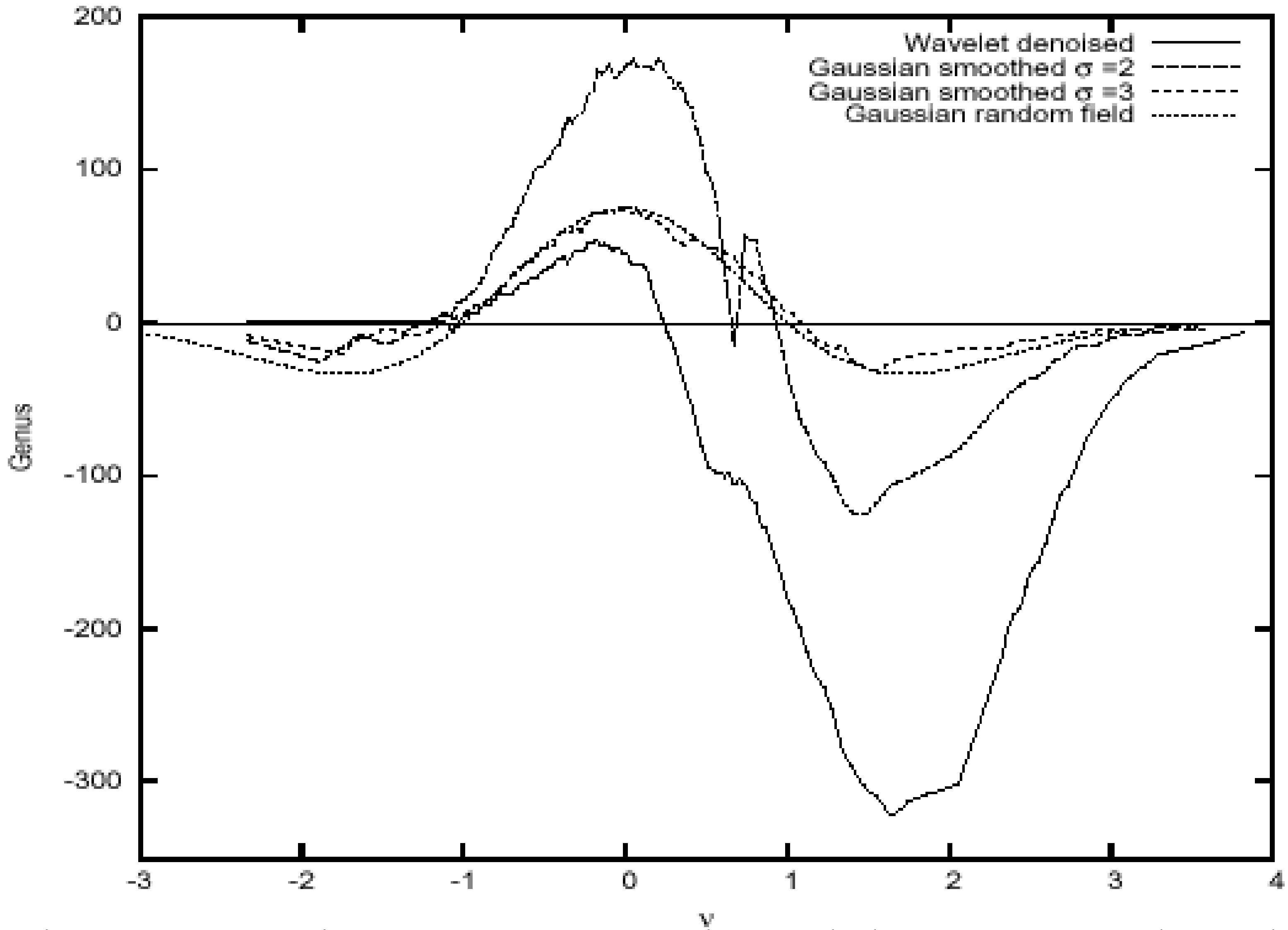
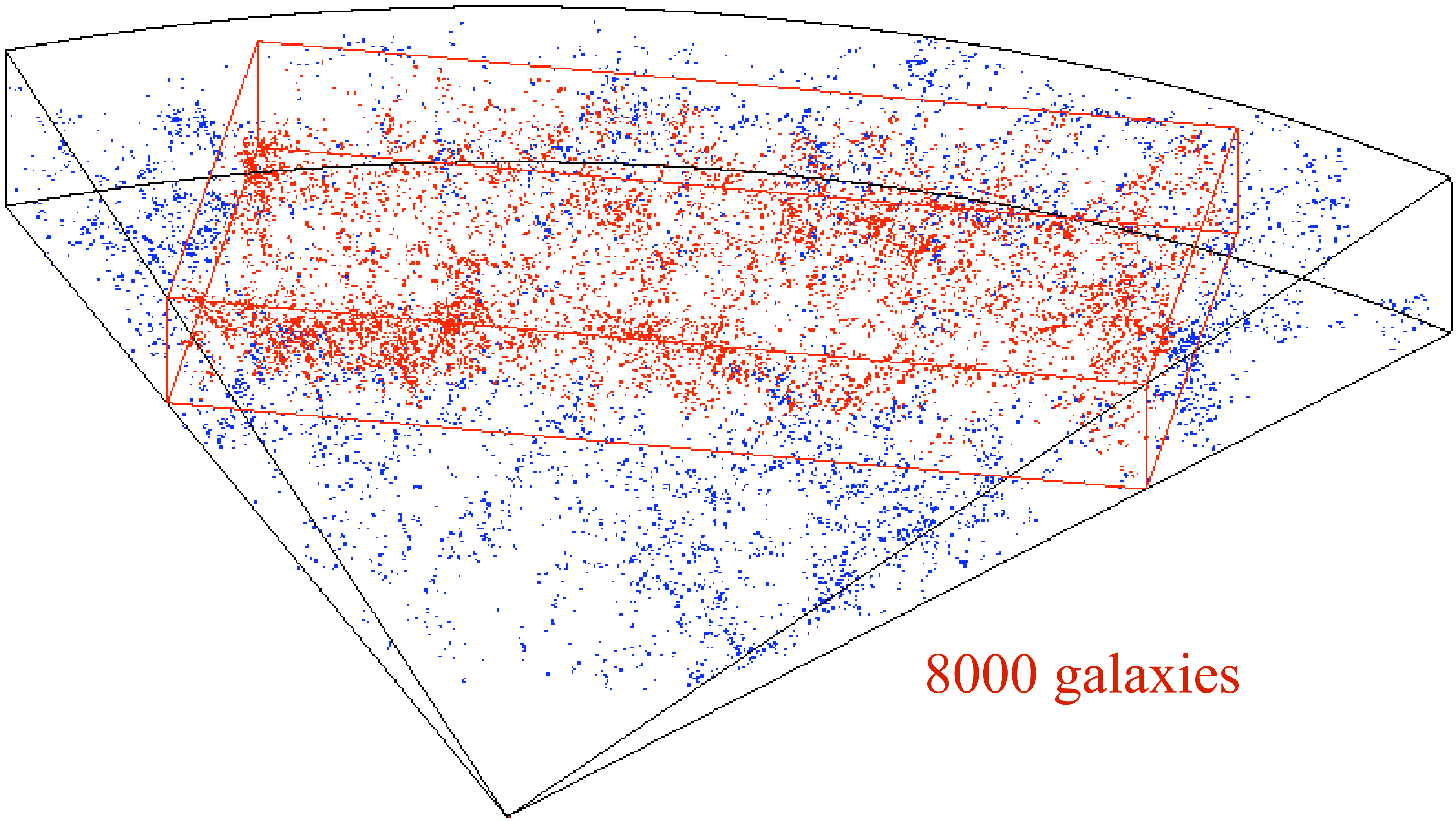


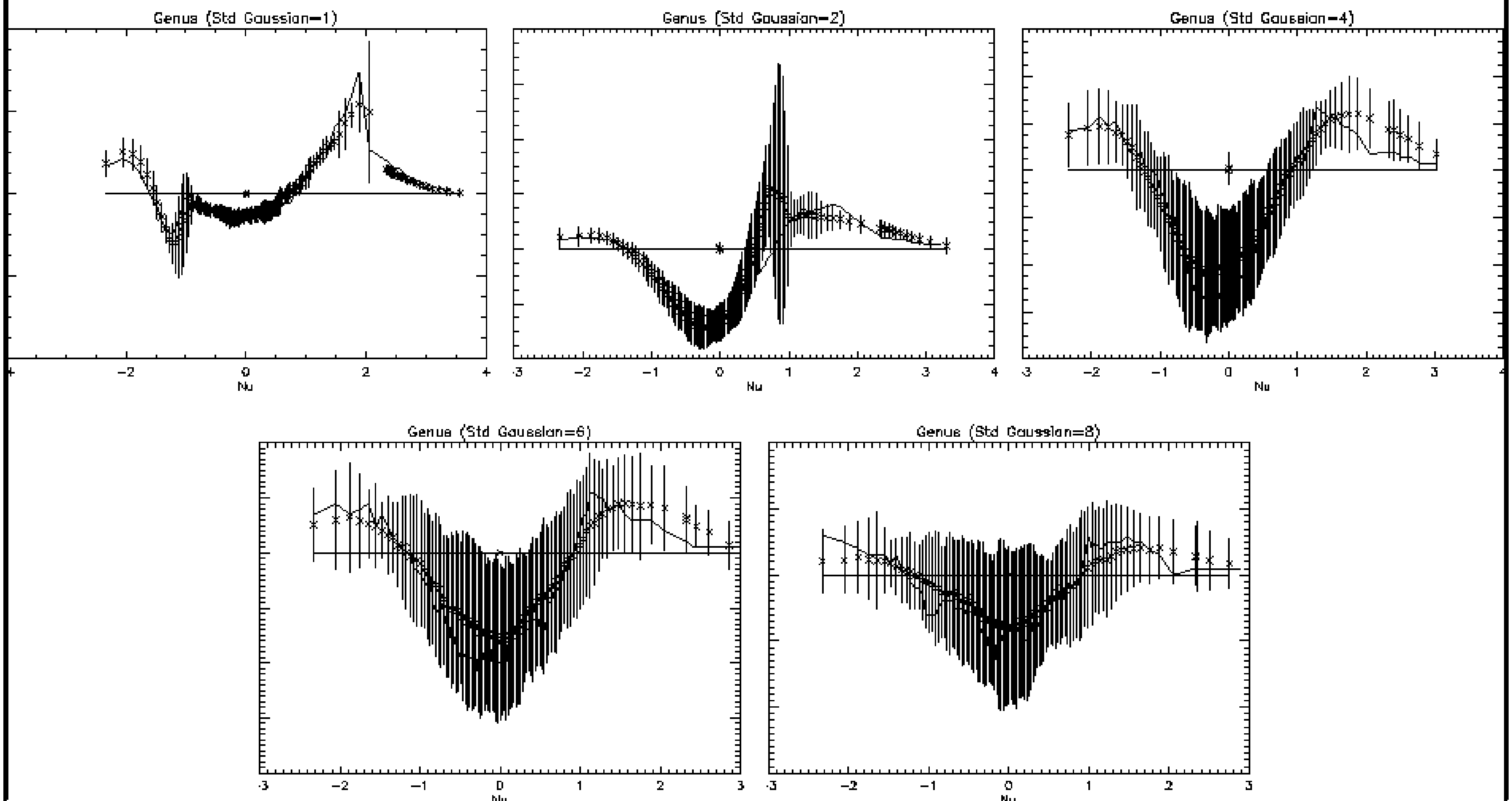
V. Martinez, J.-L. Starck, E. Saar, D.L. Donoho, P. de la Cruz, S. Paredes and S. Reynolds, "Wavelet Morphology of the Galaxy Distribution", ApJ, in press.

The genus curve of this adaptive reconstructed density field is much more informative because is unique and does not depend of the particular choice of the filter radius. Additionally, the gen curves of Gaussian-smoothed density fields mimic those of Gaussian random fields, describi thus more the properties of the filter than the real morphology of the density distribution.

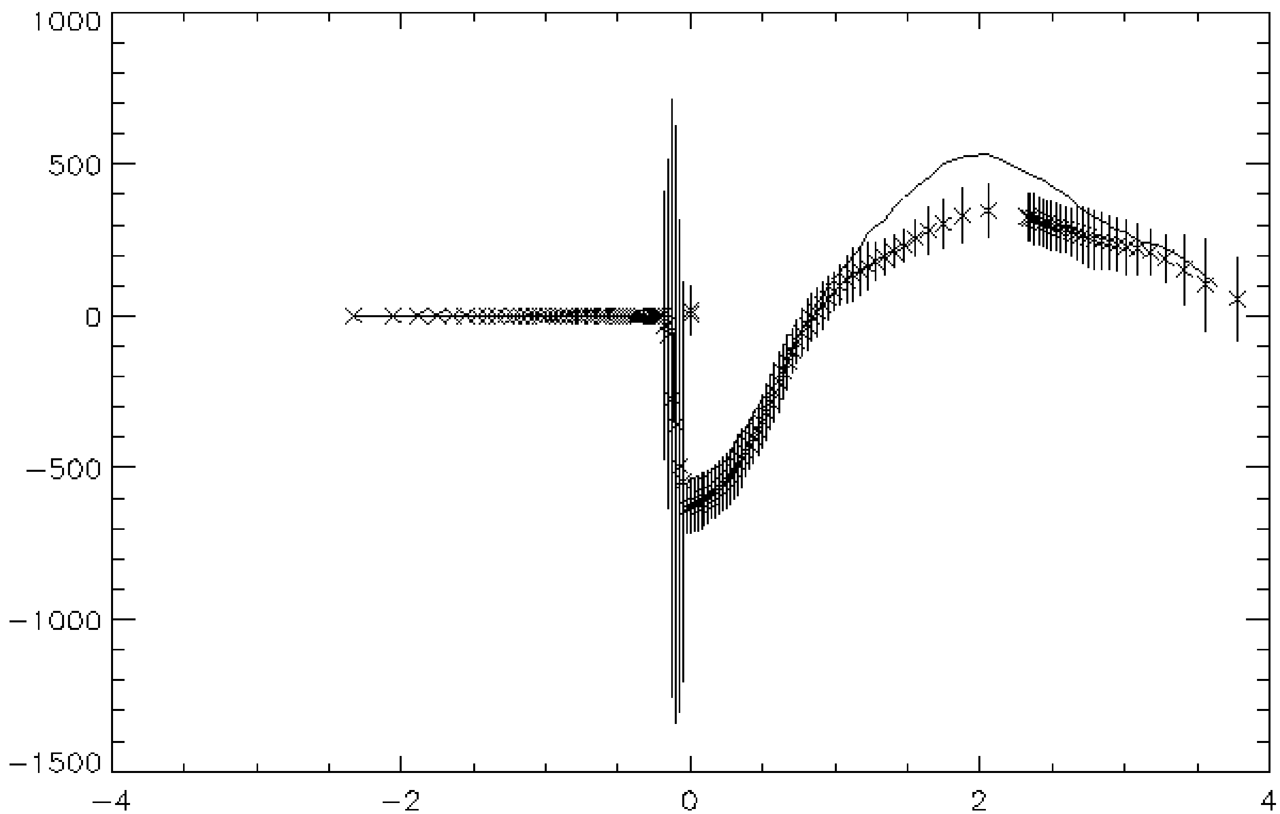
2dfGRS northern slice



Genus: 2DF and Lambda-CDM simulations



Wavelet Denoising+Genus: 2DF and Lambda-CDM Sim.



3D MULTISCALE TRANSFORMS

1) **3D WAVELET TRANSFORM: Isotropic Structures**

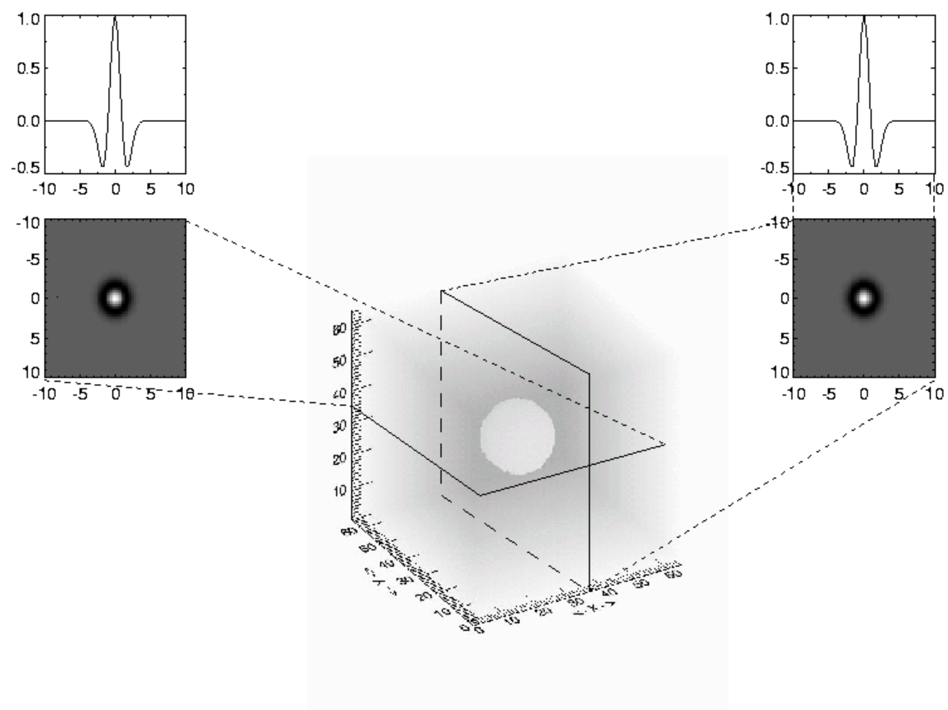
2) **3D RIDGELET TRANSFORM: Sheet like Structures**

3) **3D BEAMLET TRANSFORM: Filaments**

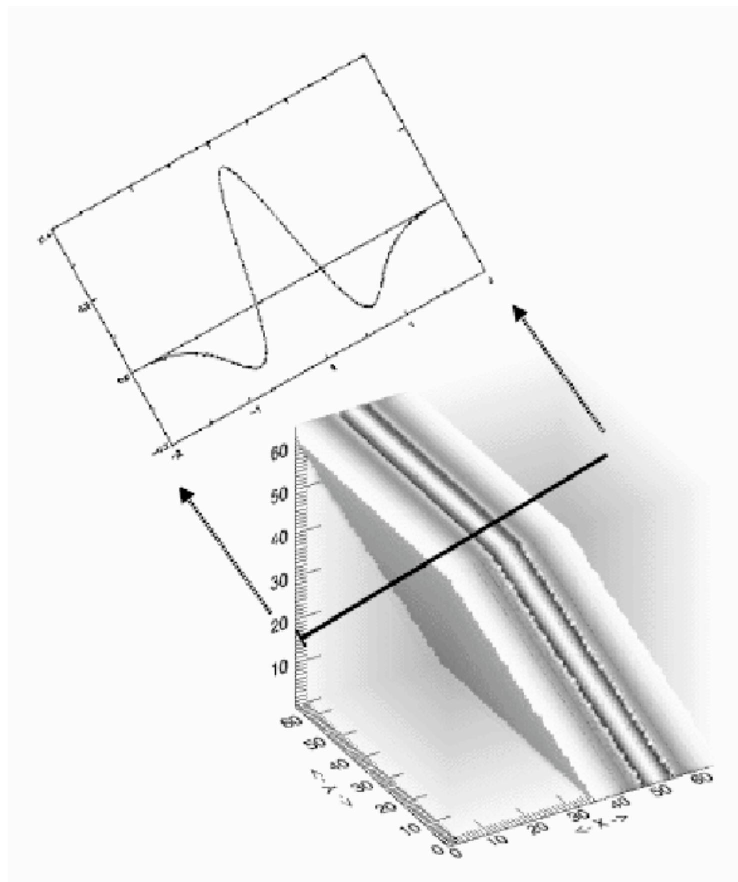
⇒ **Statistical information extraction from all transforms**

Starck et al, "Analysis of the spatial distribution of galaxies by multiscale methods", in press, 2005.

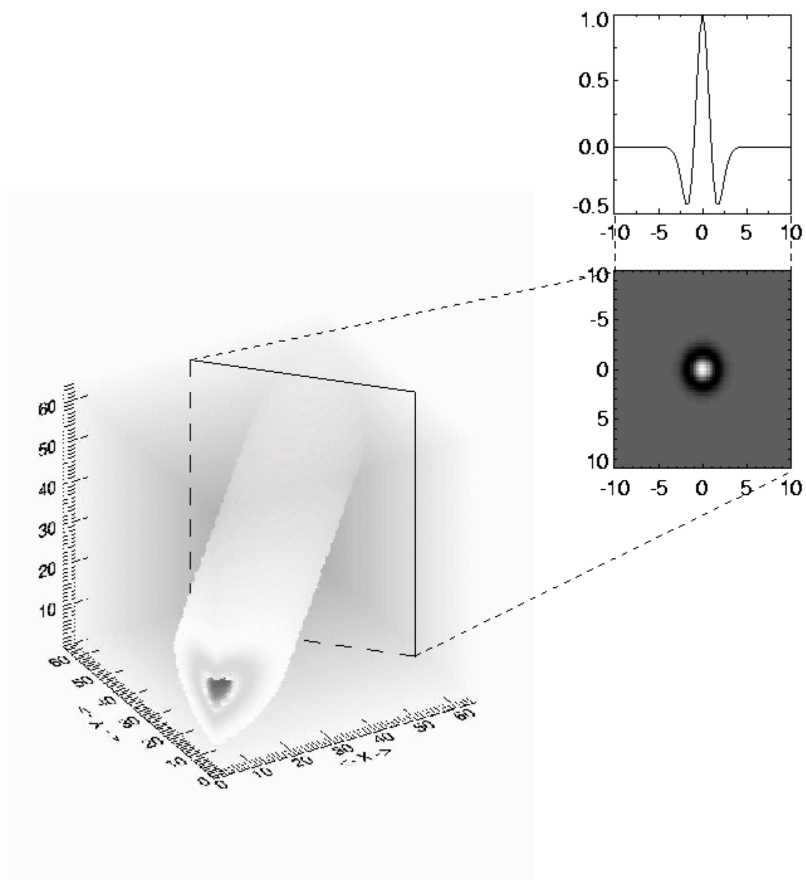
3D Wavelet Function

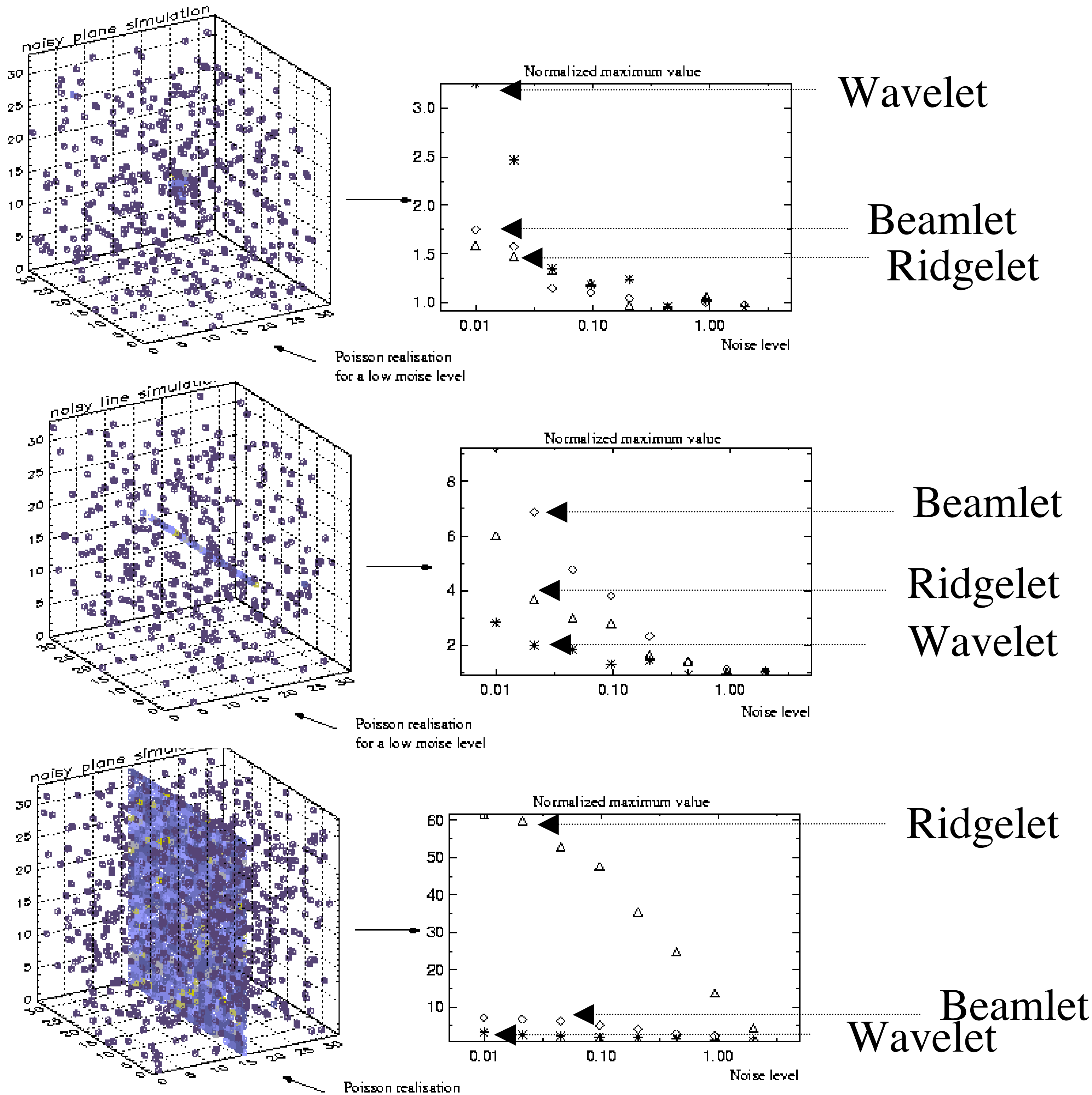


3D Ridgelet Function



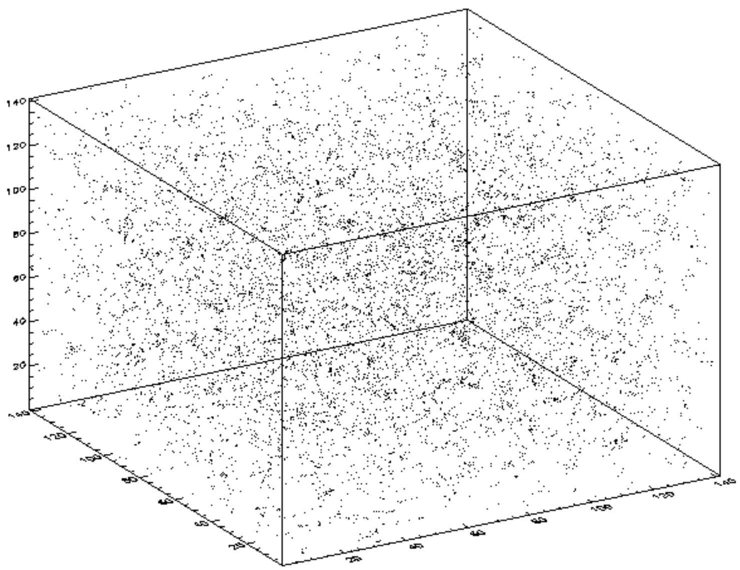
3D Beamlet Function

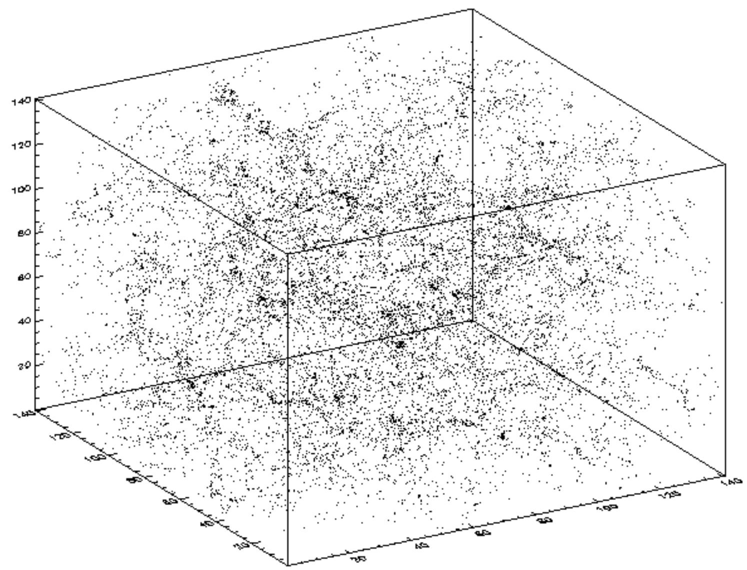


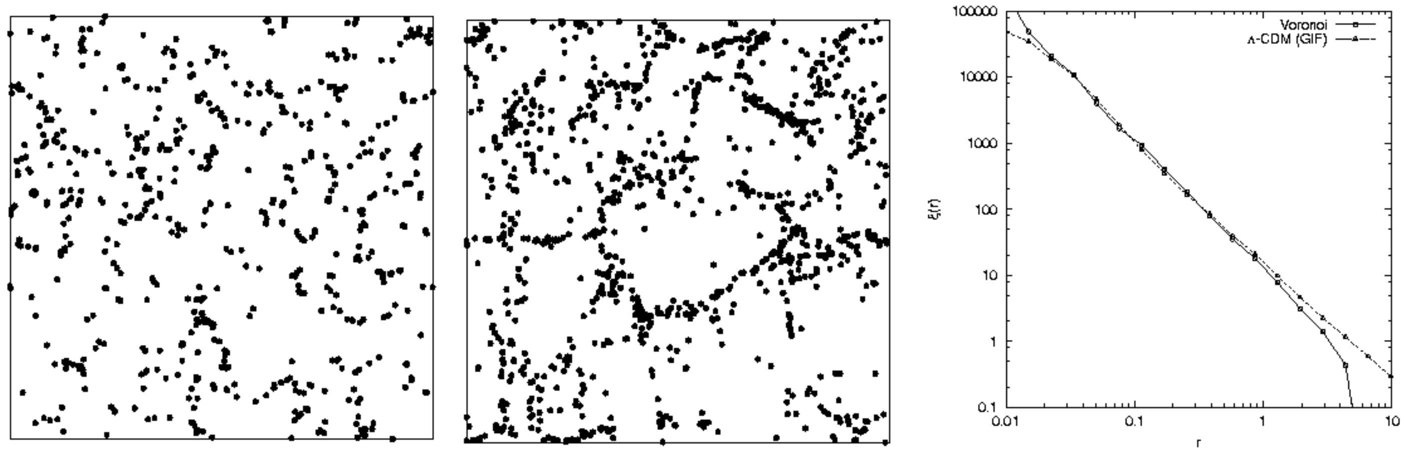


noisy plane simulation
Normalized maximum value
Noise level
Wavelet
Beamlet
Ridgelet
Poisson realisation
for a low moise level
noisy line simulation
Normalized maximum value
Noise level
Beamlet
Ridgelet
Wavelet
Poisson realisation
for a low moise level
noisy plane simulation
Normalized maximum value
Noise level
Ridgelet
Beamlet
Wavelet
Poisson realisation

Simulations

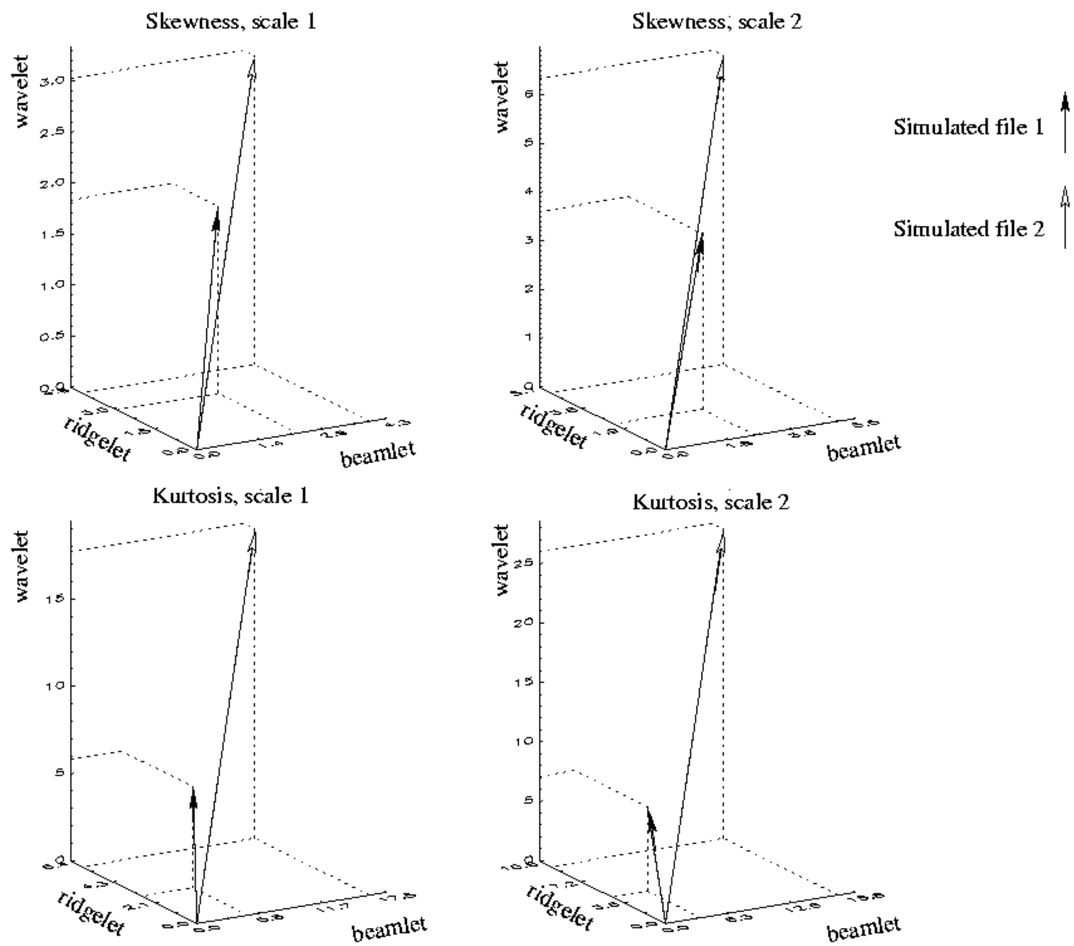






Voronoi
Λ-CDM (GIF)
ξ(r)
r

Skewness and Kurtosis

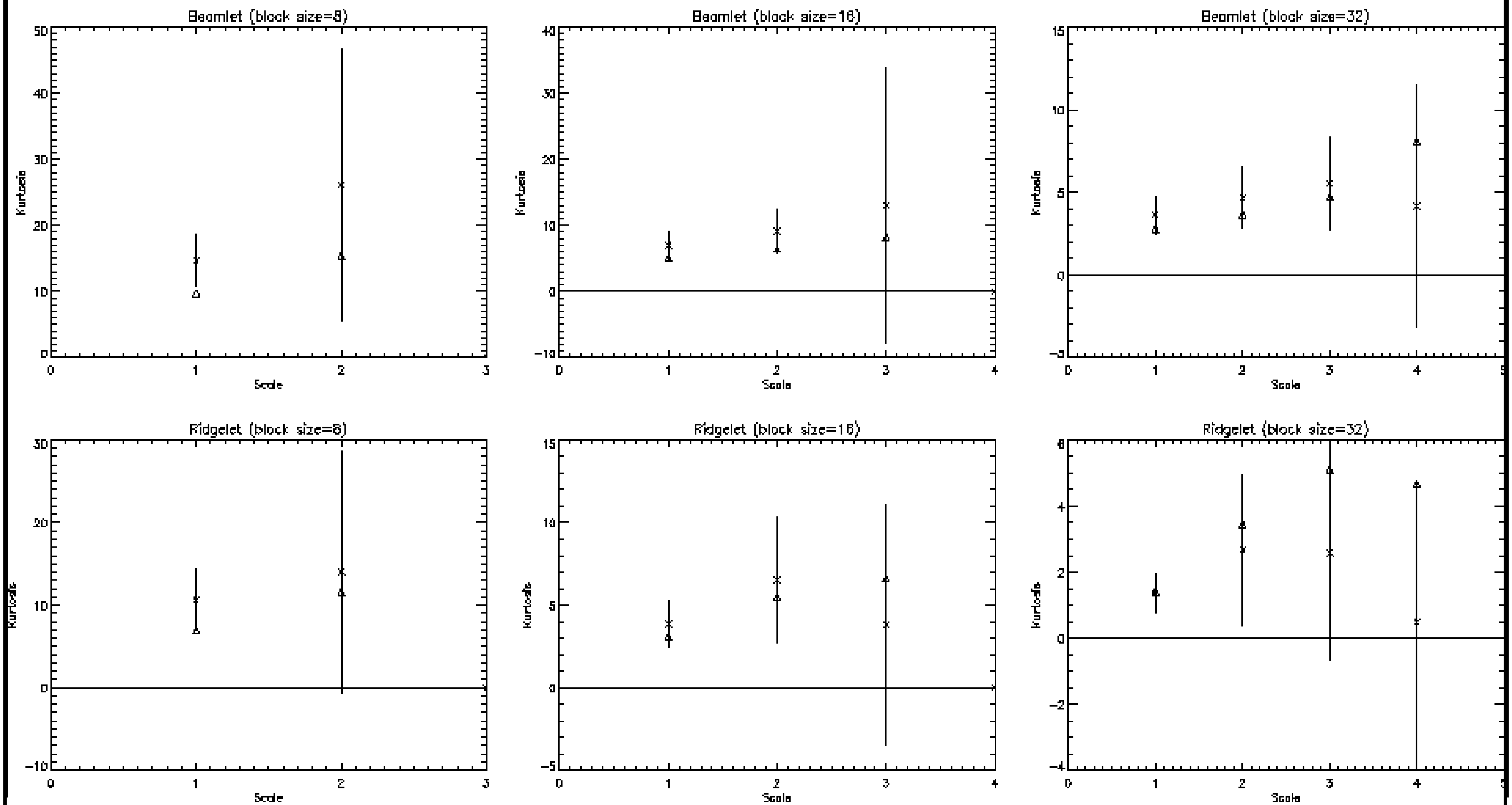


MGA and the 2DF

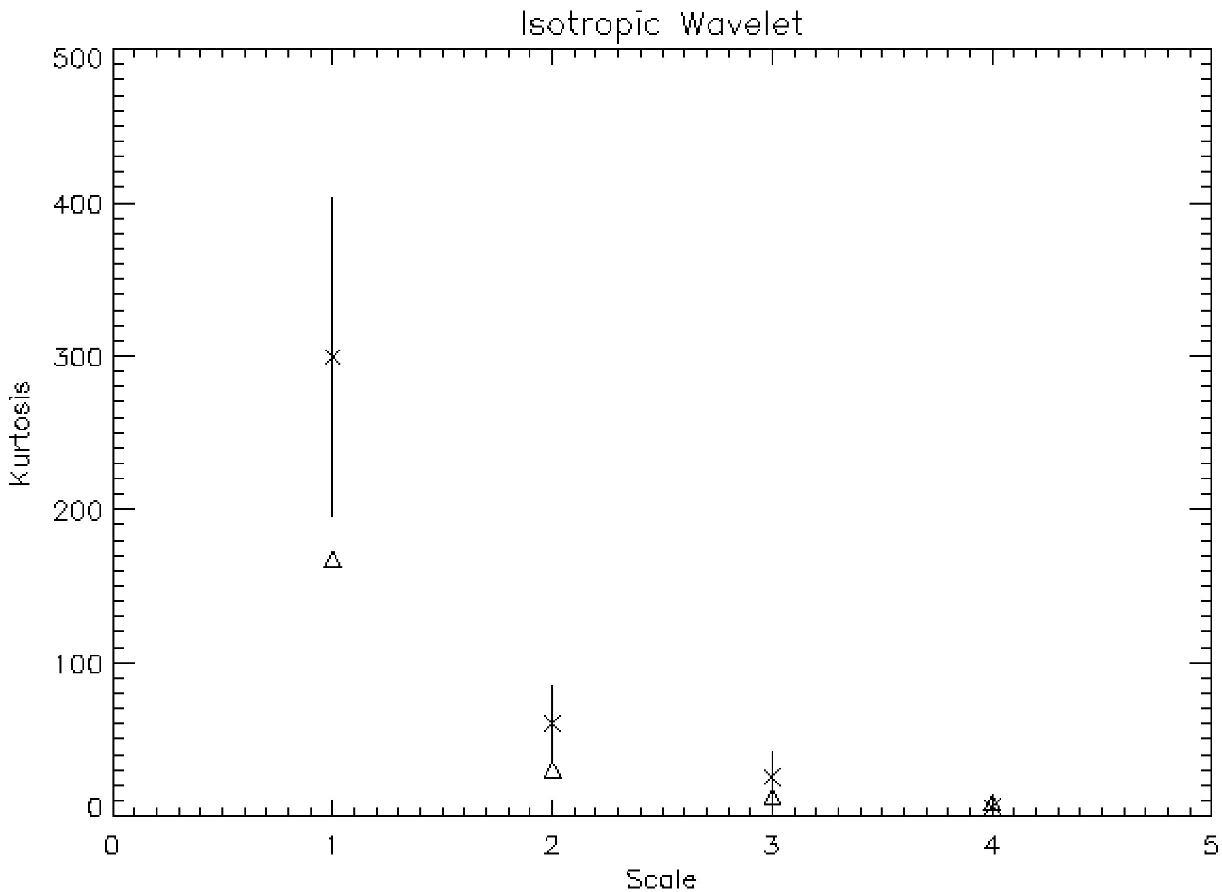
We have considered 7 transforms:

1. 3D Isotropic Wavelet Transfrom with 4 dyadic scales.
2. 3D Ridgelet Transform using a block size of 8 Mpc and two scales. Here the scale is related to the width of the ridgelet function, its length being fixed by the block size.
3. 3D Ridgelet Transform using a block size of 16 Mpc and three scales.
4. 3D Ridgelet Transform using a block size of 32 Mpc and three scales.
5. 3D Beamlet Transform using a block size of 8 Mpc and two scales. Here the scale is related to the width of the beamlet function, its length being fixed by the block size.
6. 3D Beamlet Transform using a block size of 16 Mpc and three scales.
7. 3D Beamlet Transform using a block size of 32 Mpc and three scales.

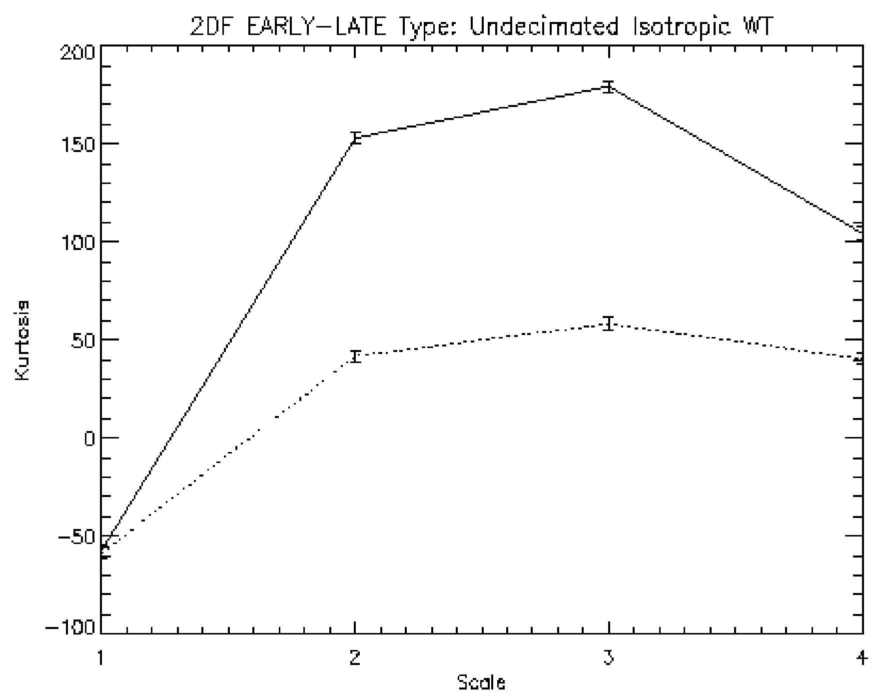
Beamlet Kurtosis and Ridgelet Kurtosis.



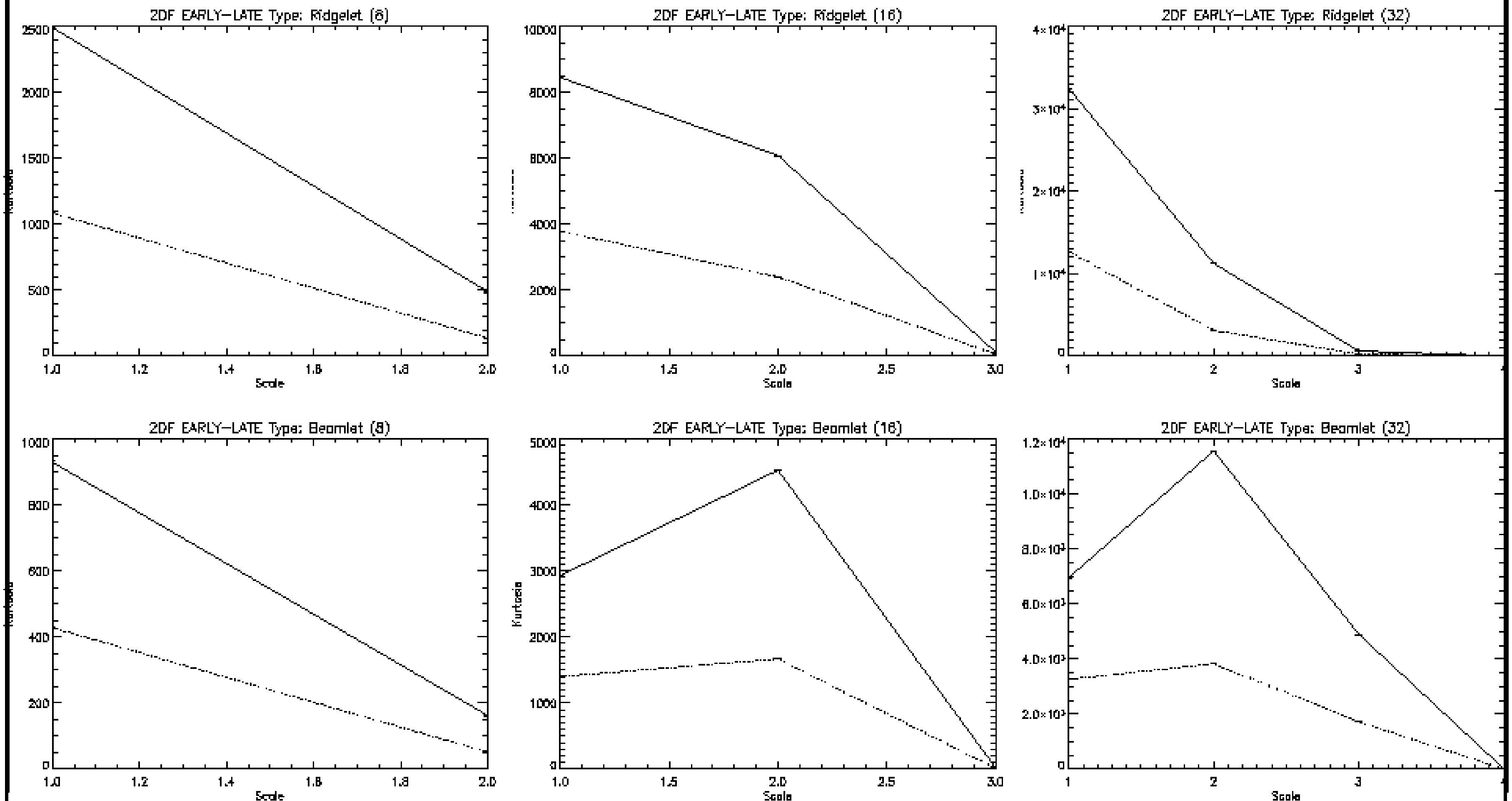
Wavelet Kurtosis.



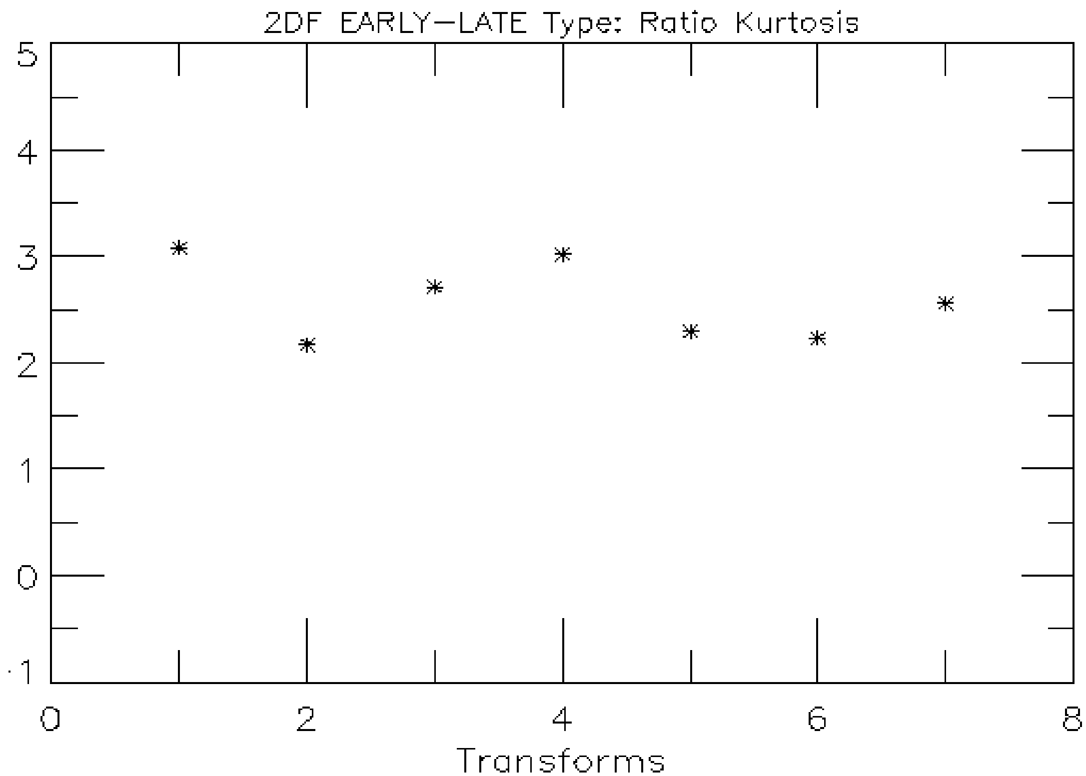
Isotropic WT Kurtosis: ETG (solid line) and LTG (dashed line).



Ridgelet and Beamlet Kurtosis: ETG (solid line) and the LTG (dashed).



Kurtosis ratio: $R_K(t) = \frac{K_{max}^{(ETG)}(t)}{K_{max}^{(LTG)}(t)}$



Conclusions

- Quantitative descriptors -being reliable, robust, unbiased, and physically interpretable- are needed to extract cosmological information from the data ==> MGA approach seems very promising.

- Using MGA on 2DF Data:

  1) The mock catalogs are NOT compatible with the data.

  2) We do not see any tendancy toward homogenity up to the scale of 32 Mpc.

  3) Early type galaxies are more clustered than late type galaxies in filaments, walls, and clusters in a similar way.

If you want to know more...