Sparsity and the Cosmic Microwave Background

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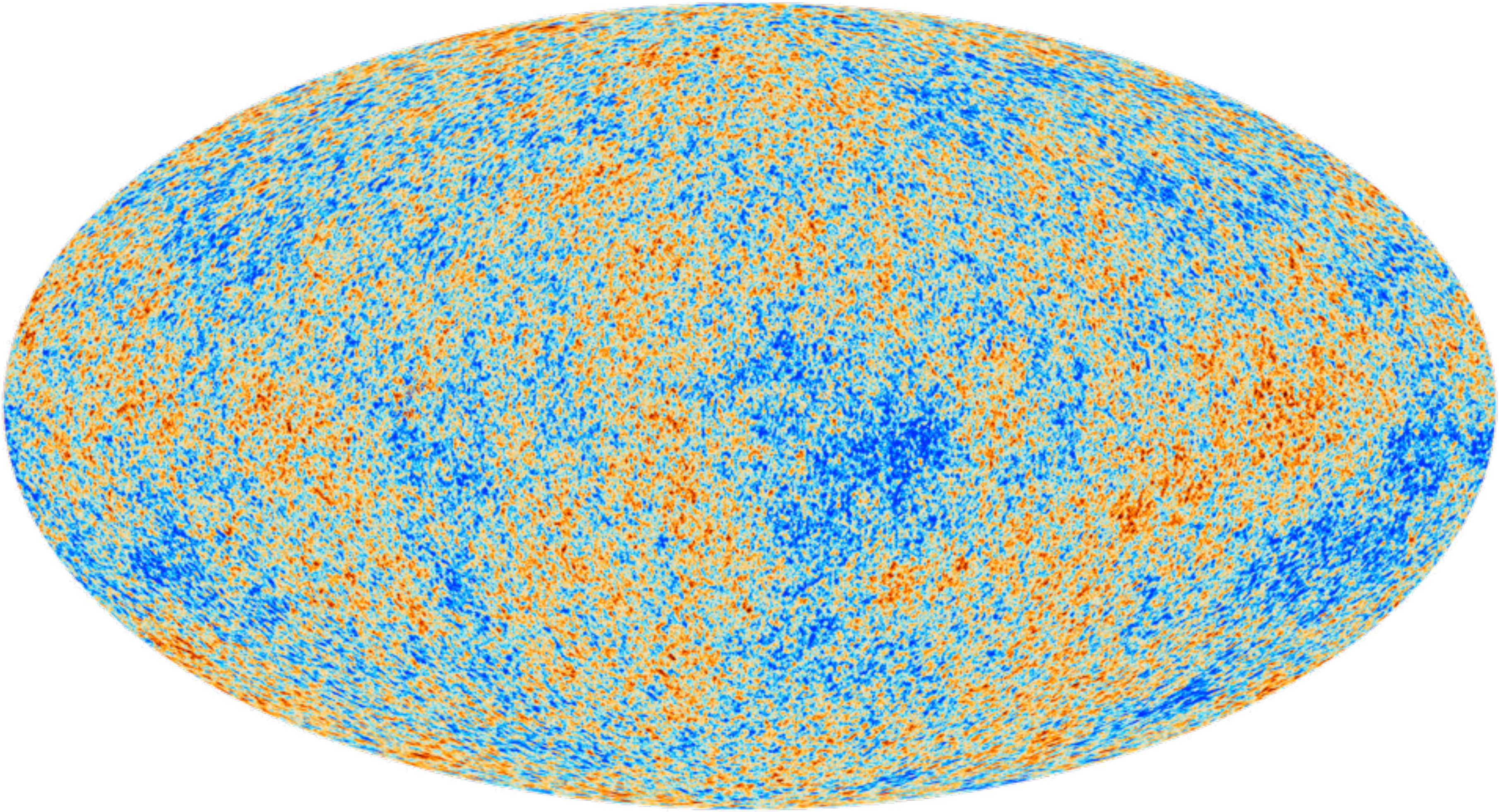
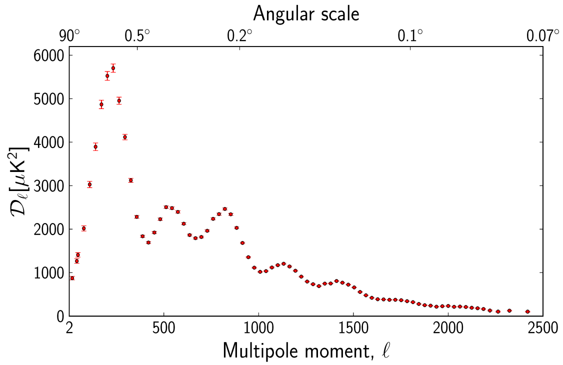
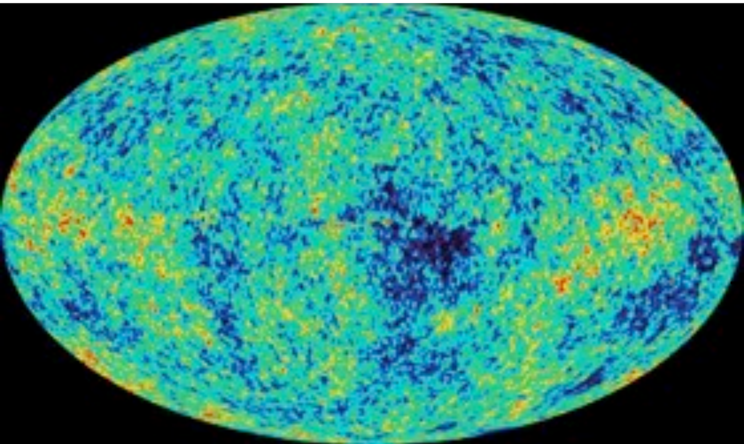
PLANCK CMB MAP



Statistical Properties of the CMB fluctuation

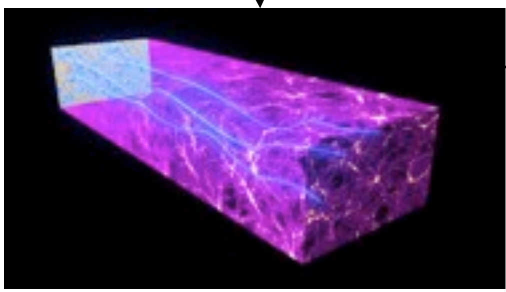




Cosmological Parameters

Search for specific signatures predicted by inflation models

constraints on inflation models (fnl)



Statistical analysis of the weak lensing effect

gravitationnal potential mapping

+

power spectrum

Large scale analysis

Topology of the univers, inflation, ISW, etc

Integrated Sachs-Wolfe Effect (ISW)

Constraint on dark energy

INVERSE PROBLEMS AND SPARSE RECOVERY

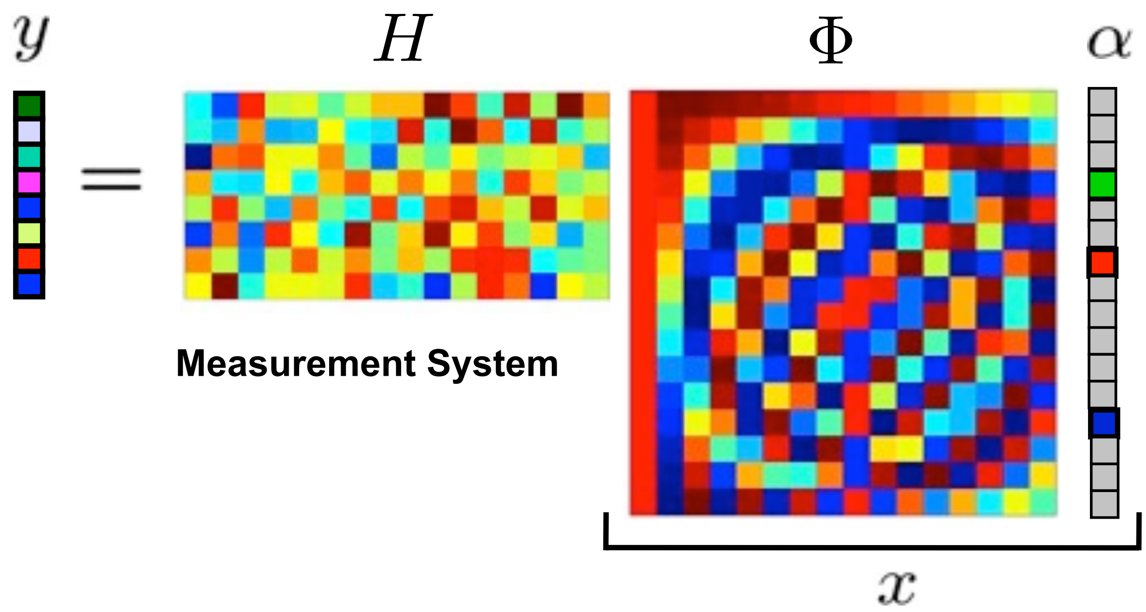
$$Y = HX + N$$

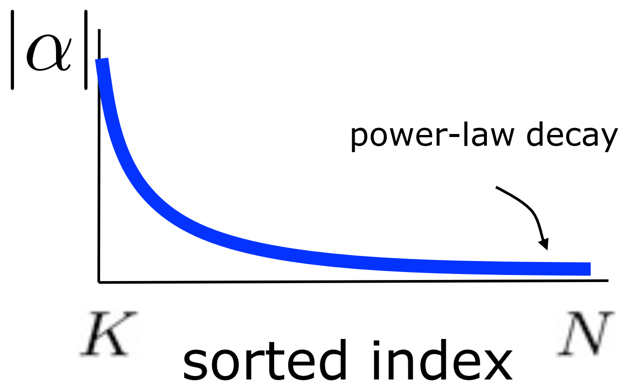
$X = \Phi\alpha$, and α is sparse

- **Denoising**
- **Deconvolution**
- **Component Separation**
- **Inpainting**
- **Blind Source Separation**
- **Minimization algorithms**
- **Compressed Sensing**

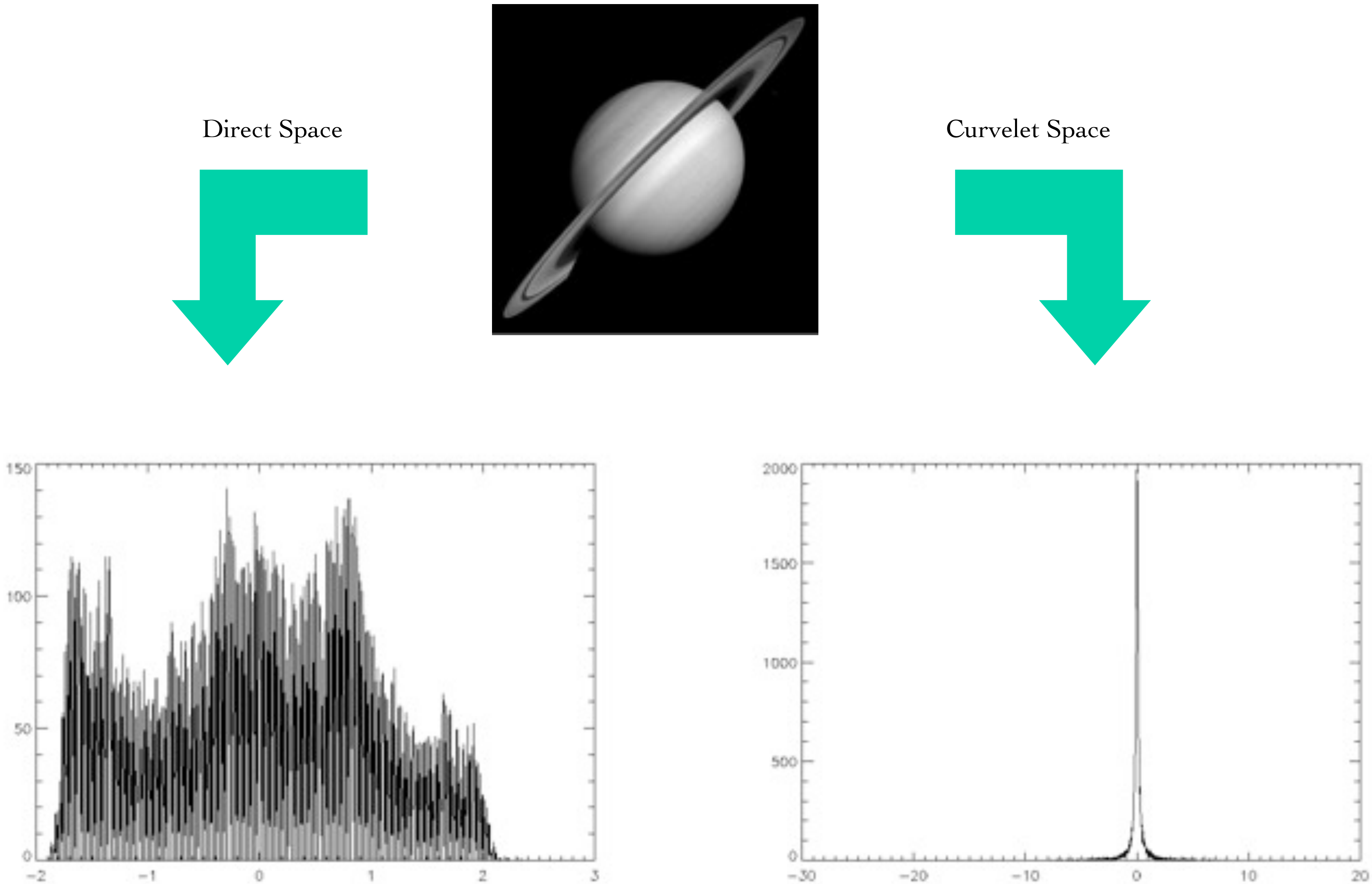
$$\min_{\alpha} \|\alpha\|_p^p \quad \text{subject to} \quad \|Y - A\Phi\alpha\|^2 \leq \epsilon$$

Very efficient recent methods now exist to solve it (proximal theory)

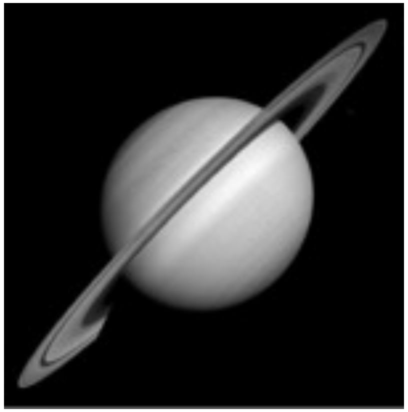




Weak Sparsity or Compressible Signals

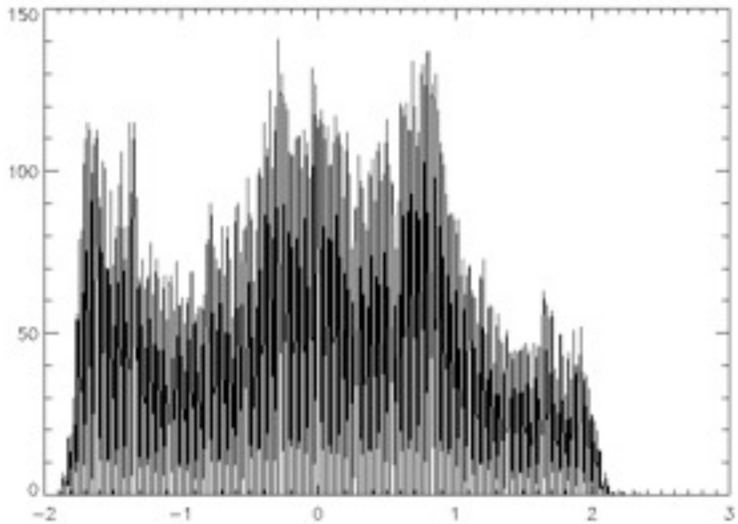


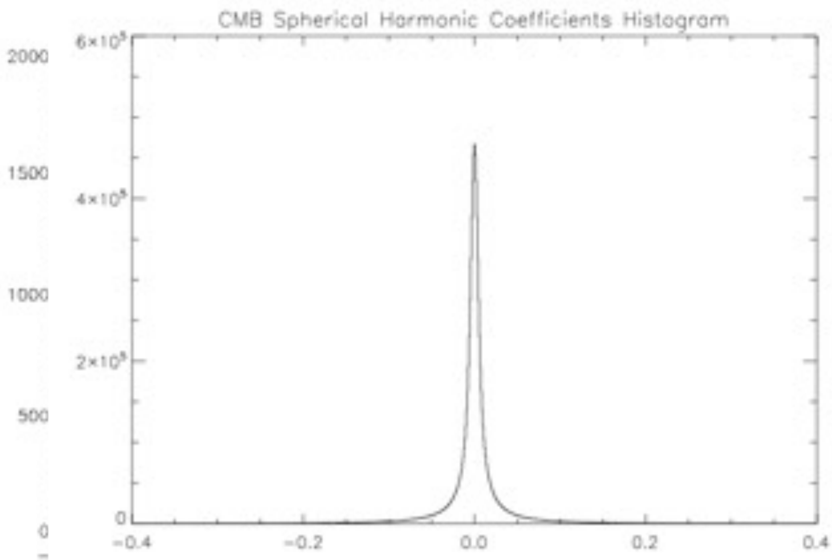
Weak Sparsity or Compressible Signals



Direct Space

Curvelet Space





Weak Sparsity or Compressible Signals

A signal *s* (*n* samples) can be represented as sum of weighted elements of a given dictionary

Dictionary (basis, frame)

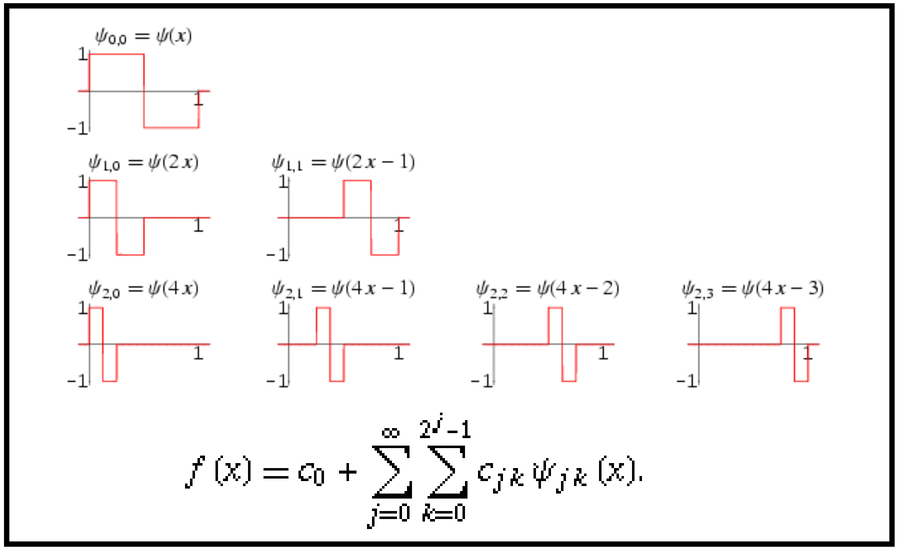
$$\Phi = \{\phi_1, \ldots, \phi_K\}$$

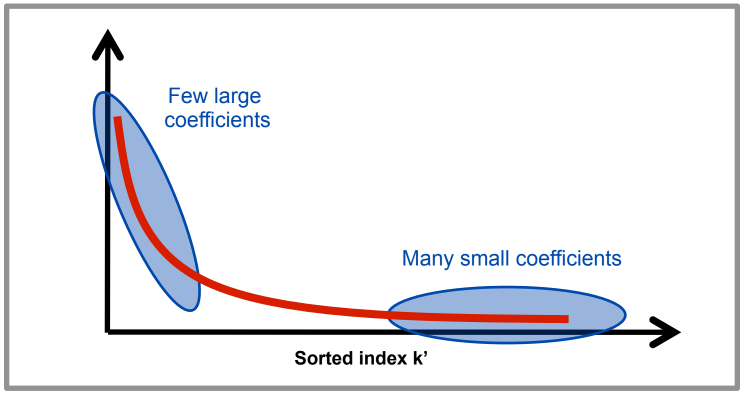
Atoms

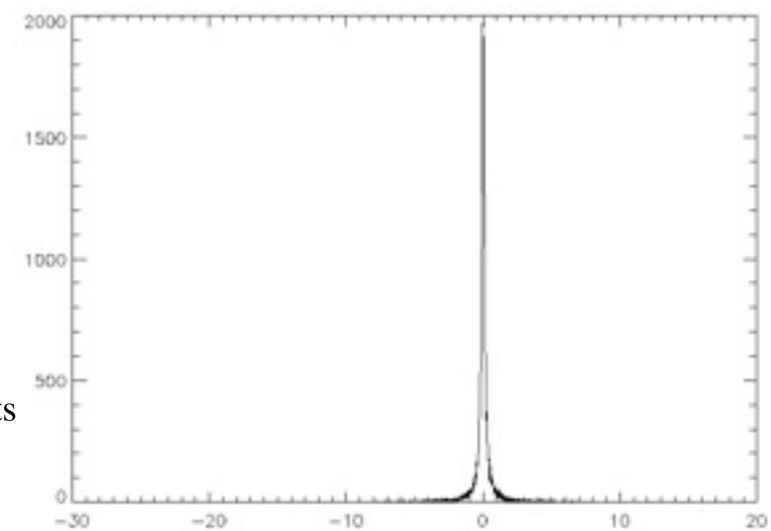
$$s = \sum_{k=1}^{K} \alpha_k \phi_k = \Phi\alpha$$

coefficients

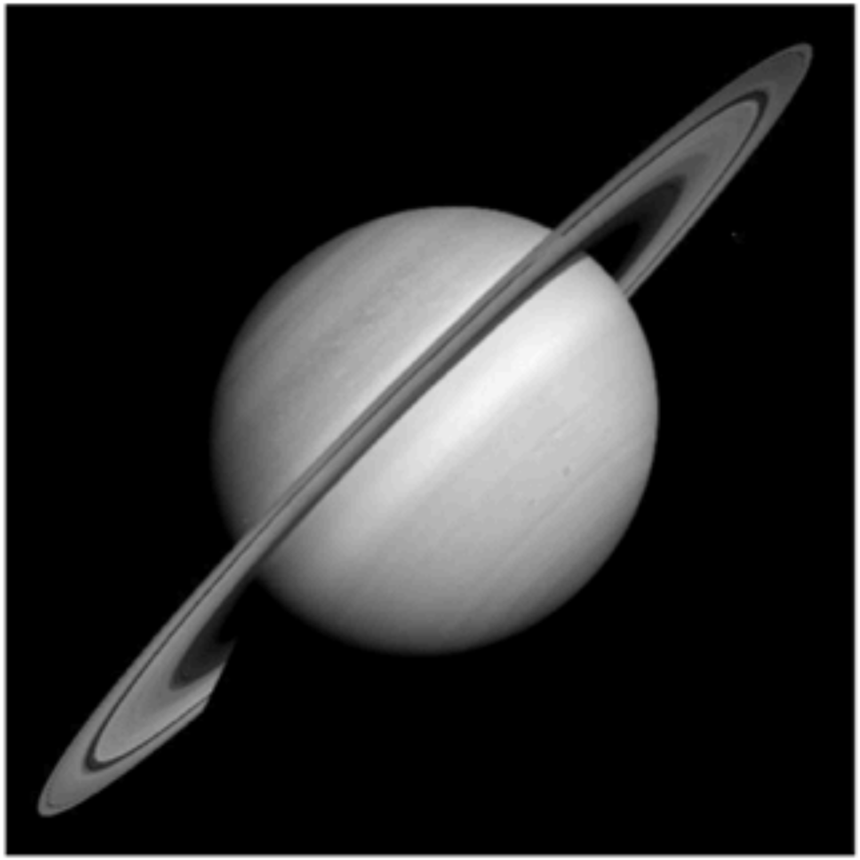
Ex: Haar wavelet







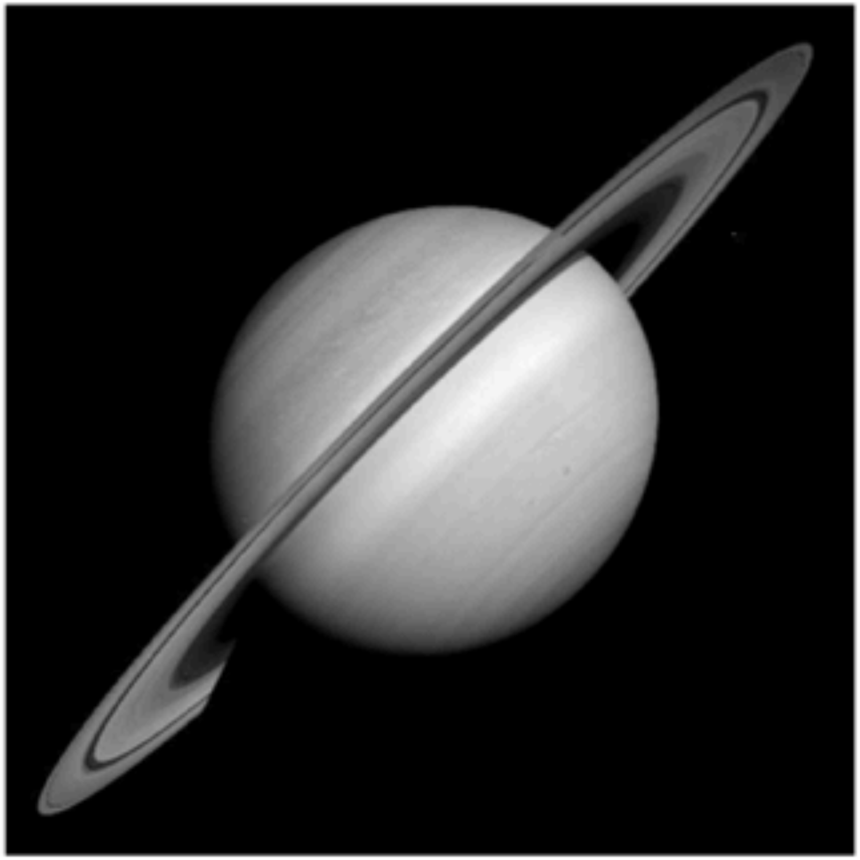
- Fast calculation of the coefficients
- Analyze the signal through the statistical properties of the coefficients
- Approximation theory uses the sparsity of the coefficients

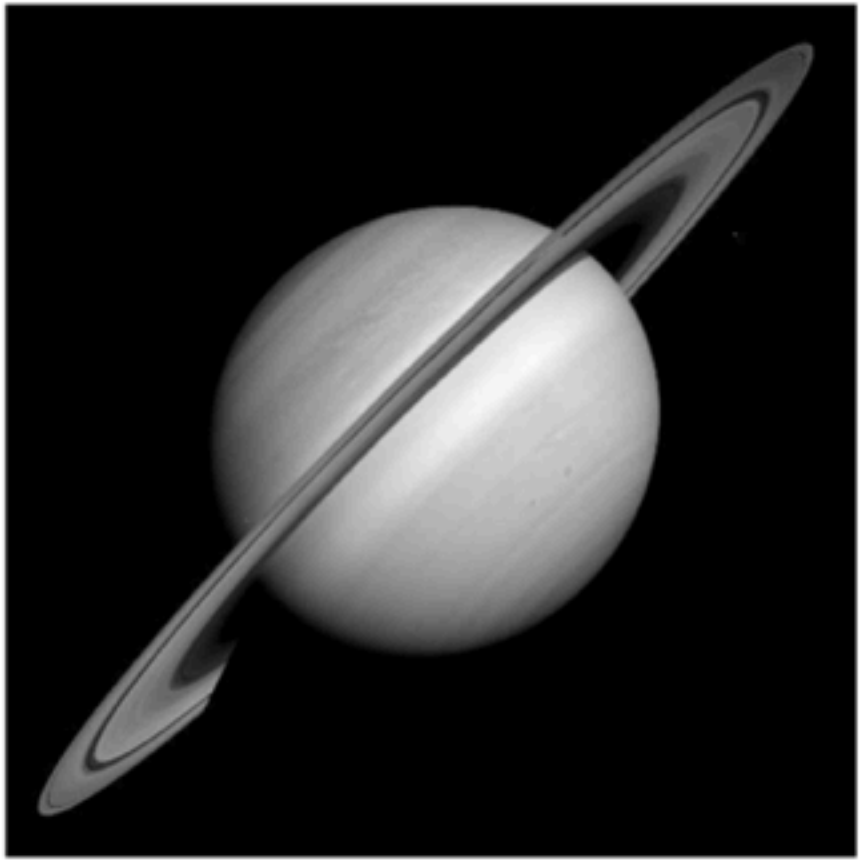


1% largest coefficients in wavelet space
(the others are set to 0)

The wavelet coefficients encode edges and large scale information.

Wavelet transform

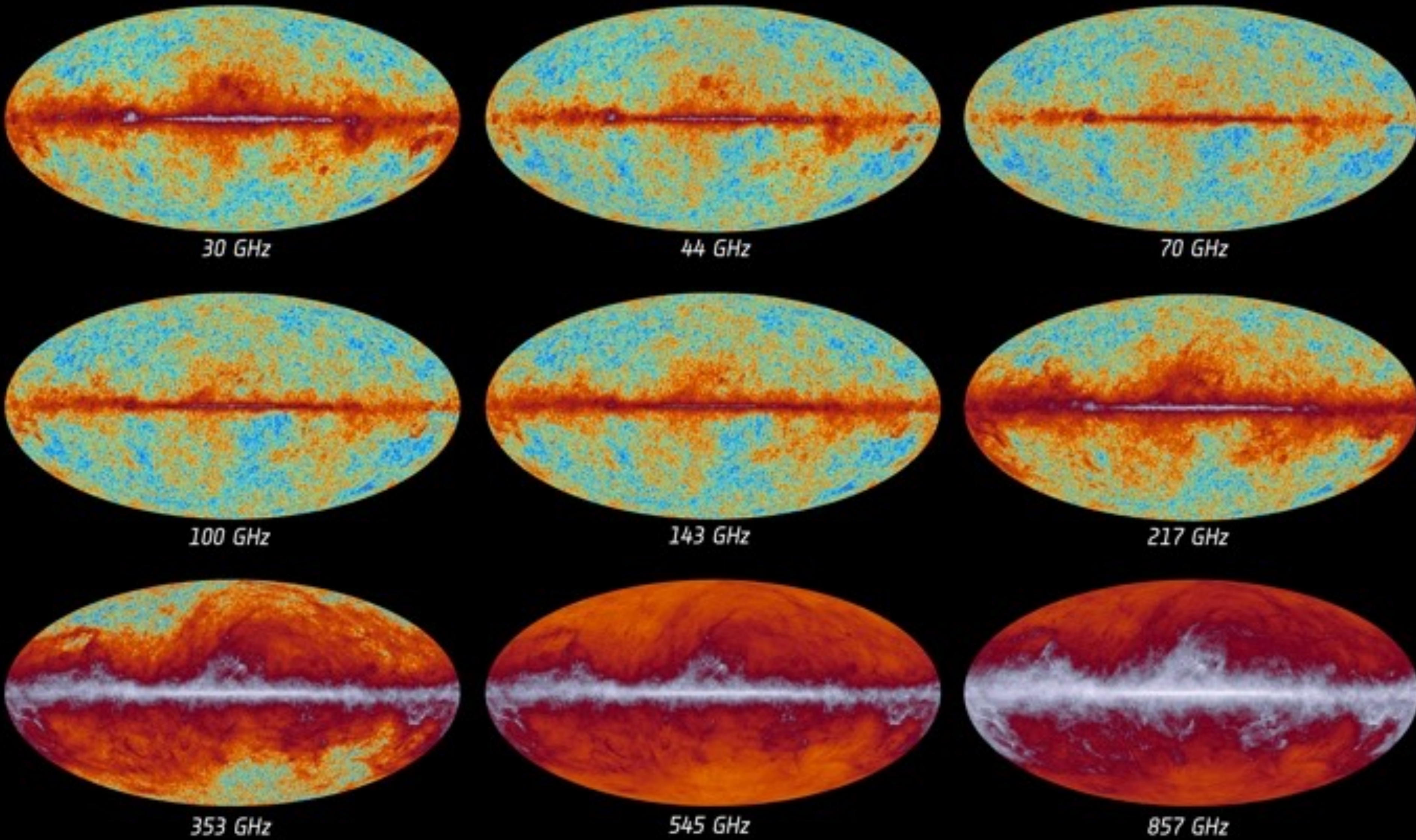


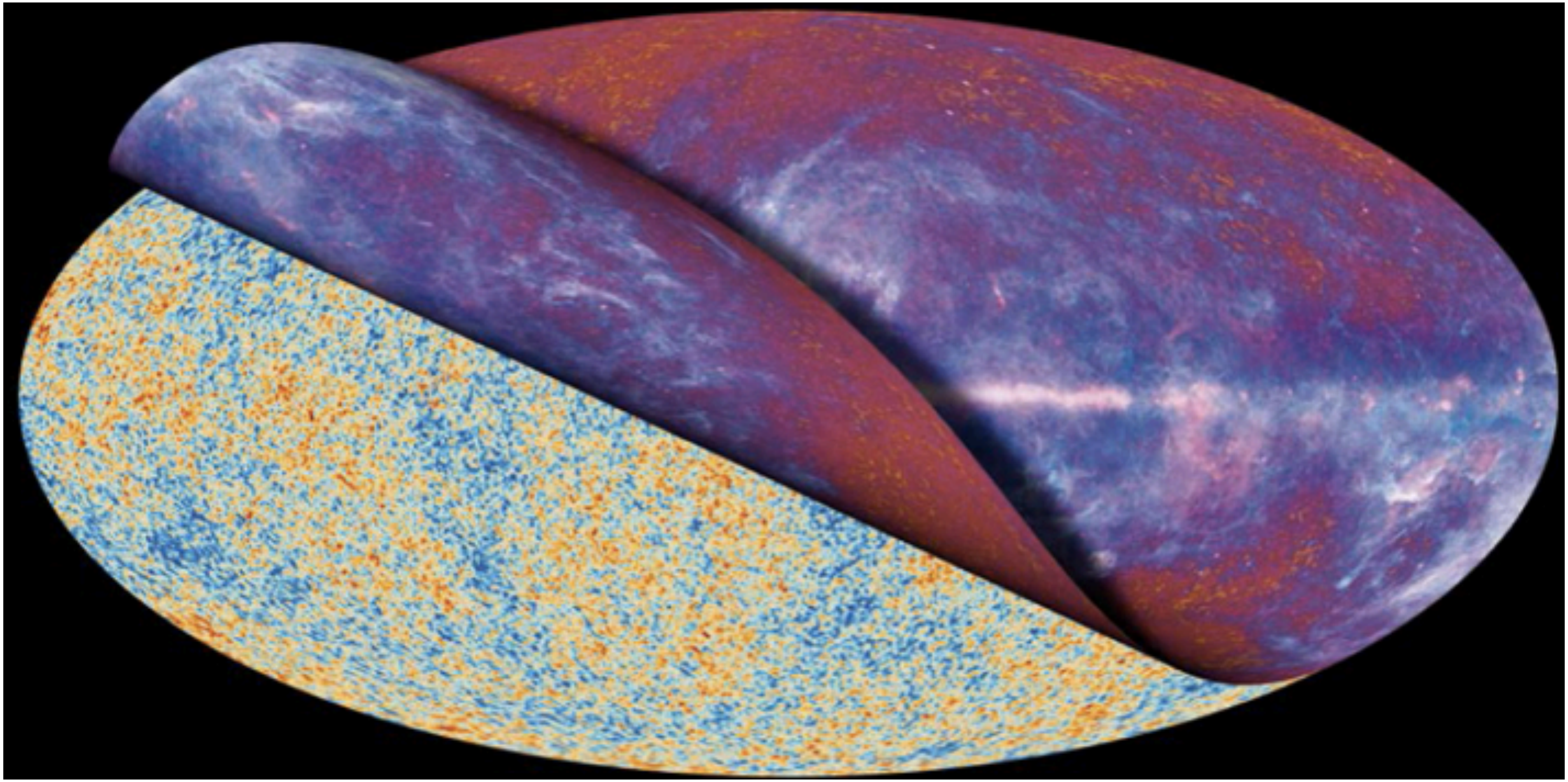


1% of the wavelet coefficients concentrate 99.96% of the energy: This can be used as a *prior.*

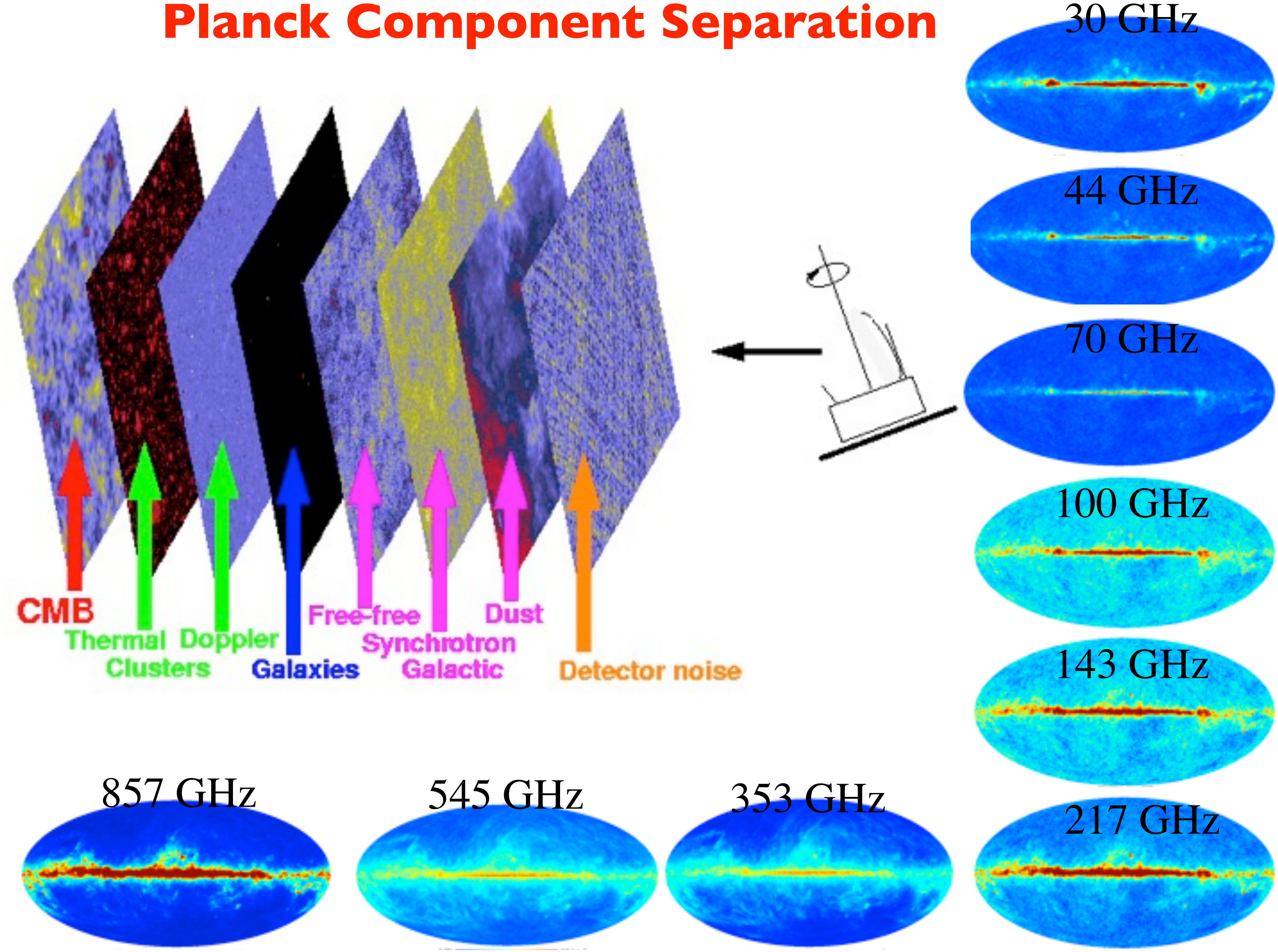
Reconstruction, after throwing away 99% of the wavelet coefficients planck

The sky as seen by Planck





Planck Component Separation



Component Separation: more problems

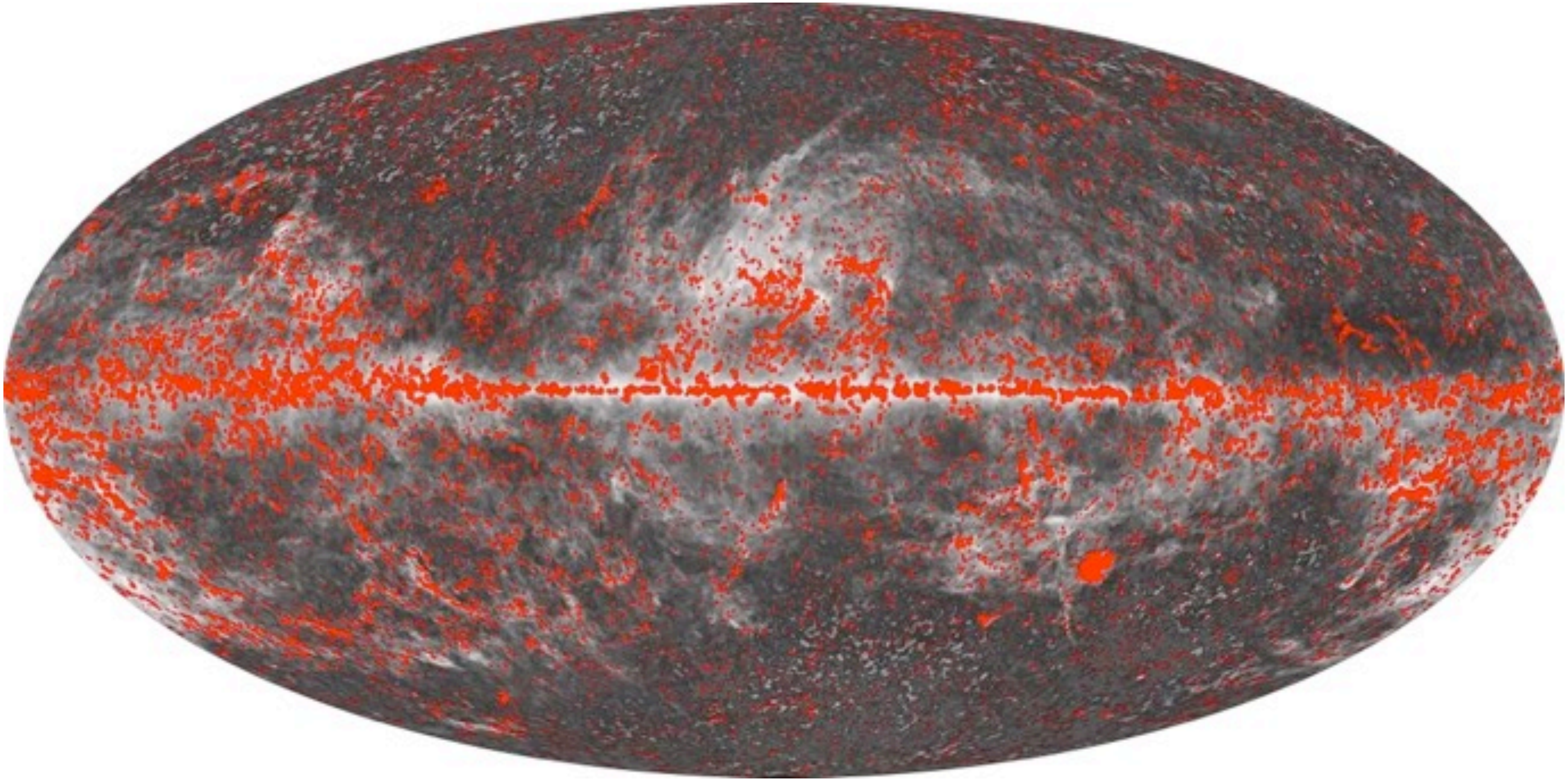
The beam: $$\forall i;\ x_i = b_i \star \left(\sum_j a_{ij} s_j\right) + n_i$$

Globally: $$\mathbf{X} = \mathcal{H}\left(\mathbf{AS}\right) + \mathbf{N}$$

where $\mathcal{H}$ is the multichannel convolution operator

Spectral behavior varies spatially for some components (dust, synchroton):

Detected Compact Sources in Planck

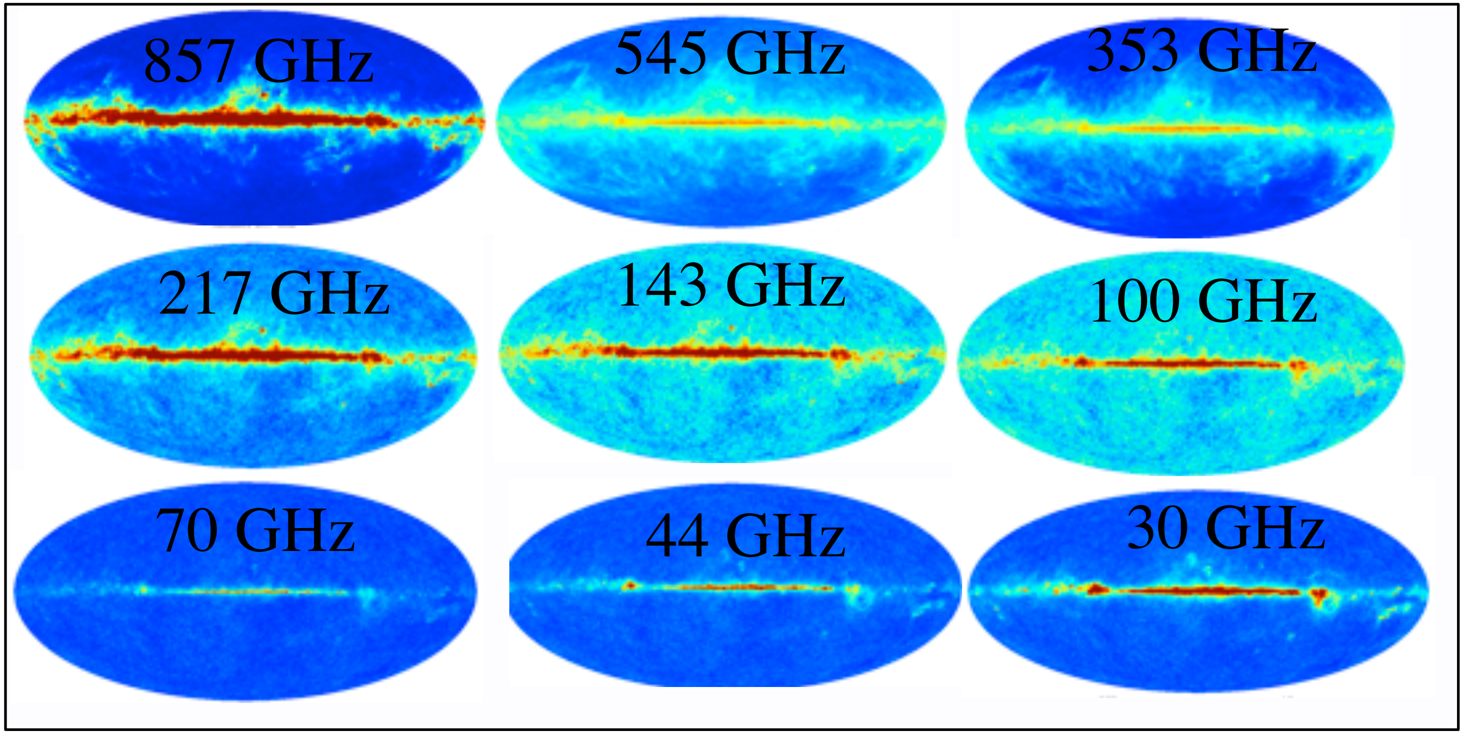


Component Separation

Sky emission

Y = A X

Instrumental effects



Component Separation Pipeline

- Point sources processing: Mask+[inpainting] or fitting.
  Mask: Commander, Sevem
  Fitting: NILC, SMICA

- Resolution: 1) Downgrade the frequency maps at the same resolution
  Commander: 40amin
  Sevem: 10 and 7 acmin
  2) Deconvolution to 5acmin: SMICA-NILC

- Choice of channels: Commander (30-353GHz), NILC (44-857GHz), Sevem and SMICA (30-857GHz).

- Separation principle

- Full sky modelling (Commander): MODEL with 4 components: CMB, low-frequency emission, CO emission and thermal dust emission.
- Template fitting (Sevem) in two regions: Clean the 100 and 143 Ghz map by:

$$T_c(\boldsymbol{x}, \nu) = d(\boldsymbol{x}, \nu) - \sum_{j=1}^{n_t} \alpha_j t_j(\boldsymbol{x}),$$

where templates are difference maps (30−44), (44−70), (545−353) and (857−545).

Component Separation

- Separation principle

- Internal Linear Combination (**ILC**), used by WMAP :

  - CMB spectrum is assumed to be known: a

  - Modelling: $$X = as + R$$

Solution ILC : $$\hat{s} = \text{Argmin}_s \left(X - as\right) R_X^{-1} \left(X - as\right)^T$$

$$\hat{s} = \frac{1}{a^T R_X^{-1} a} a^T R_X^{-1} X$$

Nilc = ILC in the wavelet domain
one ILC per wavelet scale and per region. No localization at the coarsest scales and uo to 20 regions at the finest scale.
Smica = ILC in spherical harmonic domain
+ modeling of the covariance matrix at low l,(l < 1500)

Component Separation

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Well known in statistics as the BLUE (Best Linear Unbiased Estimator) method.

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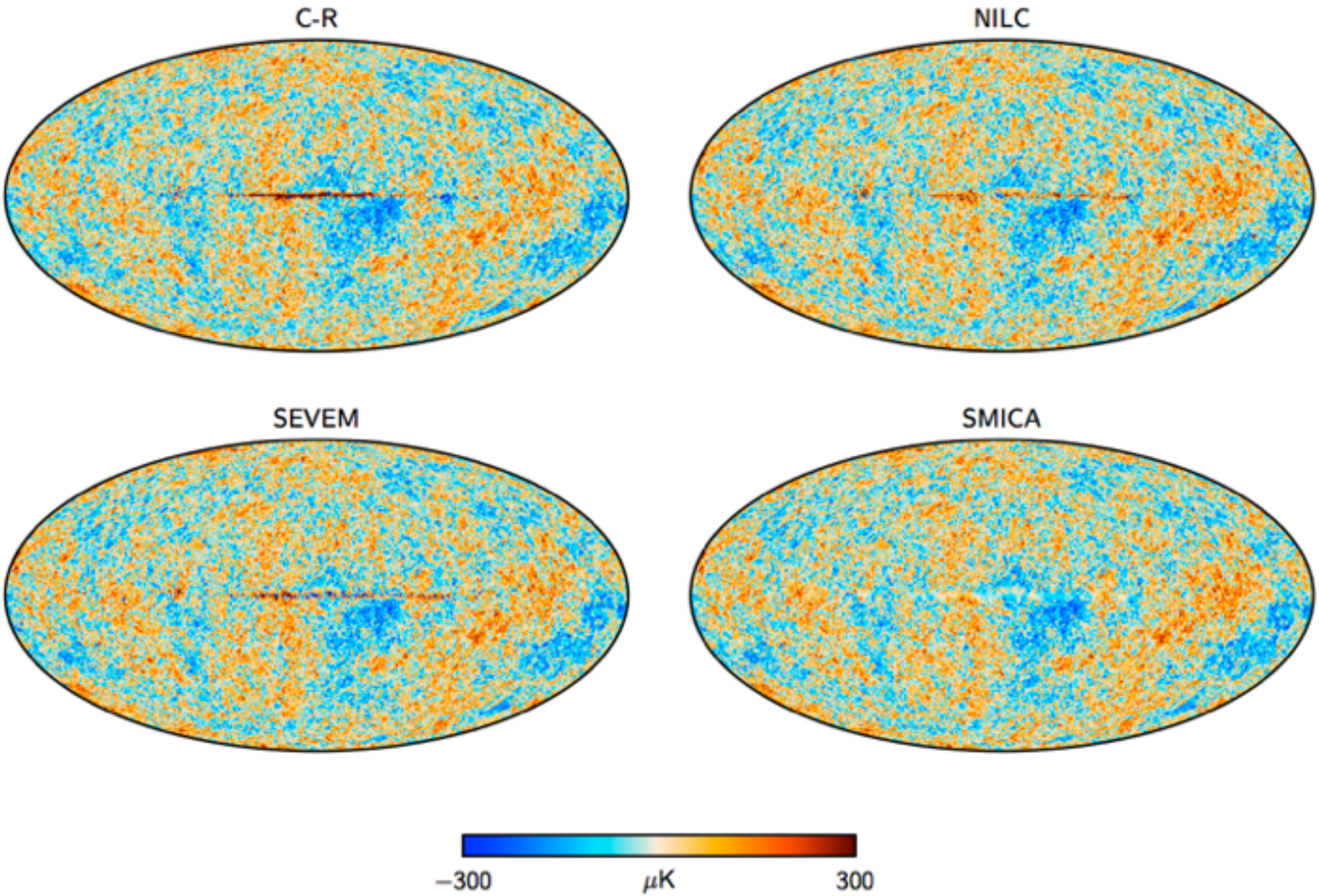
Smica = ILC in spherical harmonic domain
+ modeling of the covariance matrix at low l,(l < 1500)

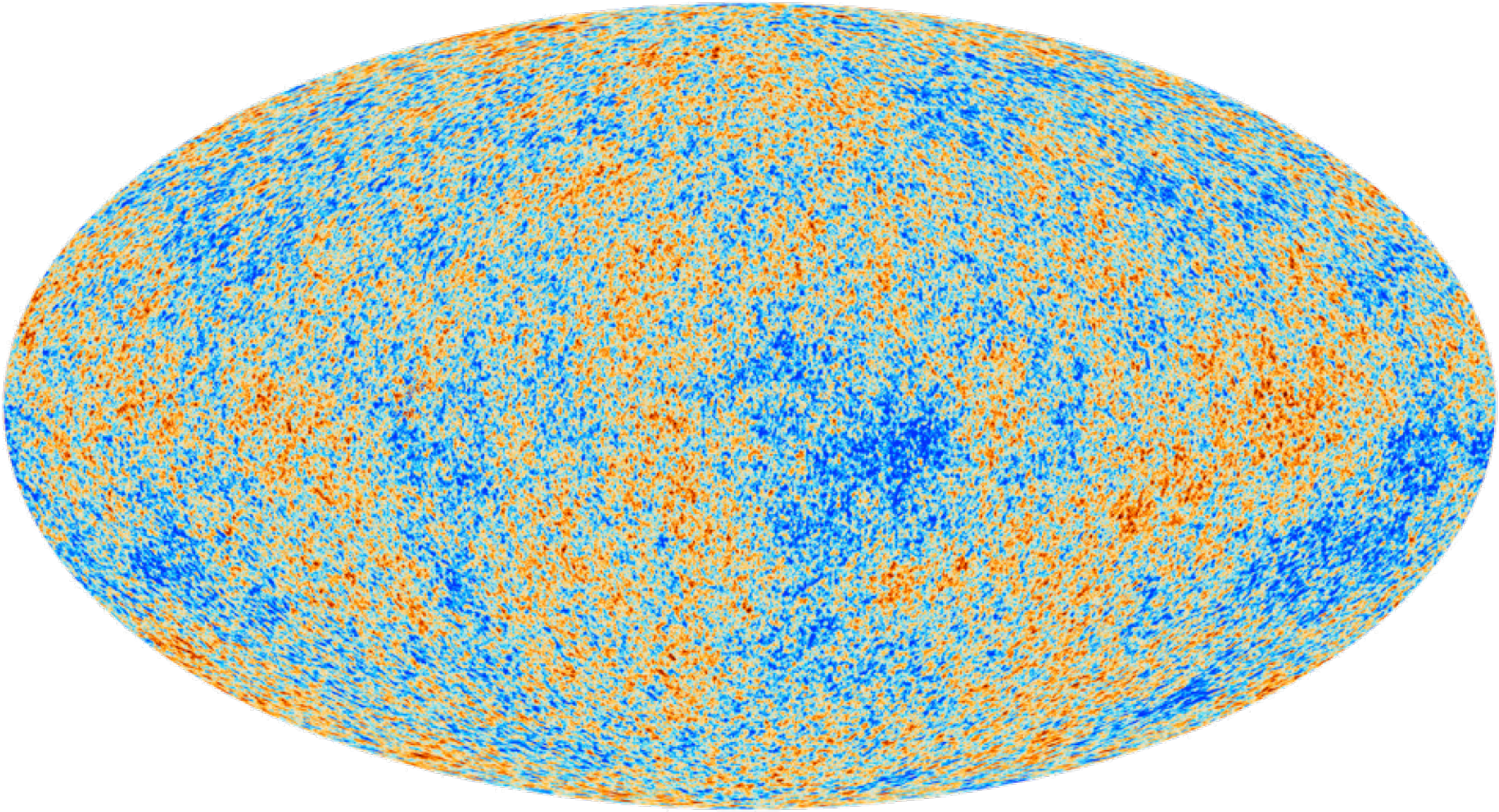
Commander-Ruler, Sevem, NILC, Smica

Planck Collaboration: *Planck* 2013 results. XII. Component separation



INPAINTING

Constraint Realization Inpainting

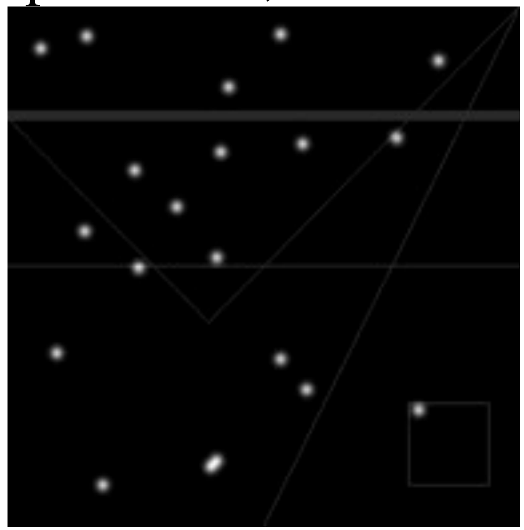


Sparsity & Morphological Diversity
Morphological Component Analysis (MCA)

•J.-L. Starck, M. Elad, and D.L. Donoho, Redundant Multiscale Transforms and their Application for Morphological Component Analysis, Advances in Imaging and Electron Physics, 132, 2004.

•J.-L. Starck, M. Elad, and D.L. Donoho, Image Decomposition Via the Combination of Sparse Representation and a Variational Approach, IEEE Trans. on Image Proces., 14, 10, pp 1570--1582, 2005.

Sparsity Model: we consider a signal as a sum of K components s_k, each of them being sparse in a given dictionary :



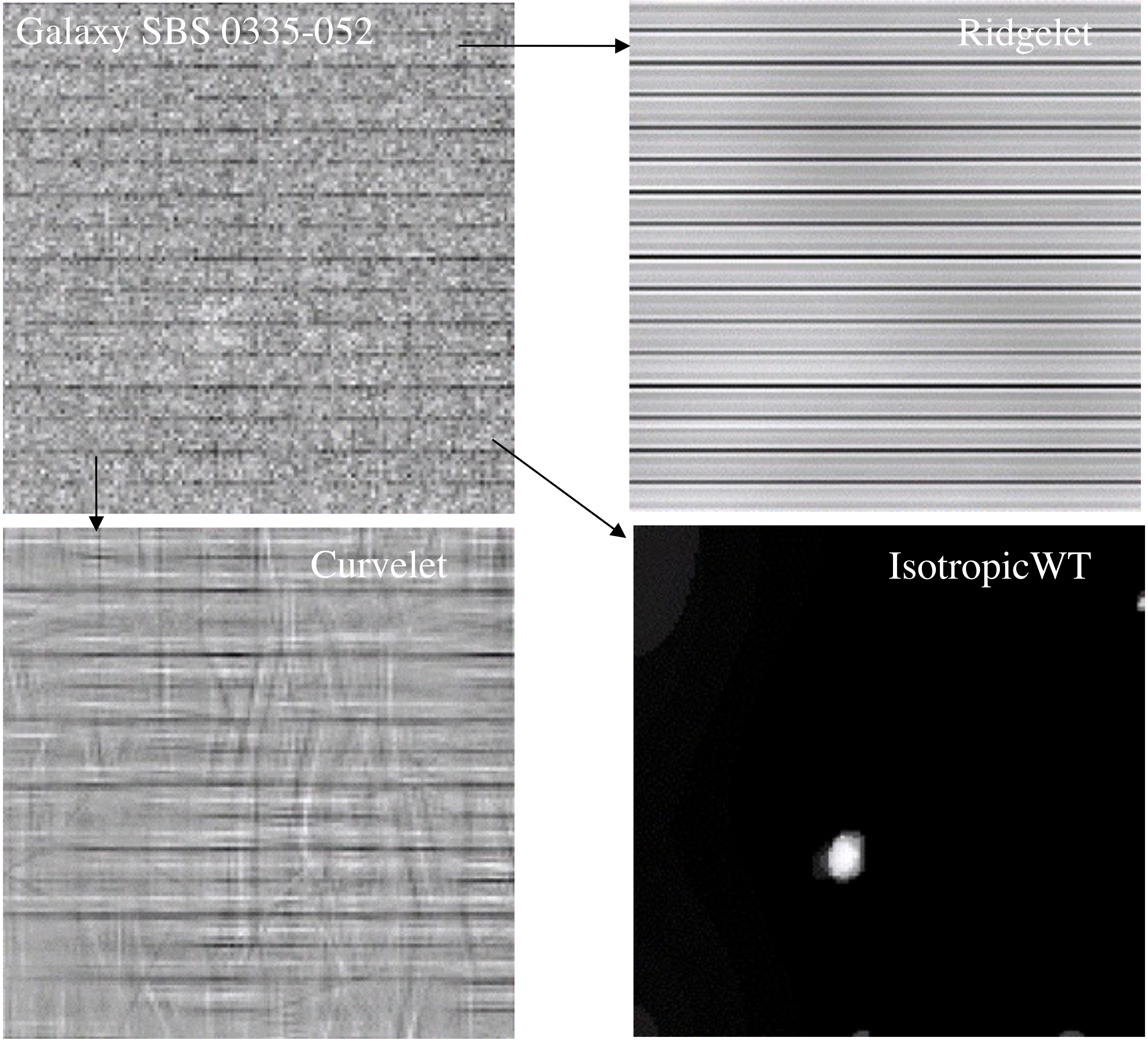
$$Y = X_1 + X_2$$

X_1 can be well approximated with few coefficients in a given domain.

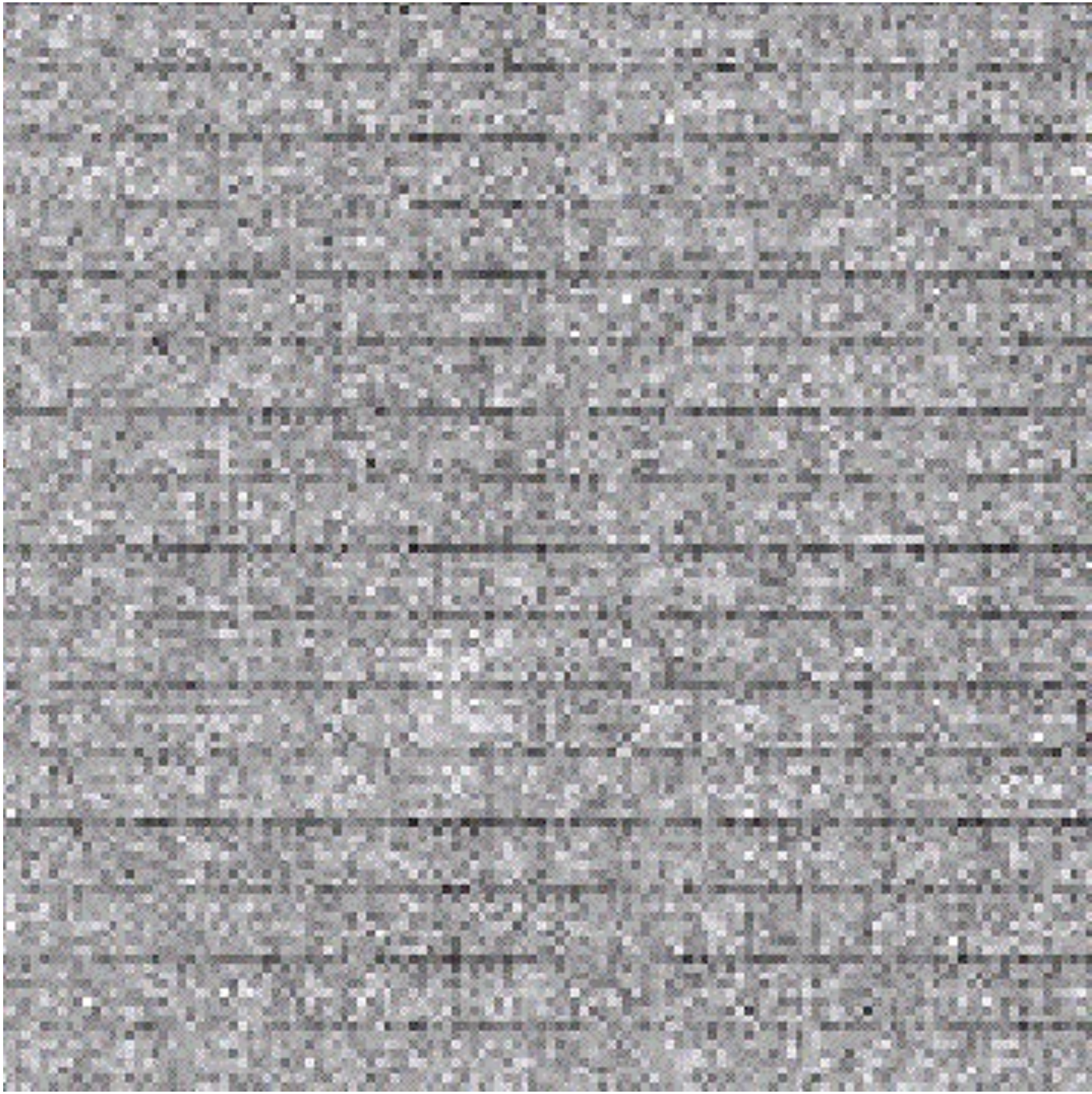
X_2 can be well approximated with few coefficients in **another** domain.

$$min_{X_1, X_2} \parallel Y - (X_1 + X_2) \parallel^2 + C_1(X_1) + C_2(X_2)$$

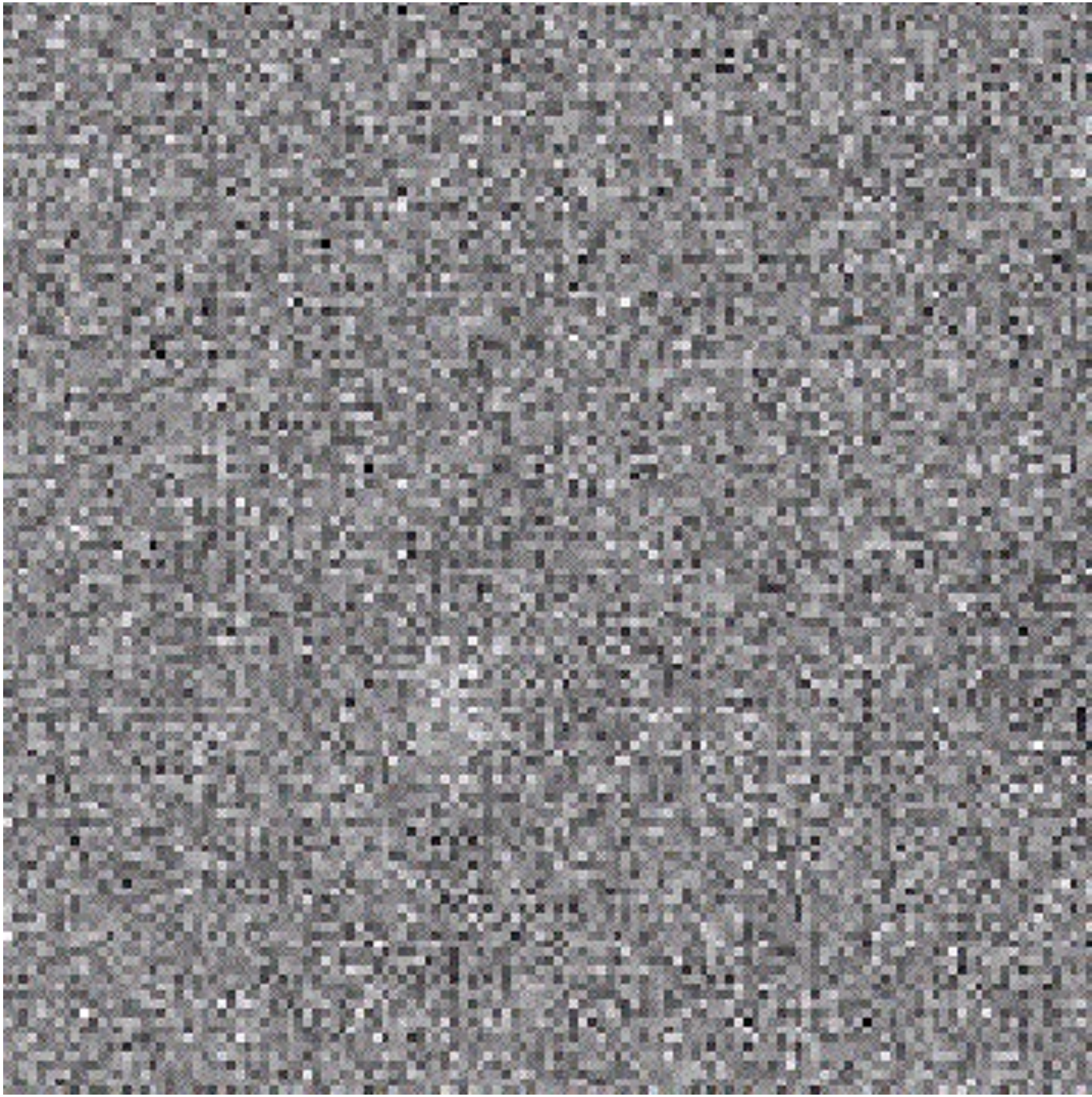
$$C_1(X_1) = \parallel \Phi_1 X_1 \parallel_1 \qquad C_2(X_2) = \parallel \Phi_2 X_2 \parallel_1$$


Galaxy SBS 0335-052
Ridgelet
Curvelet
IsotropicWT

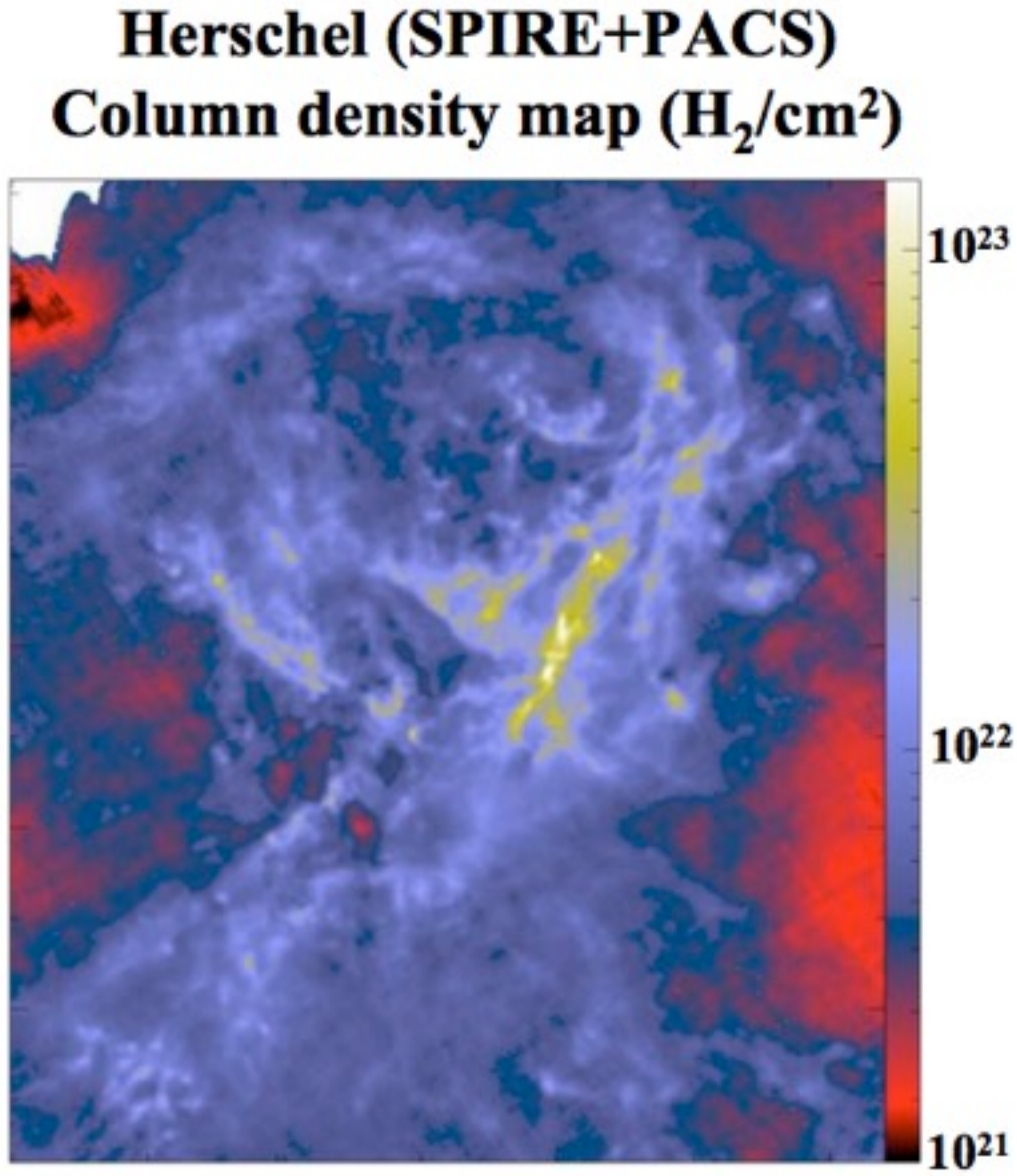
Galaxy SBS 0335-052
10 micron
GEMINI-OSCIR



Galaxy SBS 0335-052
10 micron
GEMINI-OSCIR



Revealing the structure of one of the nearest infrared dark clouds (Aquila Main: d ~ 260 pc)



Dense cores form primarily in filaments

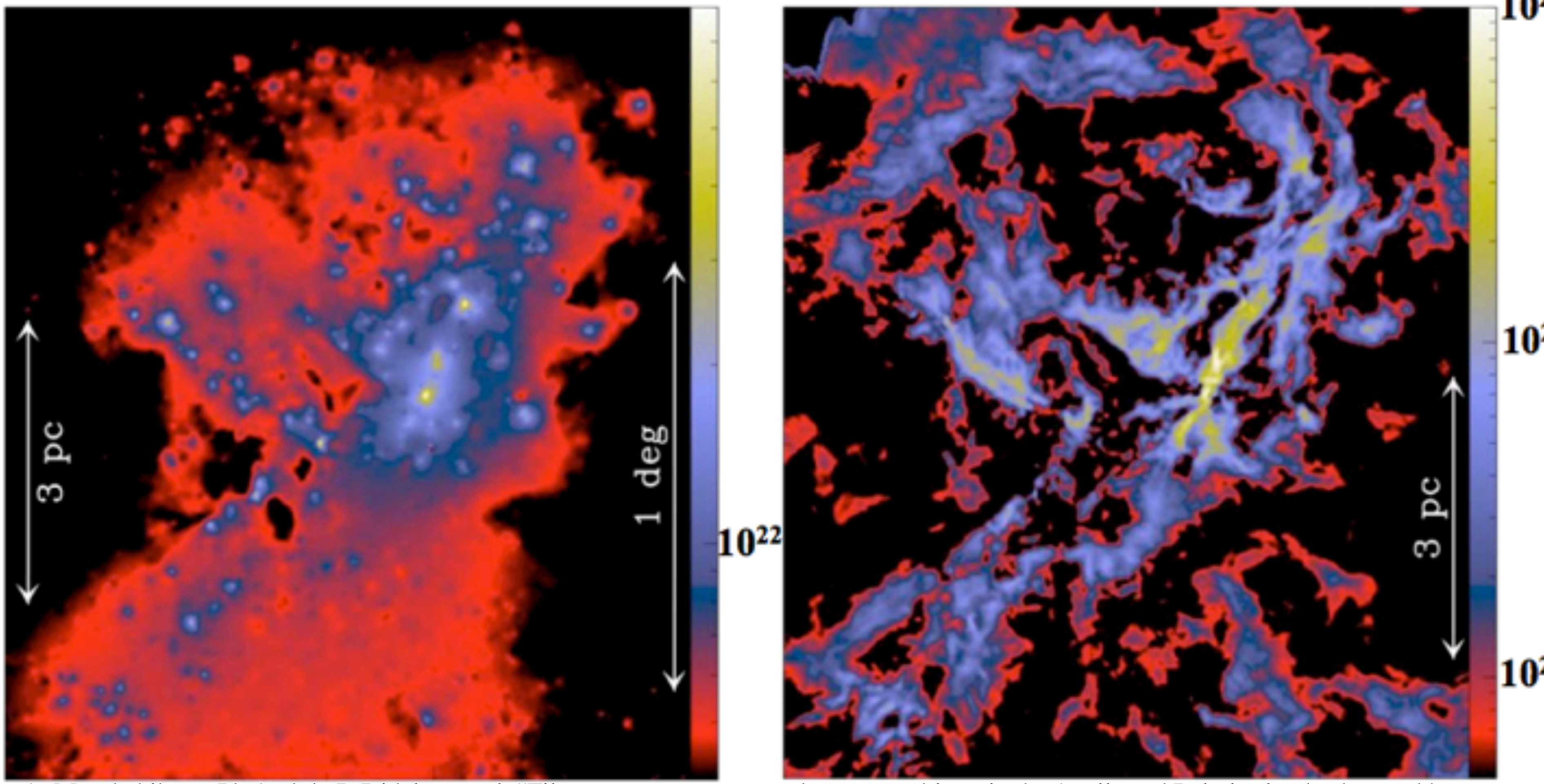
Morphological Component Analysis:

***Herschel* Column density map** (P. Didelon based on Starck et al. 2003)

=

Cores / Filaments

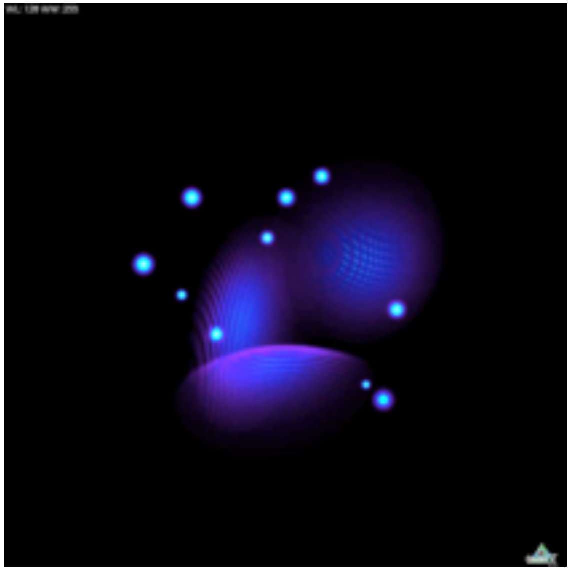
Wavelet component (H_2/cm^2) + Curvelet component (H_2/cm^2)



A. Menshchikov, Ph.André. P. Didelon, et al, "Filamentary structures and compact objects in the Aquila and Polaris clouds observed by Herschel", A&A, 518, id.L103, 2010.

3D MCA

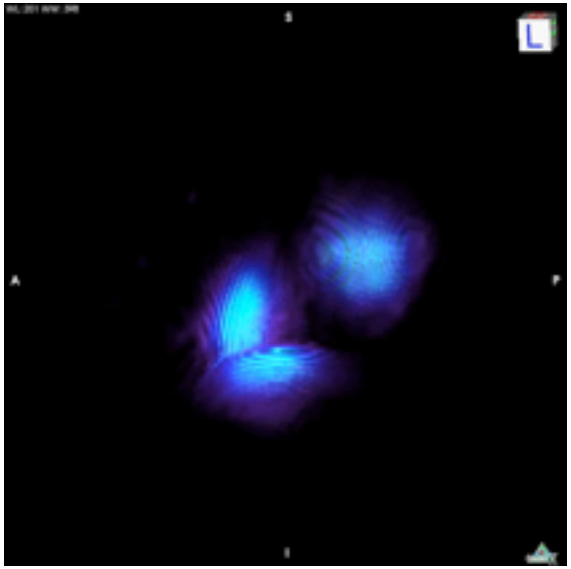
Original (3D shells + Gaussians)



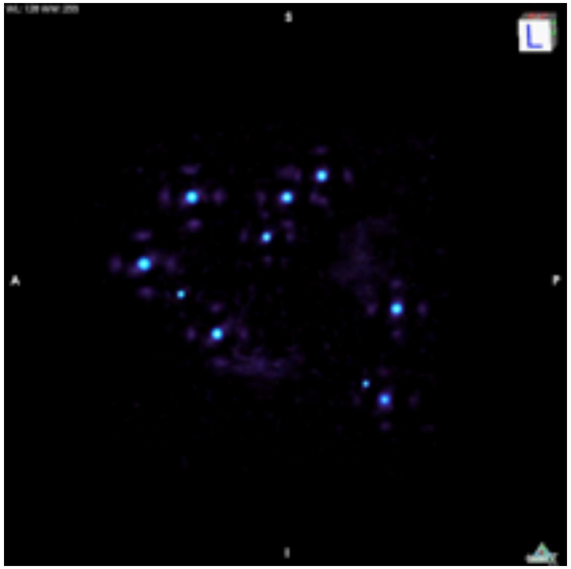
Dictionary

RidCurvelets + 3D UDWT.

Shells



Gaussians



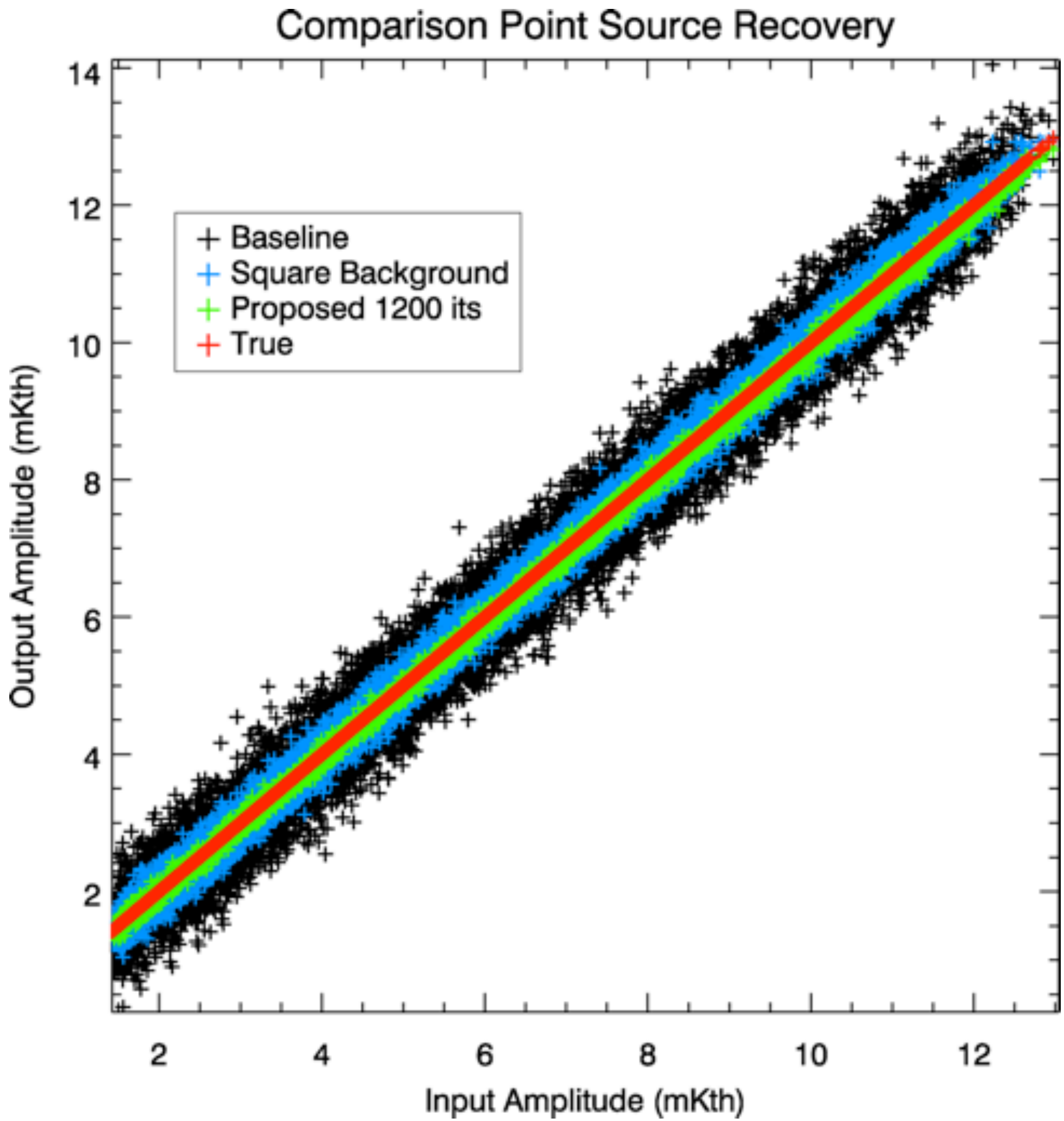
A, Woiselle, J.L. Starck, M.J. Fadili, "3D Data Denoising and Inpainting with the Fast Curvelet transform", **J. of Mathematical Imaging and Vision (JMIV)**, 39, 2, pp 121-139, 2011.

Morphological Component Analysis & Sparse Point Source Removal

$$Y = X + B * P + N$$

$$\left\{\tilde{X}, \tilde{P}\right\} = \arg\min_{X,P} ||Y - X - B * P||_{\Sigma}^{2} + \lambda_1 ||P||_1 + \lambda_2 ||\mathcal{S}X||_1$$

Sureau et al, Compact Source Removal for Full-Sky CMB Data using Sparsity, ADA7, Corsica, 14-18 May 2012.
Online at http://ada7.cosmostat.org/proceedings.php, id. 14


Comparison Point Source Recovery
14
12
10
8
6
4
2
Output Amplitude (mKth)
Baseline
Square Background
Proposed 1200 its
True
2
4
6
8
10
12
Input Amplitude (mKth)

Morpho-Spectral Diversity

$$\min_\alpha \|\alpha\|_p \text{ s.t } \mathbf{X} = \sum_{\gamma\in\Gamma} \alpha_\gamma \psi_\gamma$$

$\mathbf{\Phi_A} = [\mathbf{\Phi_{A,1}}, \mathbf{\Phi_{A,2}}]$ Spatial Dictionary

$\mathbf{\Phi_S}$ Spectral Dictionary

$$\mathbf{\Psi} = [\mathbf{\Phi_{A,1}} \otimes \mathbf{\Phi_S}, \mathbf{\Phi_{A,2}} \otimes \mathbf{\Phi_S}]$$

Sparse Component Separation: the GMCA Method

A and S are estimated alternately and iteratively in two steps :

- J. Bobin, J.-L. Starck, M.J. Fadili, and Y. Moudden, "Sparsity, Morphological Diversity and Blind Source Separation", IEEE Trans. o Image Processing, Vol 16, No 11, pp 2662 - 2674, 2007.
- .J. Bobin, J.-L. Starck, M.J. Fadili, and Y. Moudden, "Blind Source Separation: The Sparsity Revolution", Advances in Imaging and Electron Physics , Vol 152, pp 221 -- 306, 2008.

$$\mathbf{X} = \mathbf{A}\mathbf{S}$$

1) Estimate S assuming A is fixed (iterative thresholding) :

$$\{S\} = \text{Argmin}_S \sum_j \lambda_j \|s_j \mathbf{W}\|_1 + \|\mathbf{X} - \mathbf{A}\mathbf{S}\|^2_{F,\mathbf{\Sigma}}$$

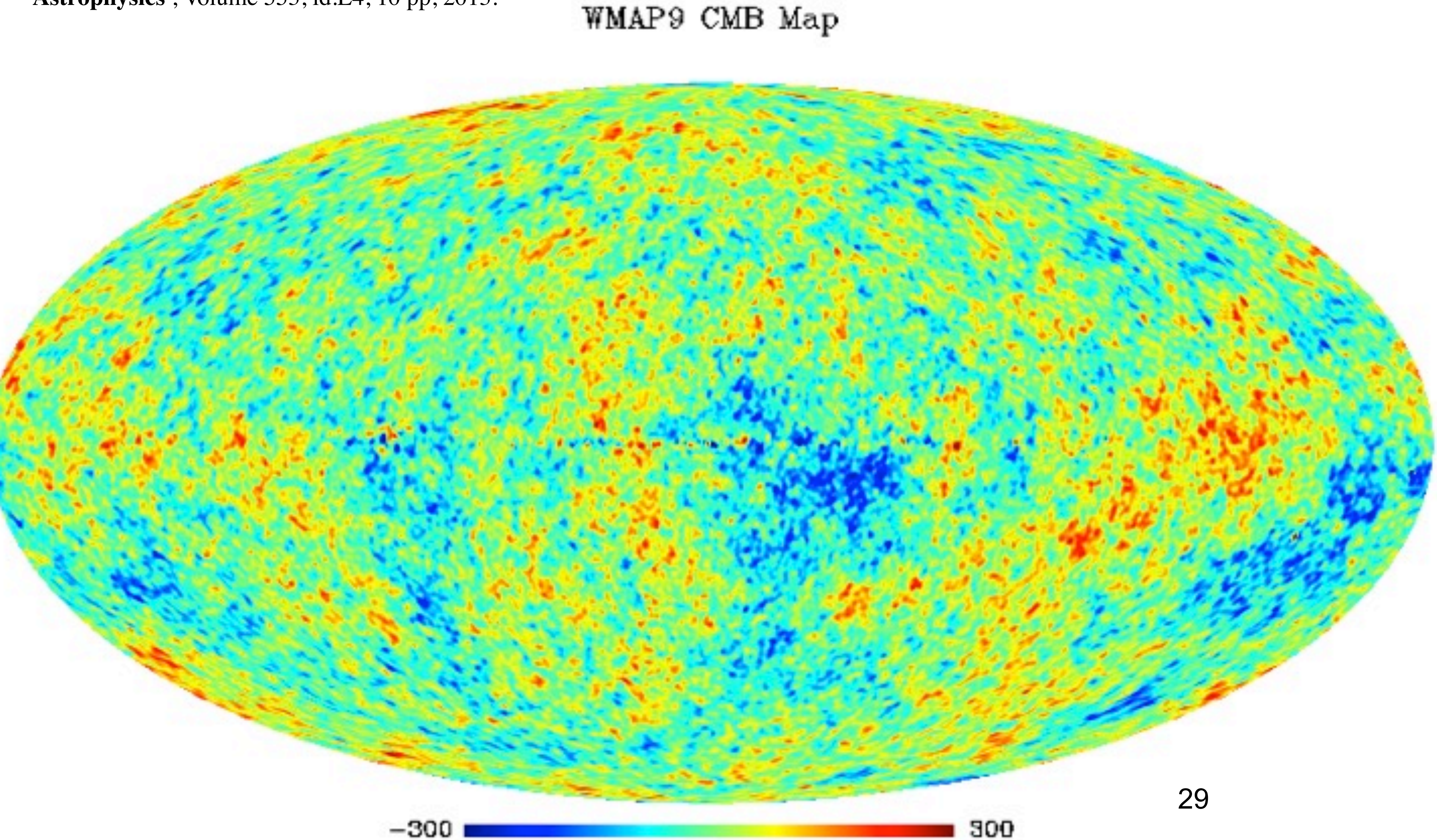
2) Estimate A assuming S is fixed (a simple least square problem) :

$$\{A\} = \text{Argmin}_A \|\mathbf{X} - \mathbf{A}\mathbf{S}\|^2_{F,\mathbf{\Sigma}}$$

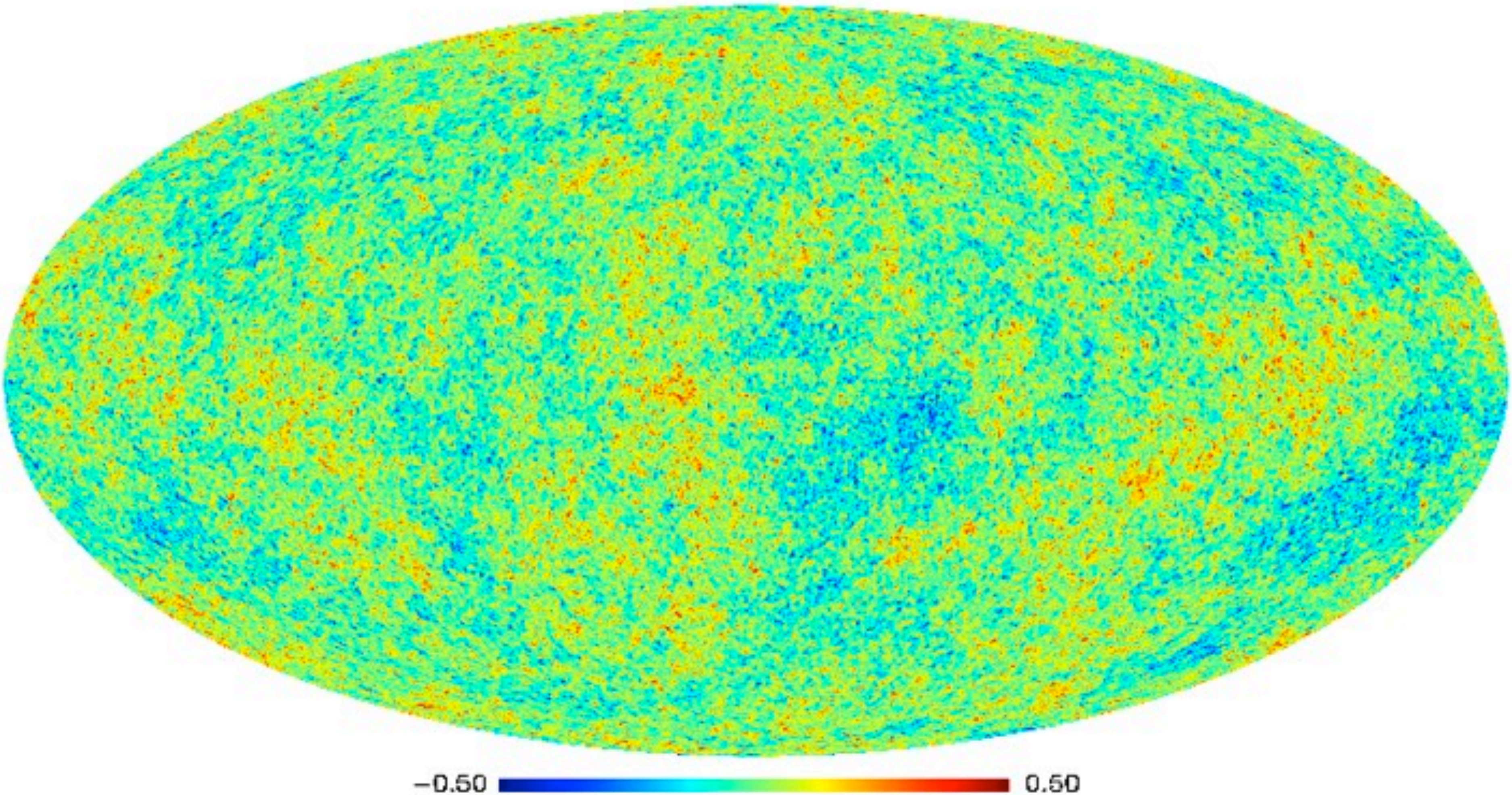
GMCA & WMAP-9yr

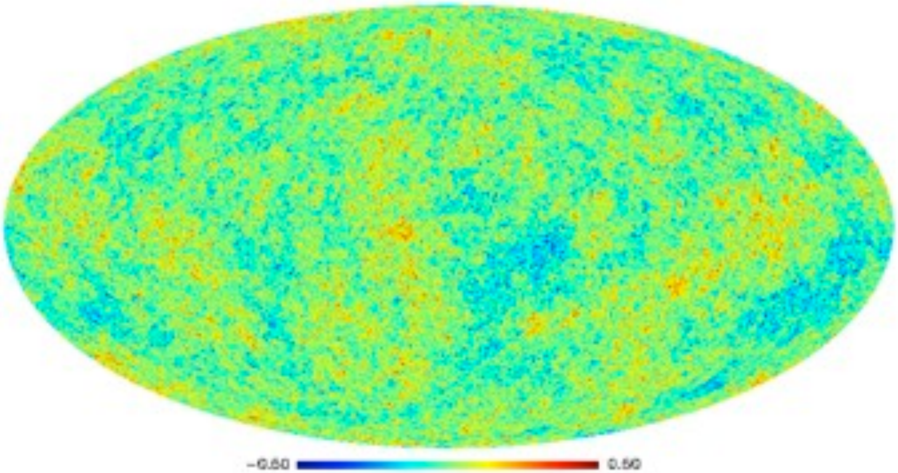
J. Bobin, J.-L. Starck, F. Sureau and S. Basak, "Sparse component separation for accurate CMB map estimation" , **Astronomy and Astrophysics** , 550, A73, 2013.

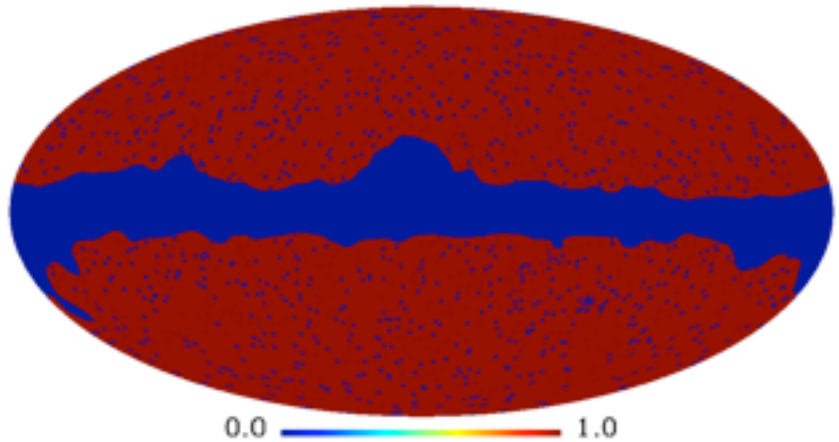
J. Bobin, F. Sureau, P. Paykari, A. Rassat, S. Basak and J.-L. Starck, "WMAP 9-year CMB estimation using sparsity", **Astronomy and Astrophysics** , Volume 553, id.L4, 10 pp, 2013.

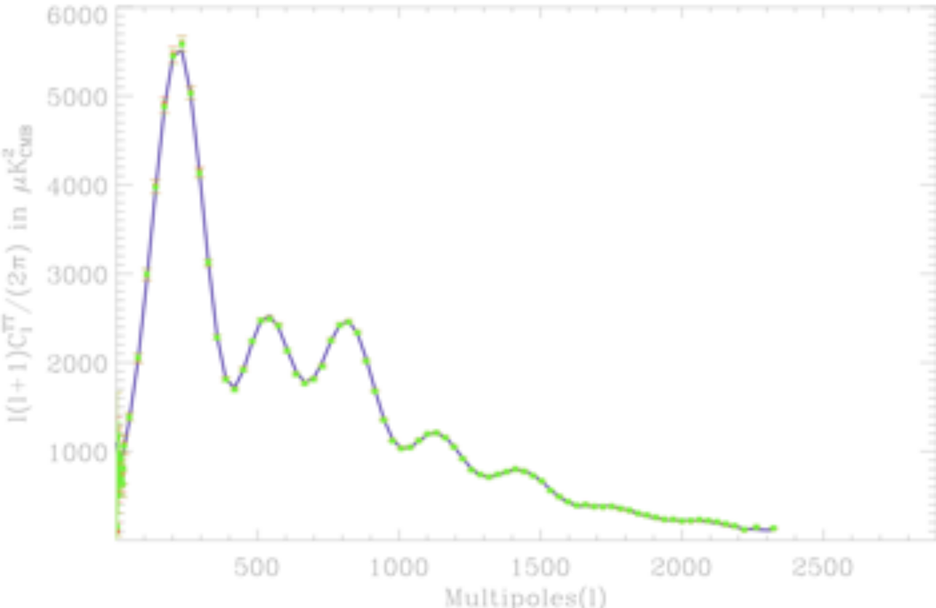


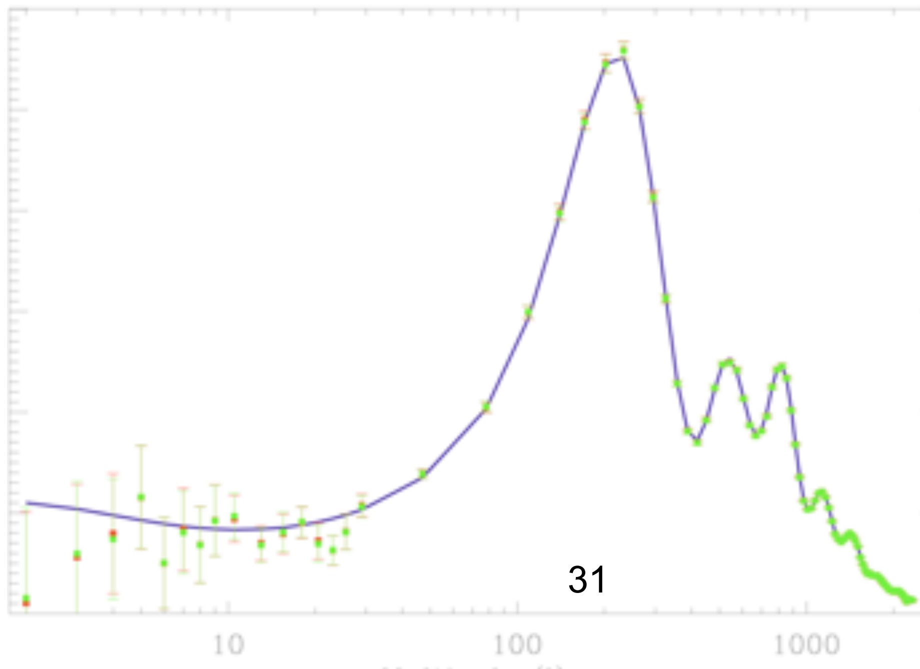
Sparse Planck Map






0.0
1.0


6000
5000
4000
3000
2000
1000
500
1000
1500
2000
2500
Multipoles(l)


10
100
1000

QUALITY MAP

Expected power in a given wavelet band :

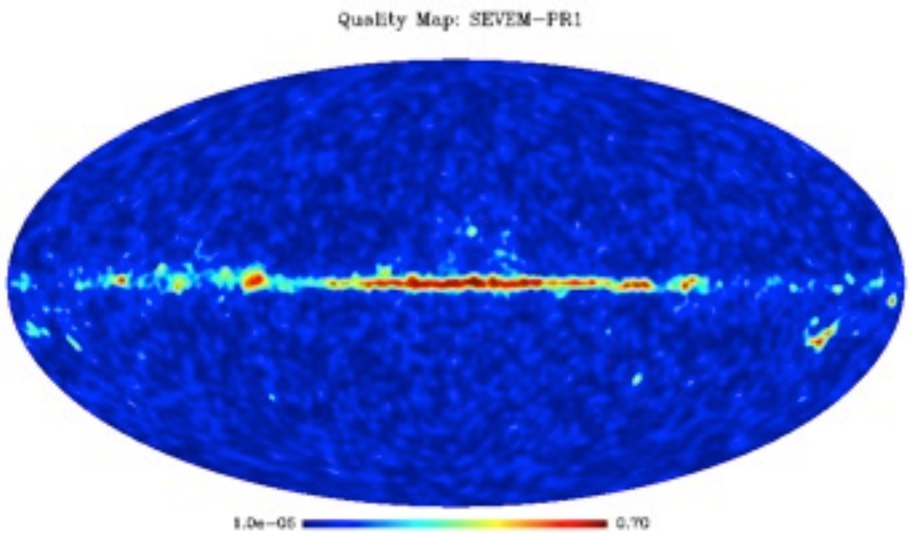
$$P_j = \frac{1}{4\pi} \sum_\ell \ell(\ell+1) \parallel a_{\ell,0}^{(\psi_j)} \parallel^2 C_\ell$$

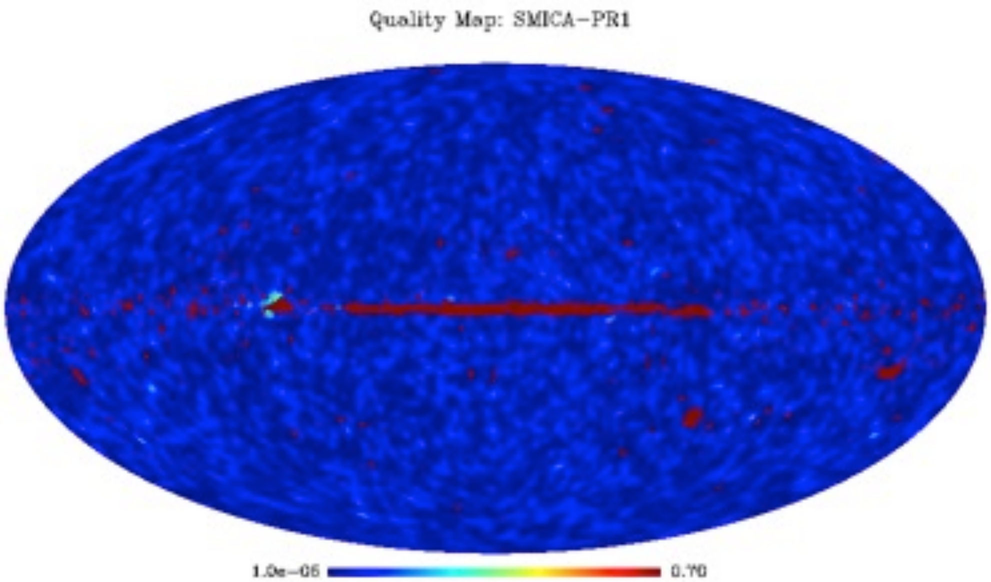
Quality coefficient :

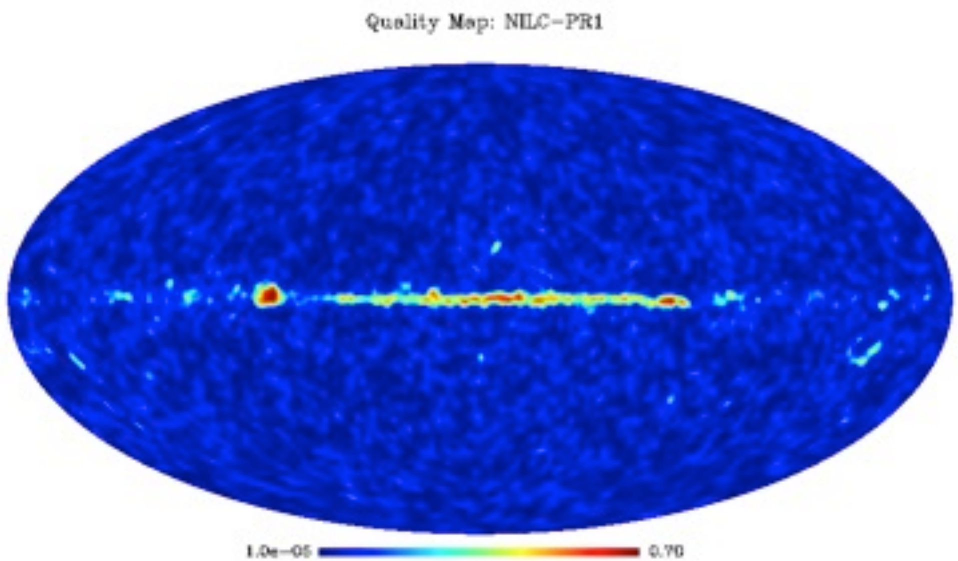
$$q_{j,k} = P_j / \left(D_{j,k} - N_{j,k}\right)$$

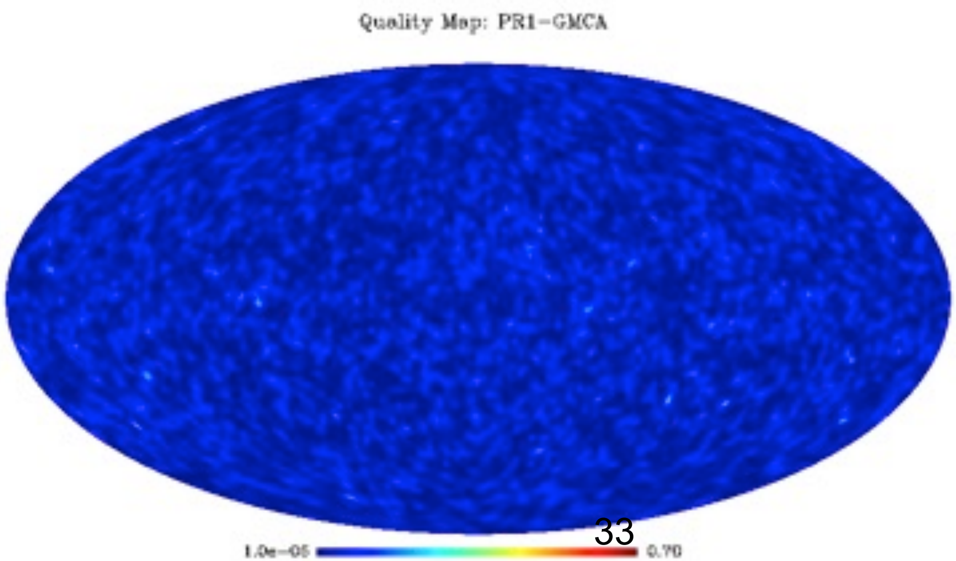
$$Q_k = 1 - \max_j q_{j,k}$$

QUALITY MAPS

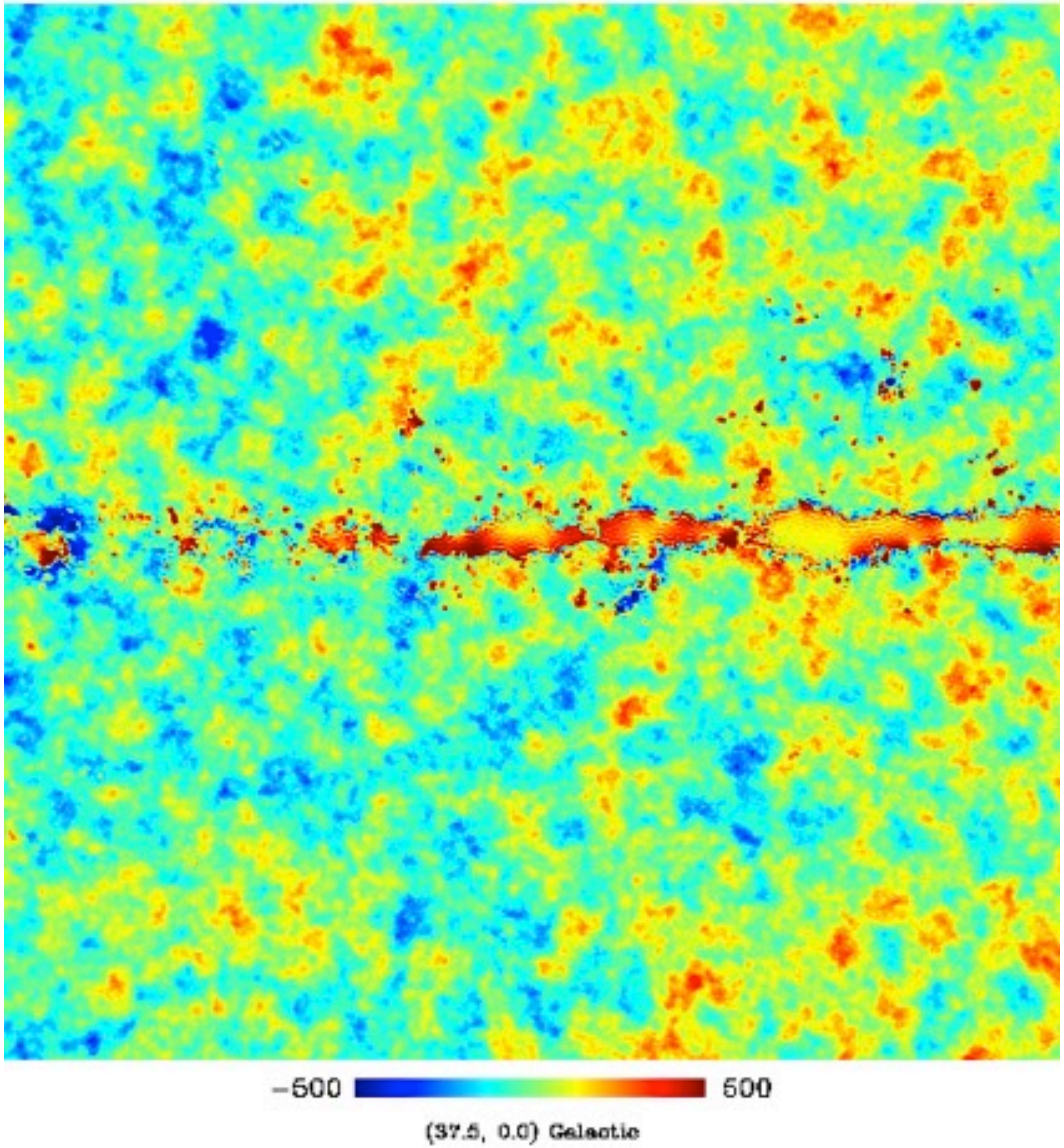




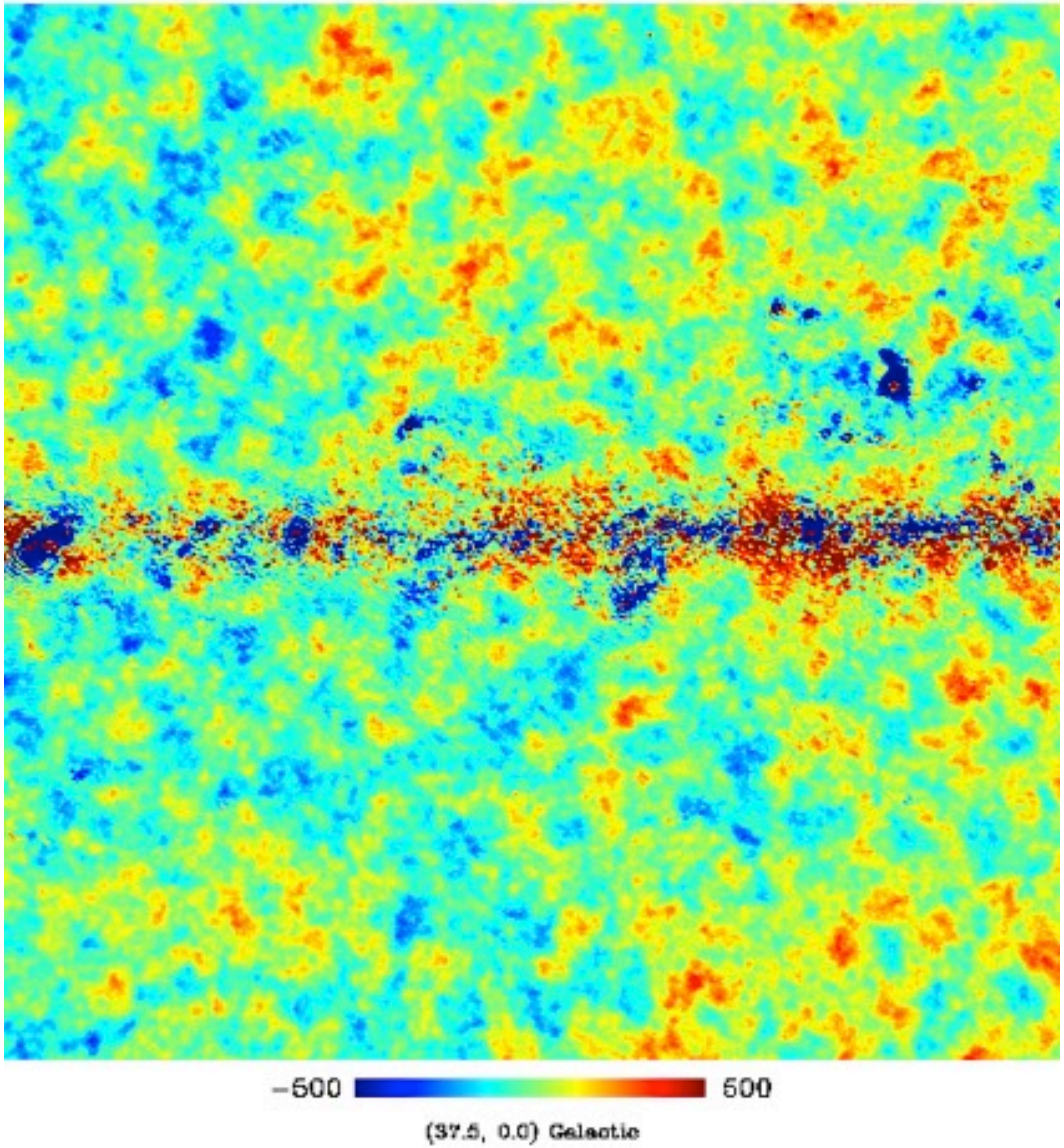


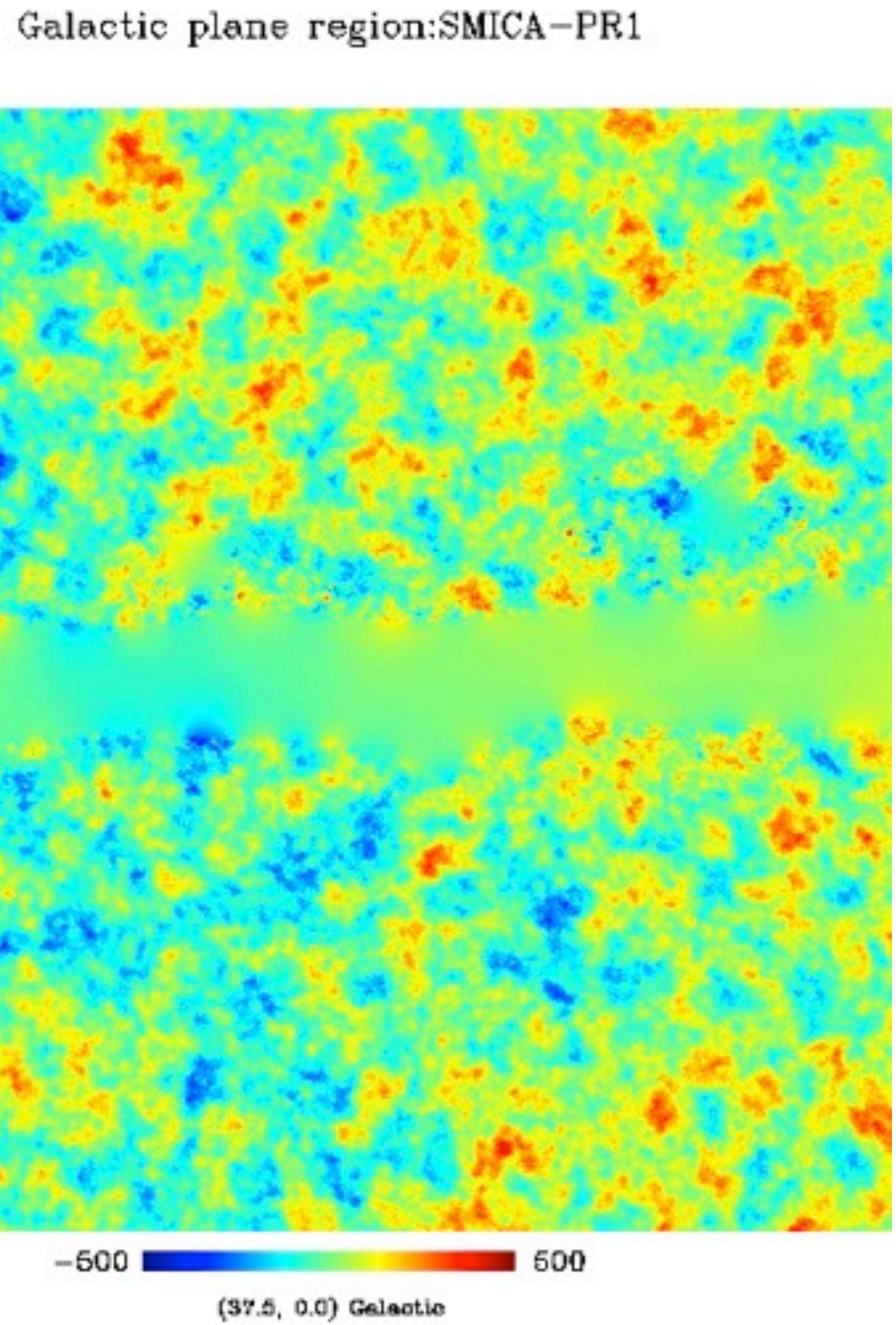


Galactic plane region:NILC−PR1

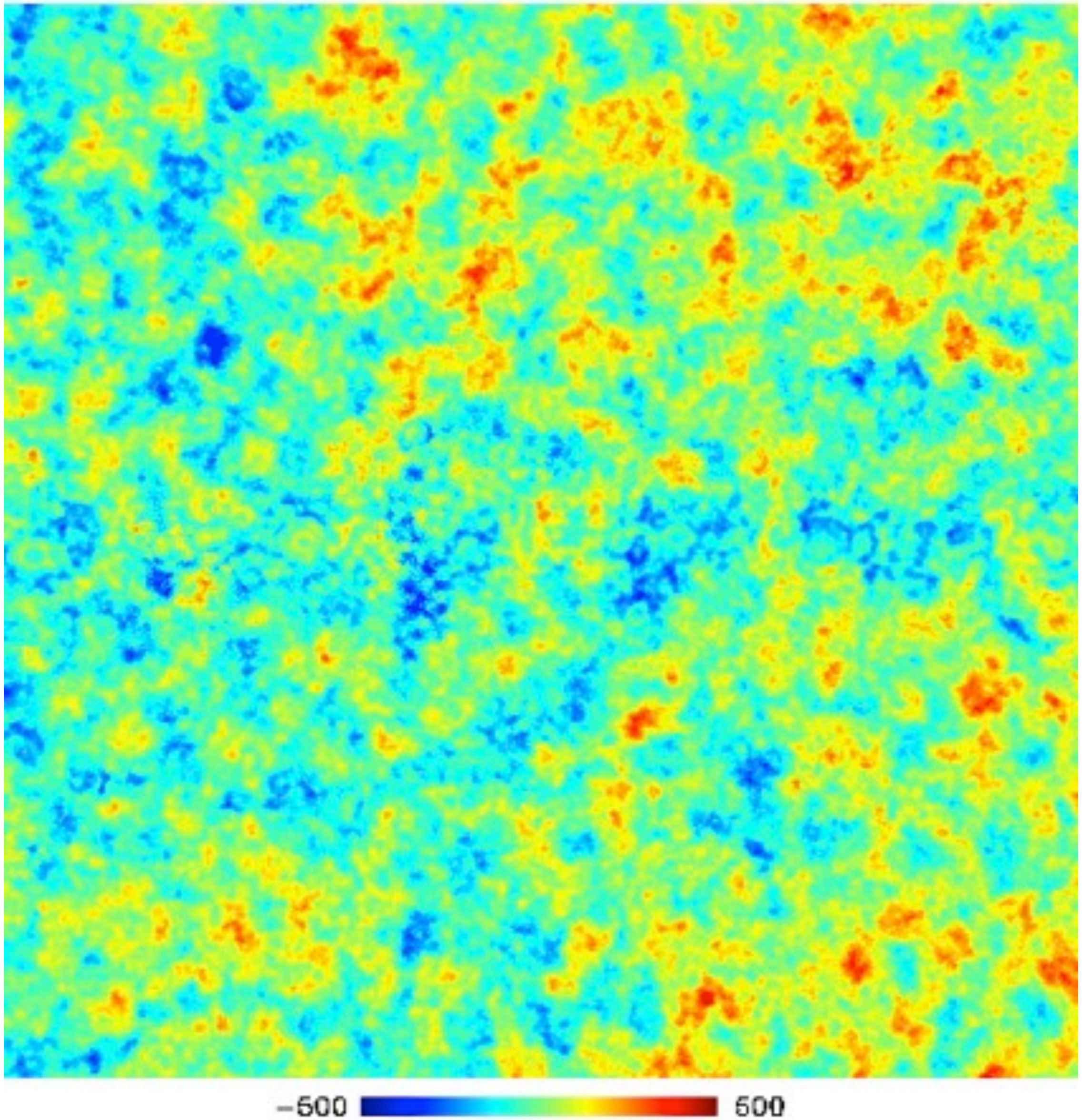


Galactic plane region:SEVEM−PR1

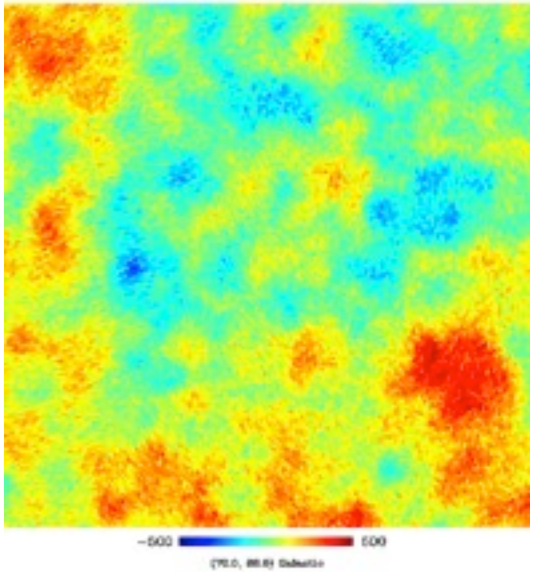



Galactic plane region:SMICA-PR1
−500
500
(37.5, 0.0) Galactic

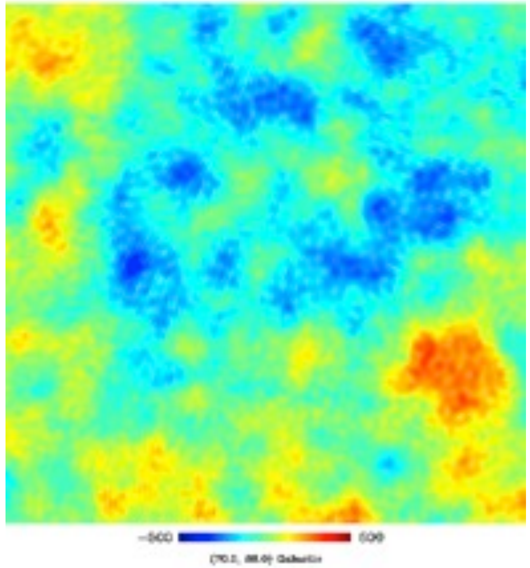
Galactic plane region:PR1-GMCA



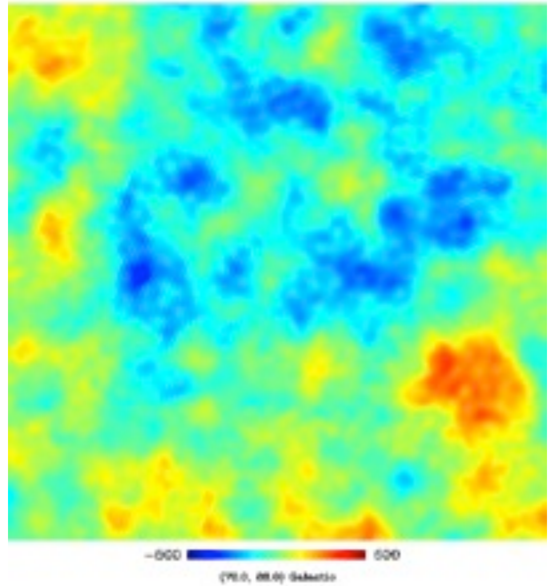
Coma: 217GHz PR1-HFI



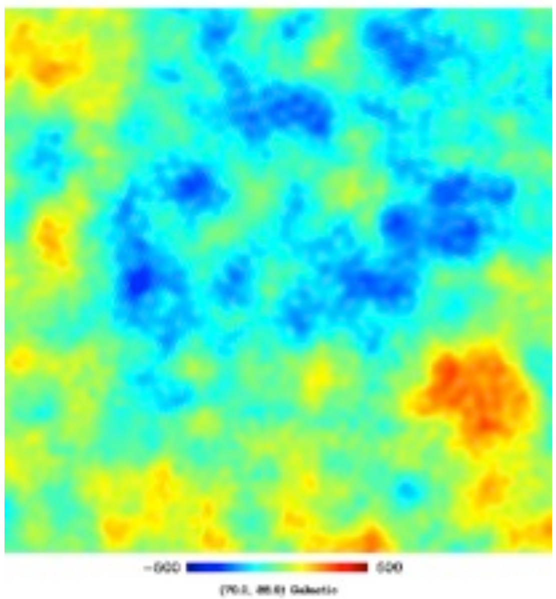
Coma: CMB Map: SMICA-PR1



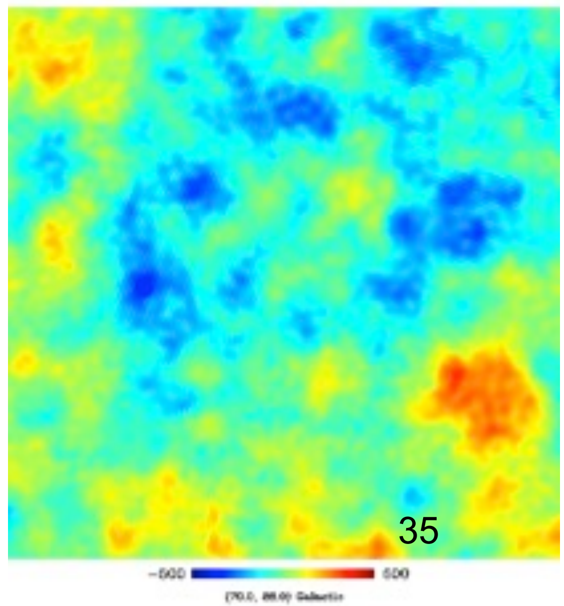
Coma: CMB Map: NILC-PR1



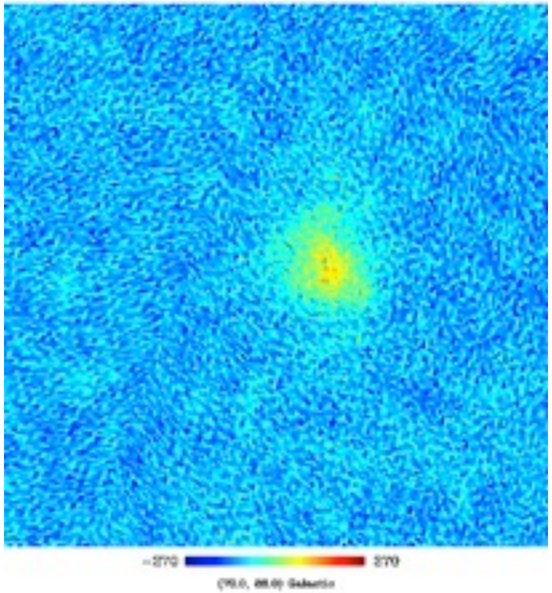
Coma: CMB Map: SEVEM-PR1



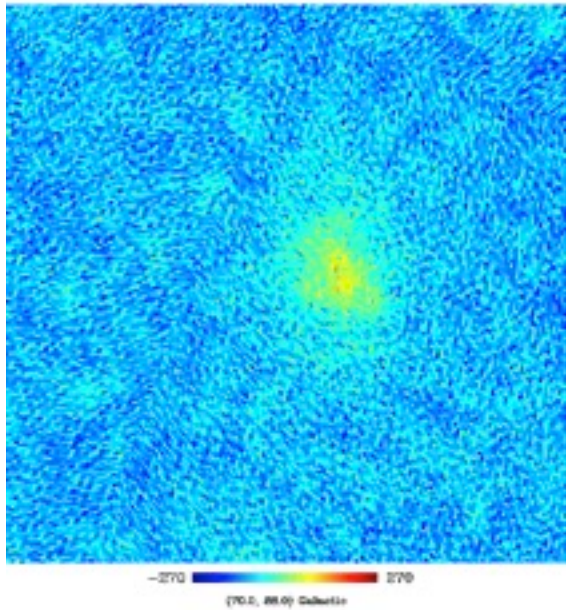
Coma: CMB Map: GMCA-PR1



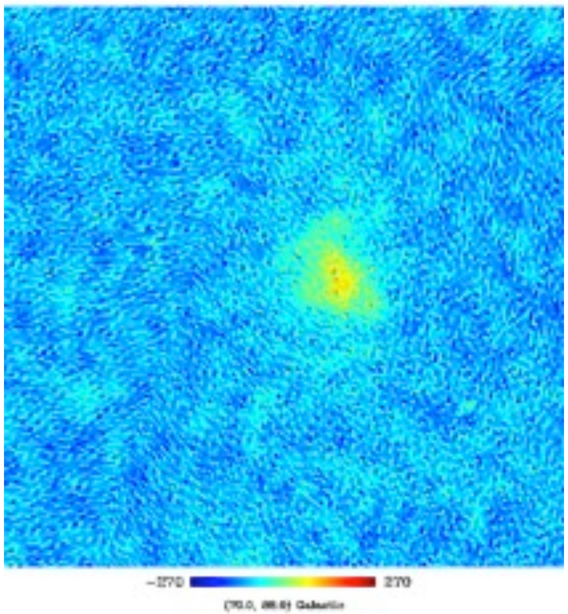
Coma: 217GHz PR1-HFI - NILC-PR1



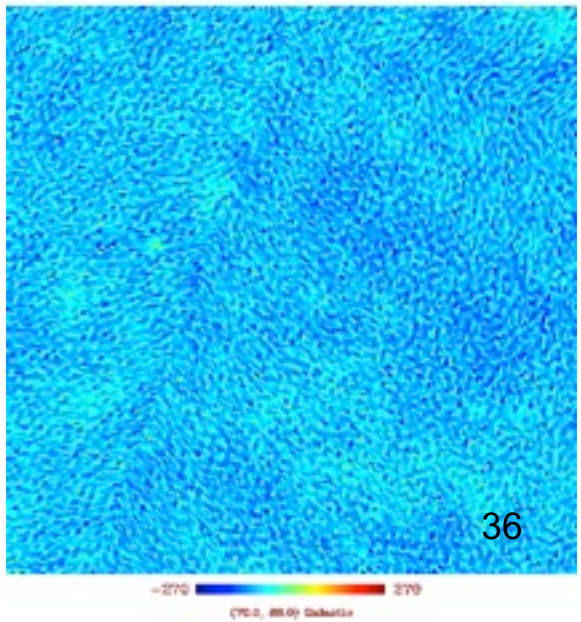
Coma: 217GHz PR1-HFI - SEVEM-PR1



Coma: 217GHz PR1-HFI - SMICA-PR1



Coma: 217GHz PR1-HFI - PR1-GMCA

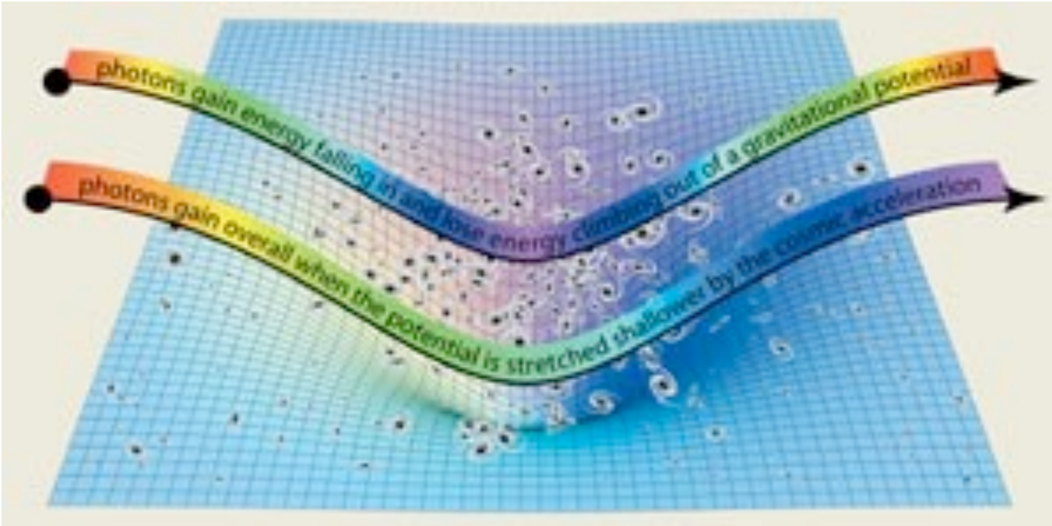


CMB & ANOMALIES

- Anomalies in WMAP CMB maps:
- Low Power in CMB Quadrupople (Hinshaw 96, Spergel 03).
- North /South Asymmetry (Erikson 04).
- Planarity of low multipoles, 'Axis of Evil' (Tegmark 03, de Oliveira-Costa 04, Land & Maguiejo 05).
- Small scale cold spot in southern hemisphere (Vielva 2004).
- Few hot spots.

Anomalies confirmed by Planck

Integrated Sachs-Wolfe Effect (ISW)



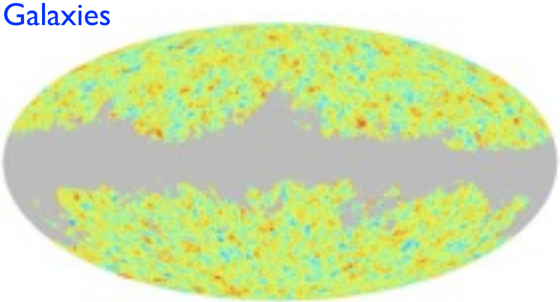
Measure of Time Variation in the Gravitational Potential on **large scales (linear)**

$$\left(\frac{\Delta T}{T}\right)_{ISW} = -2\int \frac{d\Phi}{d\eta}\, d\eta$$

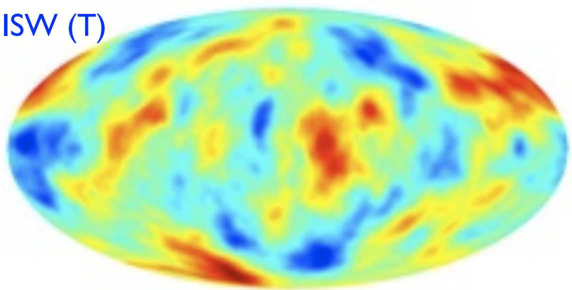
Detect by cross-correlating with local tracers of mass

Can ISW explain some of the CMB anomalies (Francis & Peacock, 2010) ?

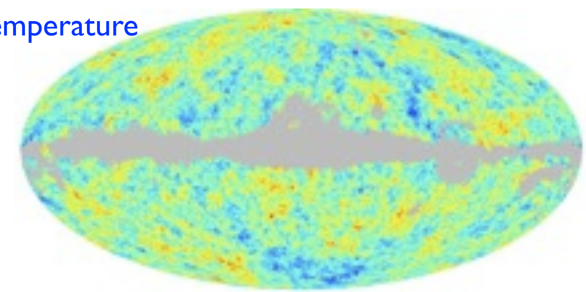
Galaxies



ISW (T)



Temperature

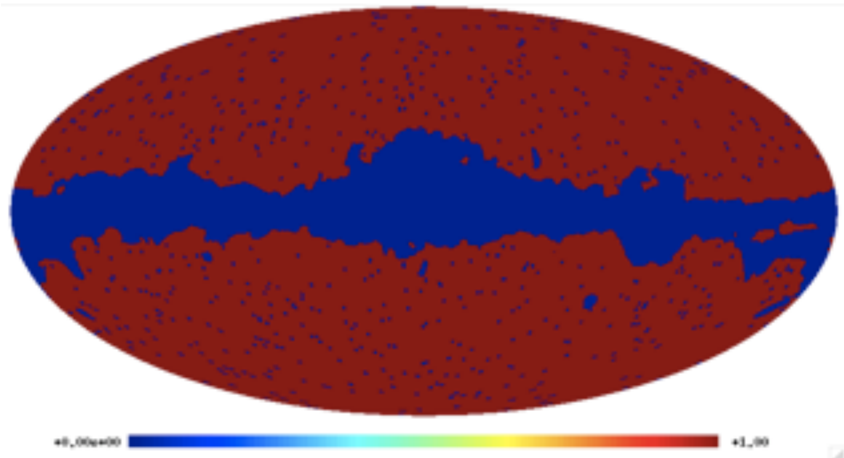


Even if you don't believe in these, you should still remove secondary anisotropies, ..., if you can.

==> Galactic Mask problem when analyzing the largest scales.

Interpolation of Missing Data: Sparse Inpainting

Where M is the mask: M(i,j) = 0 ==> missing data
M(i,j) = 1 ==> good data



$$Y = MX$$

$$\min_{\alpha} \|\alpha\|_1 \quad \text{subject to} \quad Y = M\Phi\alpha$$

$$X = \Phi\alpha \qquad \Phi = \text{Spherical Harmonics}$$

$$\|\alpha\|_1 = \sum_k \mid \alpha_k \mid$$

J.-L. Starck, A. Rassat, and M.J. Fadili, "Low-l CMB Analysis and Inpainting", **Astronomy and Astrophysics** , 550, A15, 2013.

J.-L. Starck, D.L. Donoho, M.J. Fadili and A. Rassat, "Sparsity and the Bayesian Perspective", **Astronomy and Astrophysics** , 552, A133, 2013.

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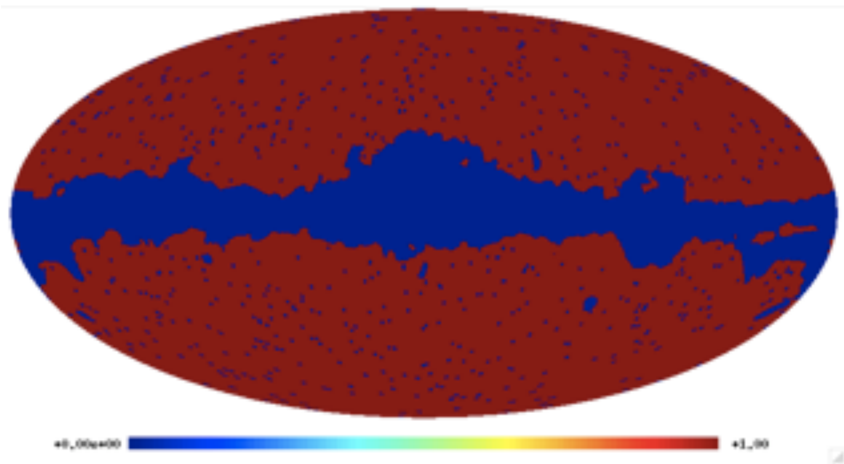

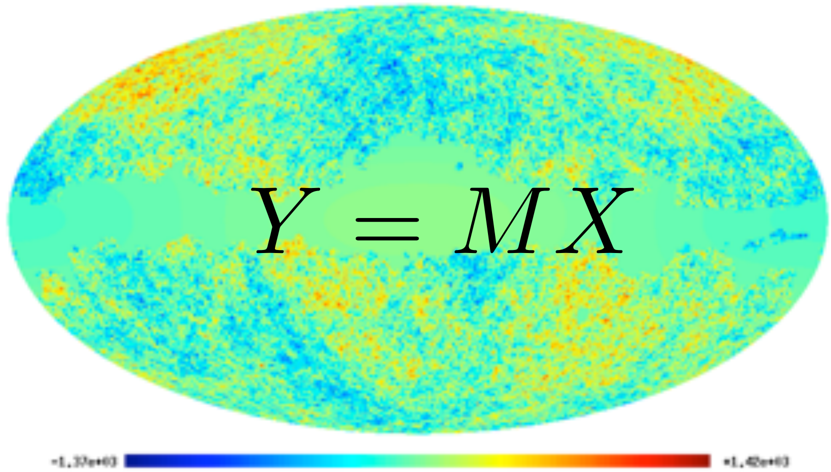

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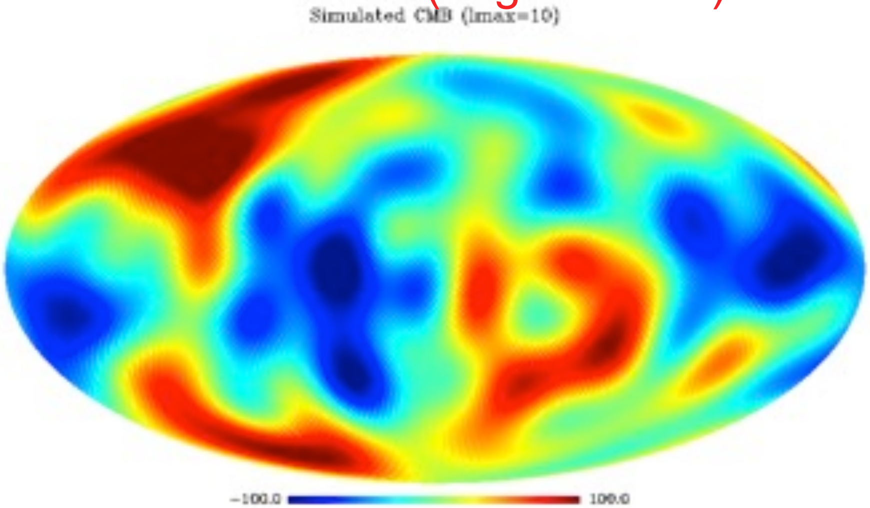
J.-L. Starck, A. Rassat, and M.J. Fadili, "Low-l CMB Analysis and Inpainting", **Astronomy and Astrophysics** , 550, A15, 2013.

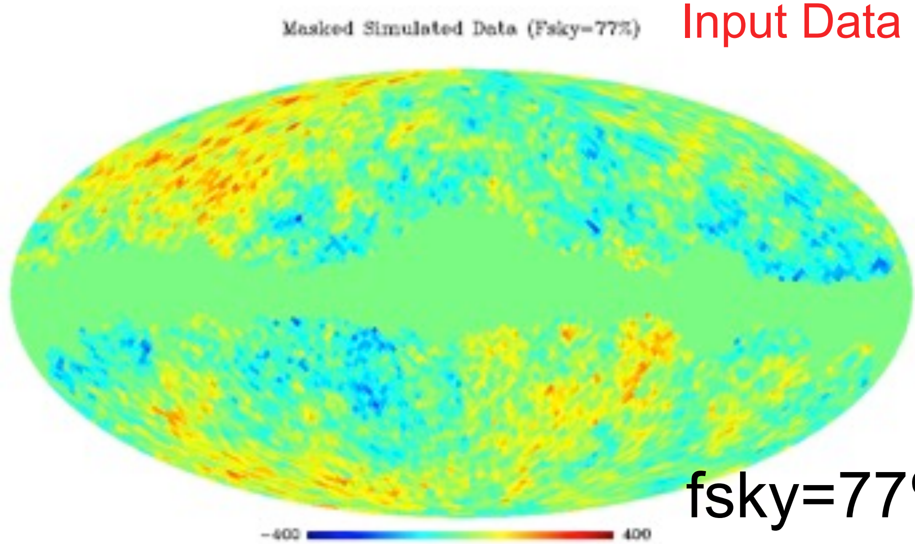
J.-L. Starck, D.L. Donoho, M.J. Fadili and A. Rassat, "Sparsity and the Bayesian Perspective", **Astronomy and Astrophysics** , 552, A133, 2013.

Large CMB Scale Analysis

Simulated CMB (largest scale)

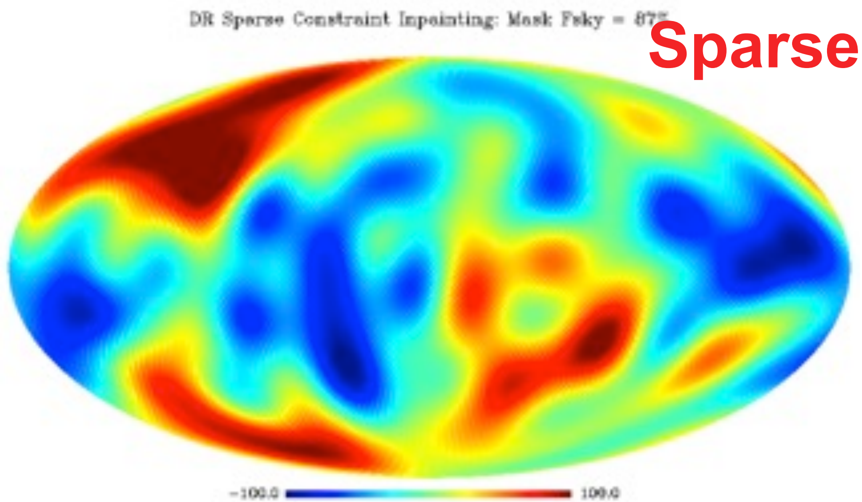


Input Data

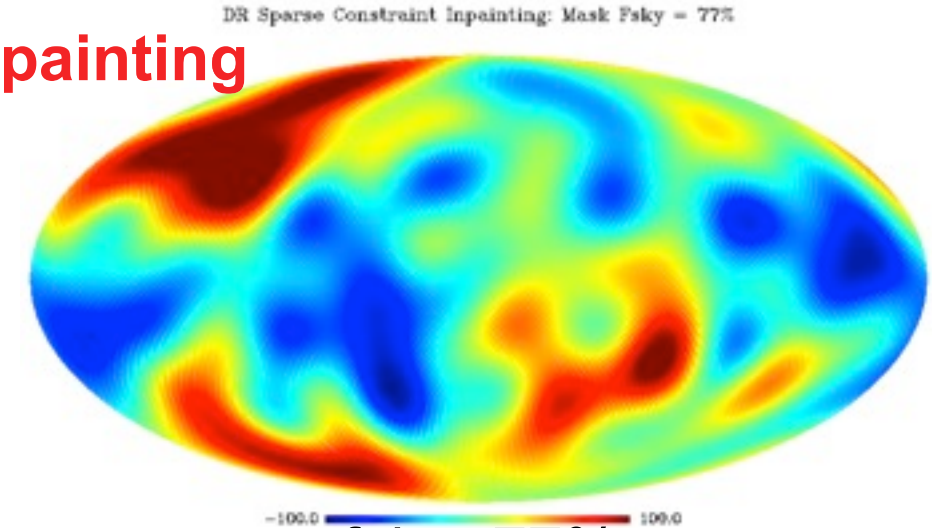


fsky=77%

Sparse Inpainting



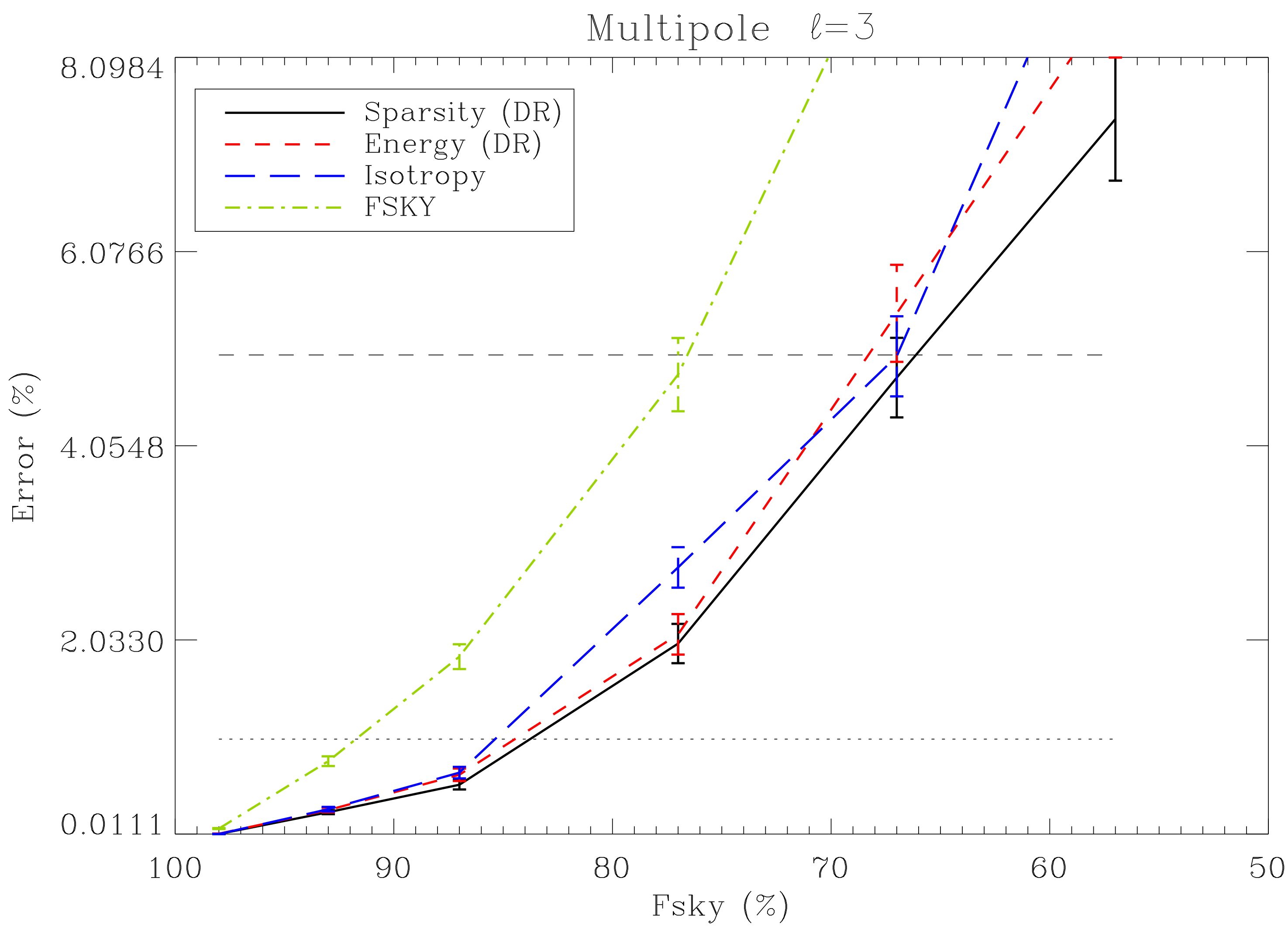
fsky=87%



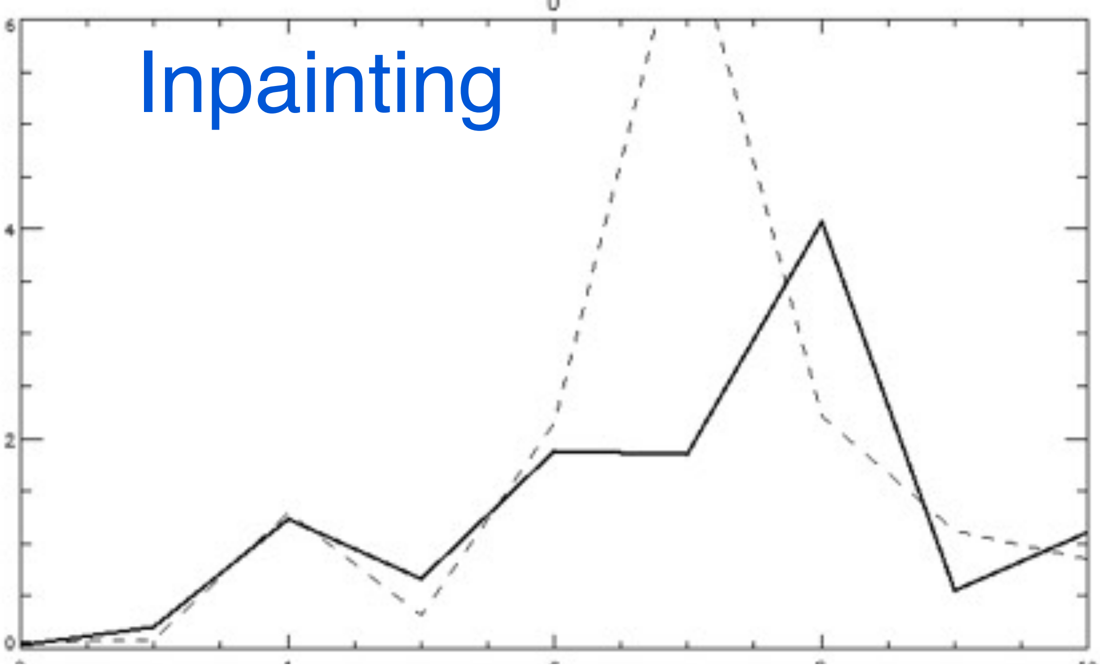
fsky=77%

J.-L. Starck, A. Rassat, and M.J. Fadili, "Low-l CMB Analysis and Inpainting", **Astronomy and Astrophysics**, 550, A15, 2013.

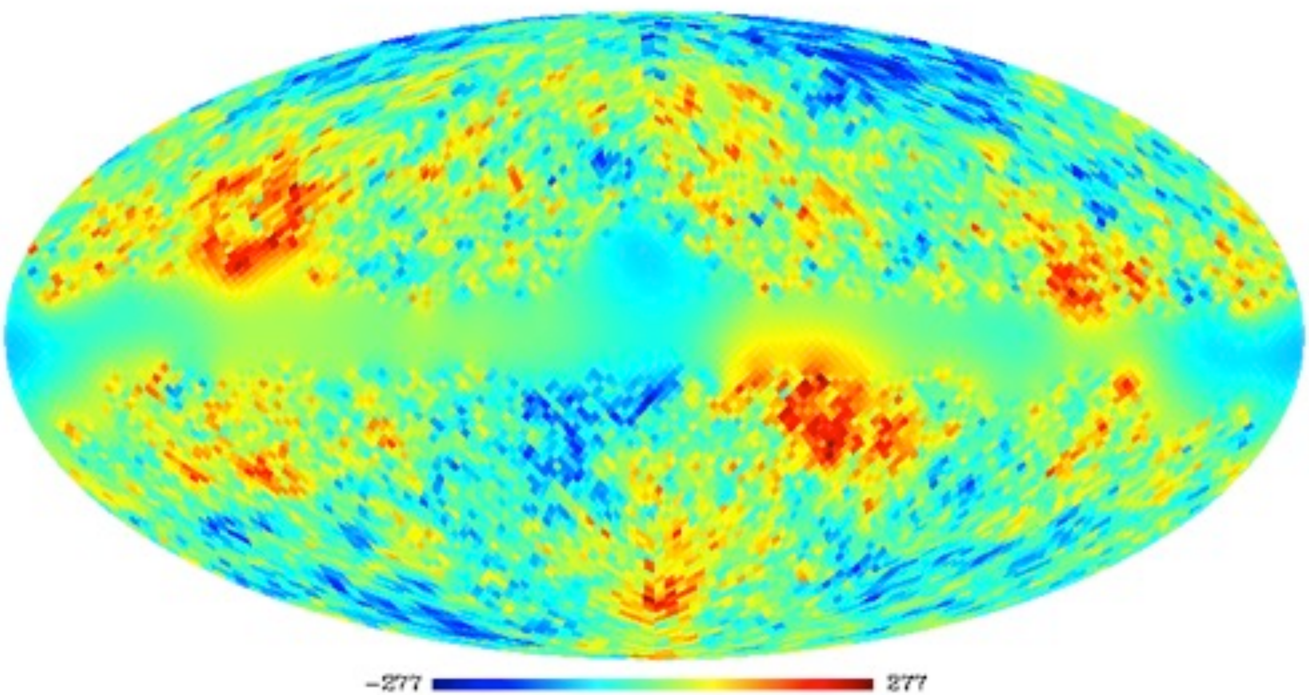
Inpainting


Multipole ℓ=3
8.0984
6.0766
4.0548
2.0330
0.0111
Error (%)
100
90
80
70
60
50
Fsky (%)
Sparsity (DR)
Energy (DR)
Isotropy
FSKY

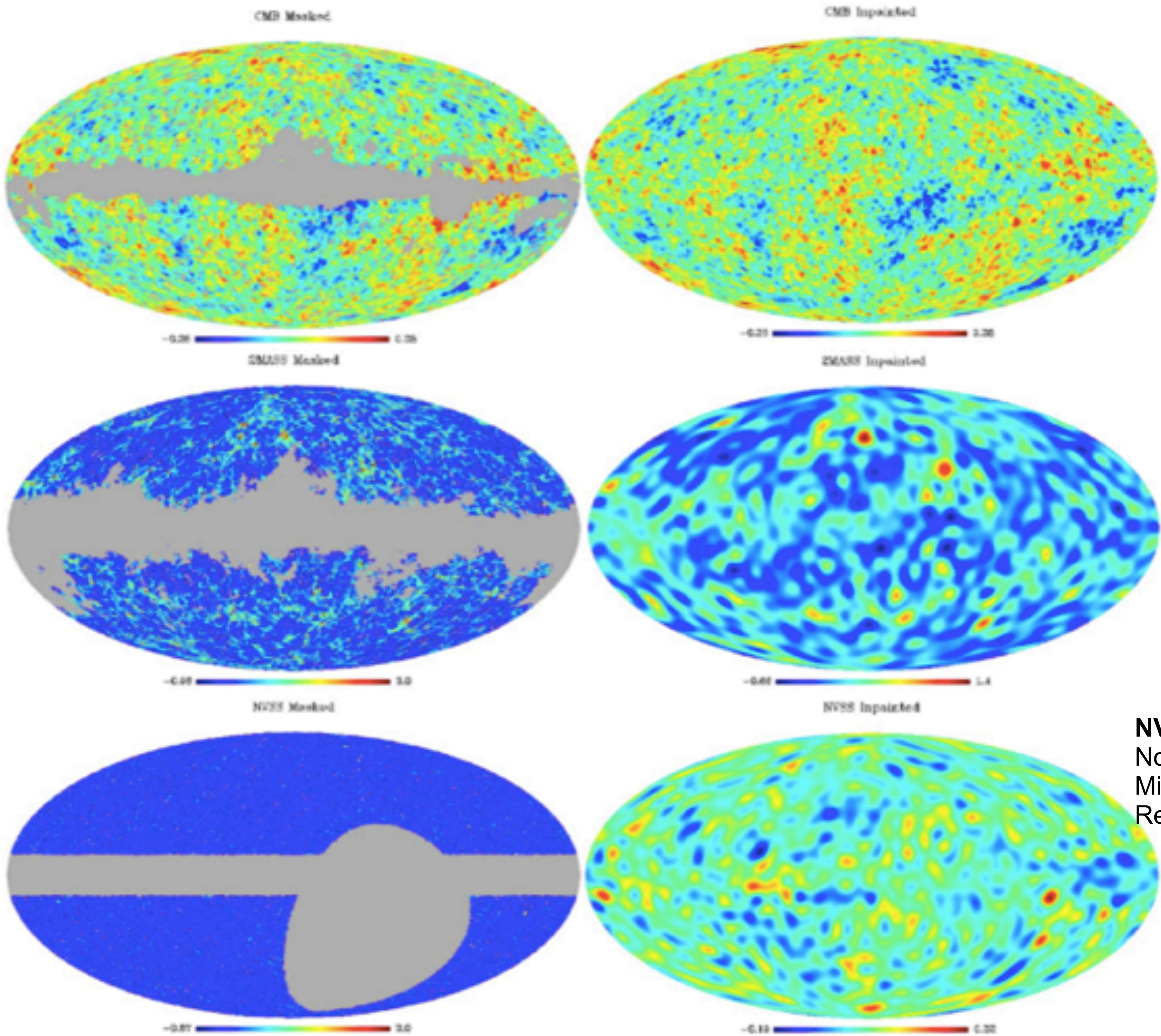
Inpainting



FFP6



Sparsity and WMAP

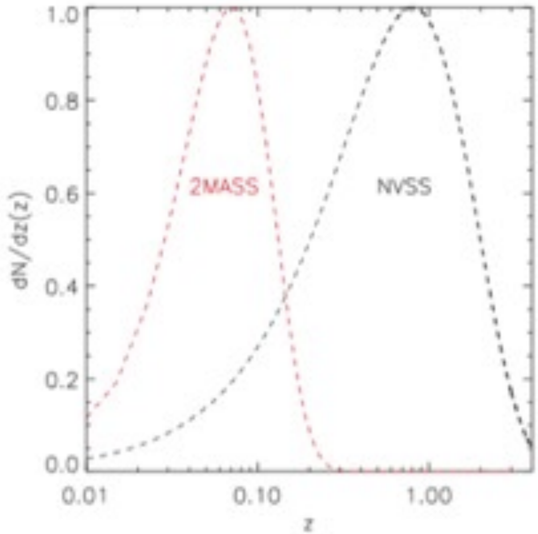


WMAP: CMB

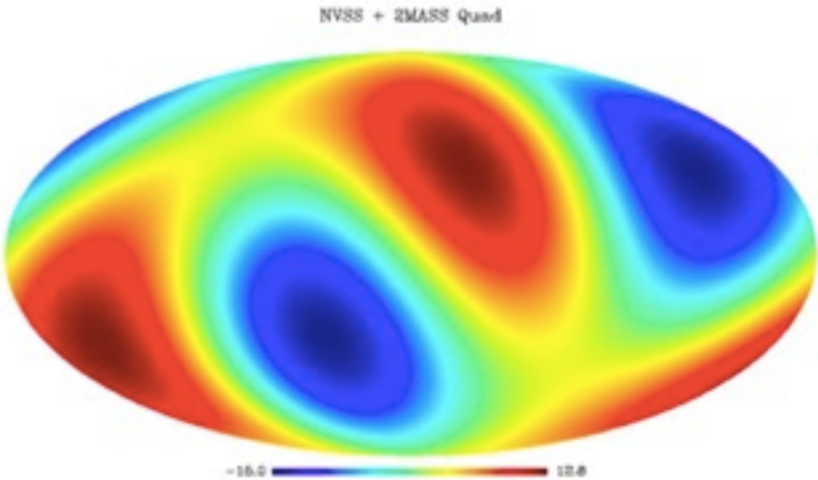
2MASS: Near IR
Make mask using dust maps

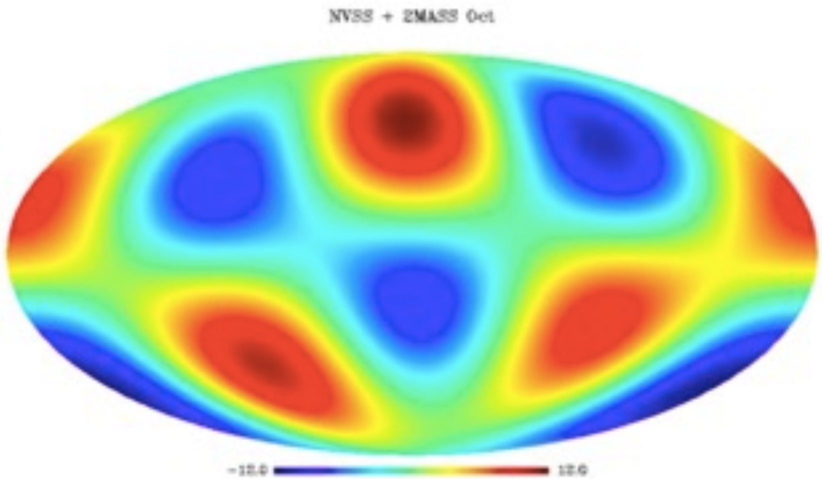
NVSS: Radio
No data for low declinations
Missing galactic plane |b| < 10
Remove bright radio sources

ISW & CMB ANOMALIES



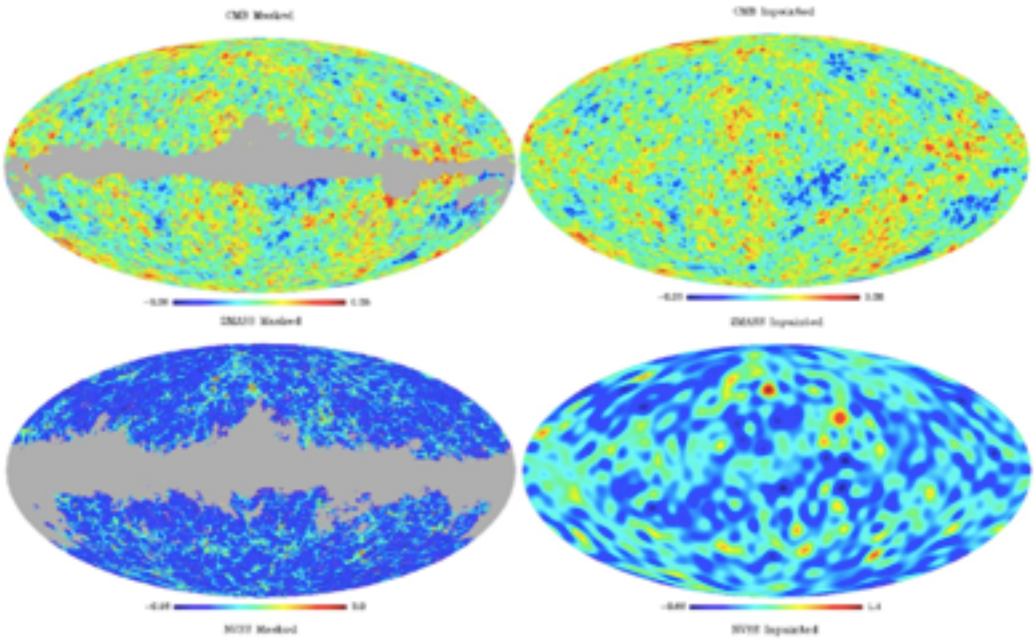
Assume independent fields
Inpaint both maps w/ respective masks
Reconstruct ISW signal from data alone
(model independent)





Reconstructed ISW temperature quad/oct due to 2MASS and NVSS galaxies

Inpainting & CMB ANOMALIES



After subtraction of ISW signal, several anomalies no longer significant

=> Quadrupole low power

=> Quad/oct anomaly.

=> Axis of Evil (AoE) statistic and even/odd mirror parity.

A. Rassat and J-L. Starck, "On Preferred Axes in WMAP Cosmic Microwave Background Data after Subtraction of the Integrated Sachs-Wolfe Effect", **Astronomy and Astrophysics** , 557, id.L1, pp 7, 2013.

A. Rassat, J-L. Starck, and F.X. Dupe, "Removal of two large scale Cosmic Microwave Background anomalies after subtraction of the Integrated Sachs Wolfe effect", **Astronomy and Astrophysics** , 557, id.A32, pp 15, 2013.

==> ISW could be a possible explanation of these anomalies in WMAP/Planck data, yet other hypotheses remain possible (e.g. exotic physics) as well.

Sparsity and CMB

- Sparsity is very efficient for
  - Inverse problems (denoising, deconvolution, etc).
  - Inpainting
  - Component Separation.
  - Wiener Wiltering.
- Next Steps
  - Polarization.
- Lessons for Future Projects
  - Importance of blind challenges.
  - Open source, at least in the consortium, has to become the norm.

iSAP Version V3.0
Interactive Sparse Astronomical Packages

Multiresolution on the Sphere: MRS/Version 3.1

J.-L. Starck, P. Abrial, Y. Moudden and M. Nguyen, ***Wavelets, Ridgelets and Curvelets on the Sphere***, *Astronomy & Astrophysics*, 446, 1191-1204, 2006.

1. Wavelet transforms
   - Continuous Wavelet Transform (Mexican Hat)
   - Orthogonal Wavelets
   - Undecimated isotropic wavelet transform (Spline, Meyer and Needlet filters).
   - Pyramidal wavelet transform
2. Ridgelet and Curvelet Transforms
3. Denoising using Wavelets and Curvelets
4. Gaussianity tests: Skewness, Kurtosis, Moment of order 5 and 6, Max, Higher Criticism
5. Astrophysical Component Separation (ICA on the Sphere): JADE, Fast ICA, GMCA.
6. Sparse Inpainting.

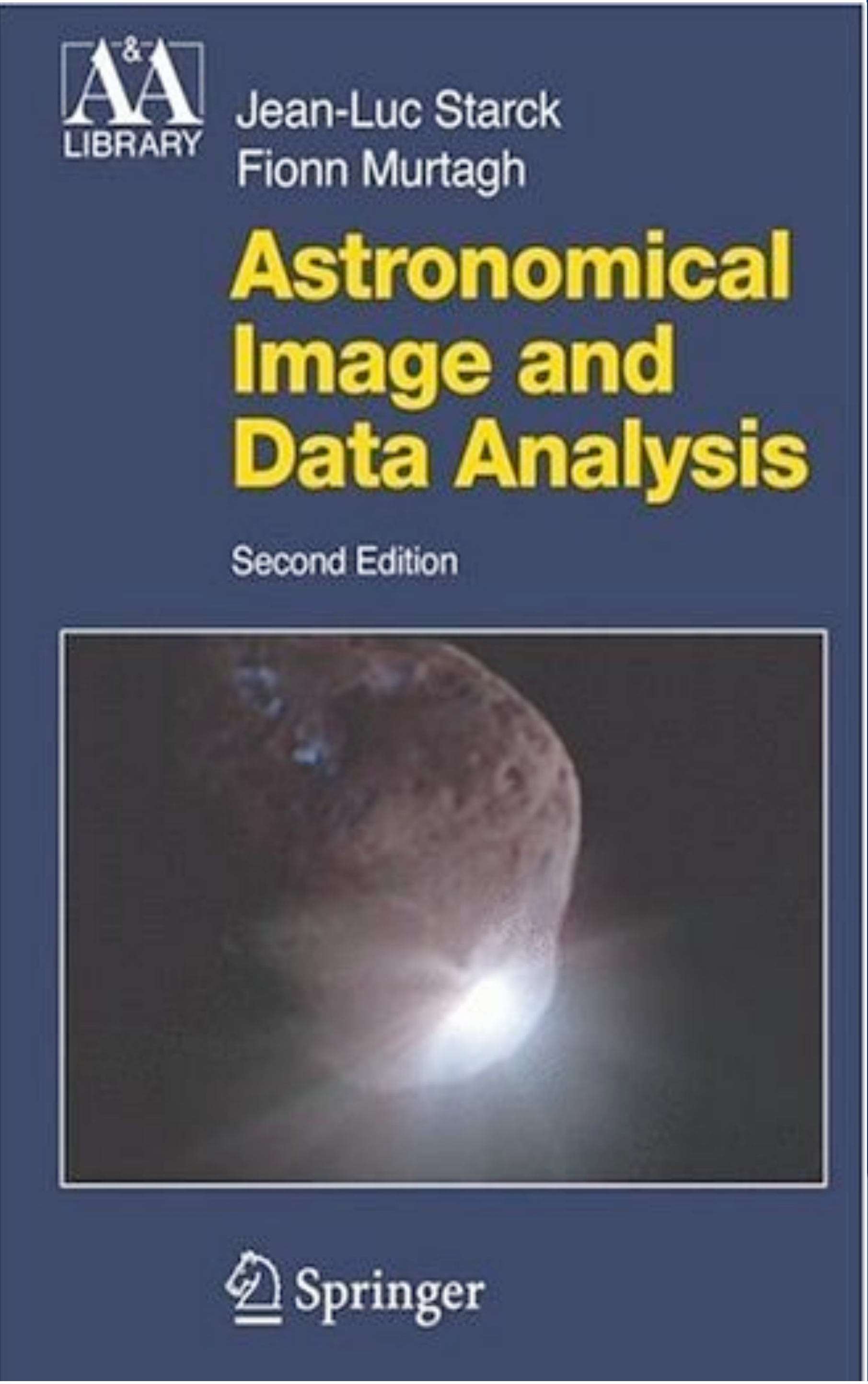
Polarized Spherical Wavelets and Curvelets: SparsePol/Version 1.0

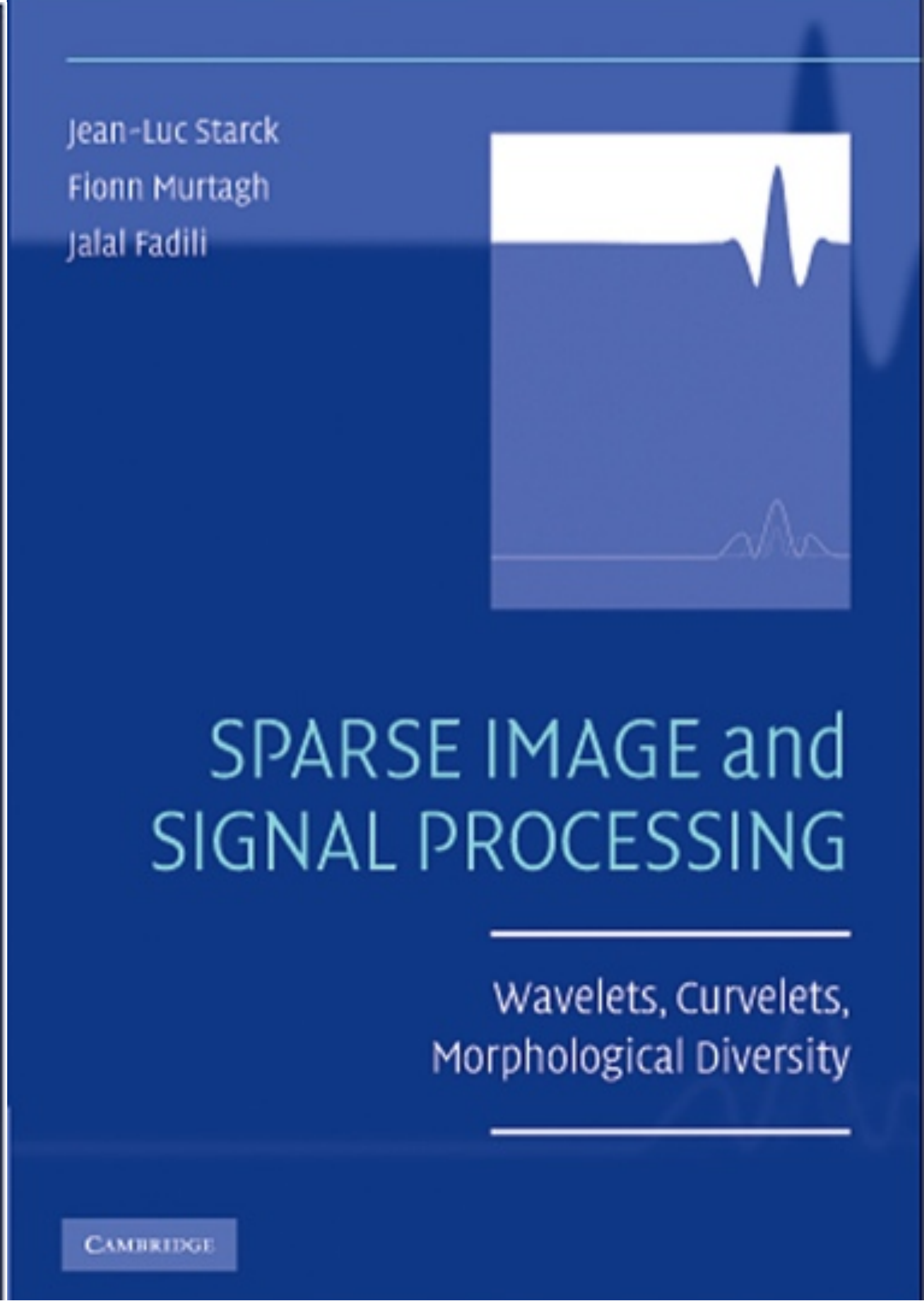
J.-L. Starck, Y. Moudden and J. Bobin, "Polarized Wavelets and Curvelets on the Sphere", Astronomy and Astrophysics, 497, 3, pp 931--943, 2009.

Multi-scale Variance Stabilizing Transform on the Sphere: MS-VSTS/Version 1.0

J. Schmitt, J.L. Starck, J.M. Casandjian, J. Fadili, I. Grenier, "Multichannel Poisson Denoising and Deconvolution on the Sphere : Application to the Fermi Gamma Ray Space Telescope, **Astronomy and Astrophysics** , 546, id.A114, pp10, 2012.

http://www.cosmostat.org/software.html


A&A LIBRARY
Jean-Luc Starck
Fionn Murtagh
Astronomical Image and Data Analysis
Second Edition
Springer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Wavelets, Curvelets, Morphological Diversity
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