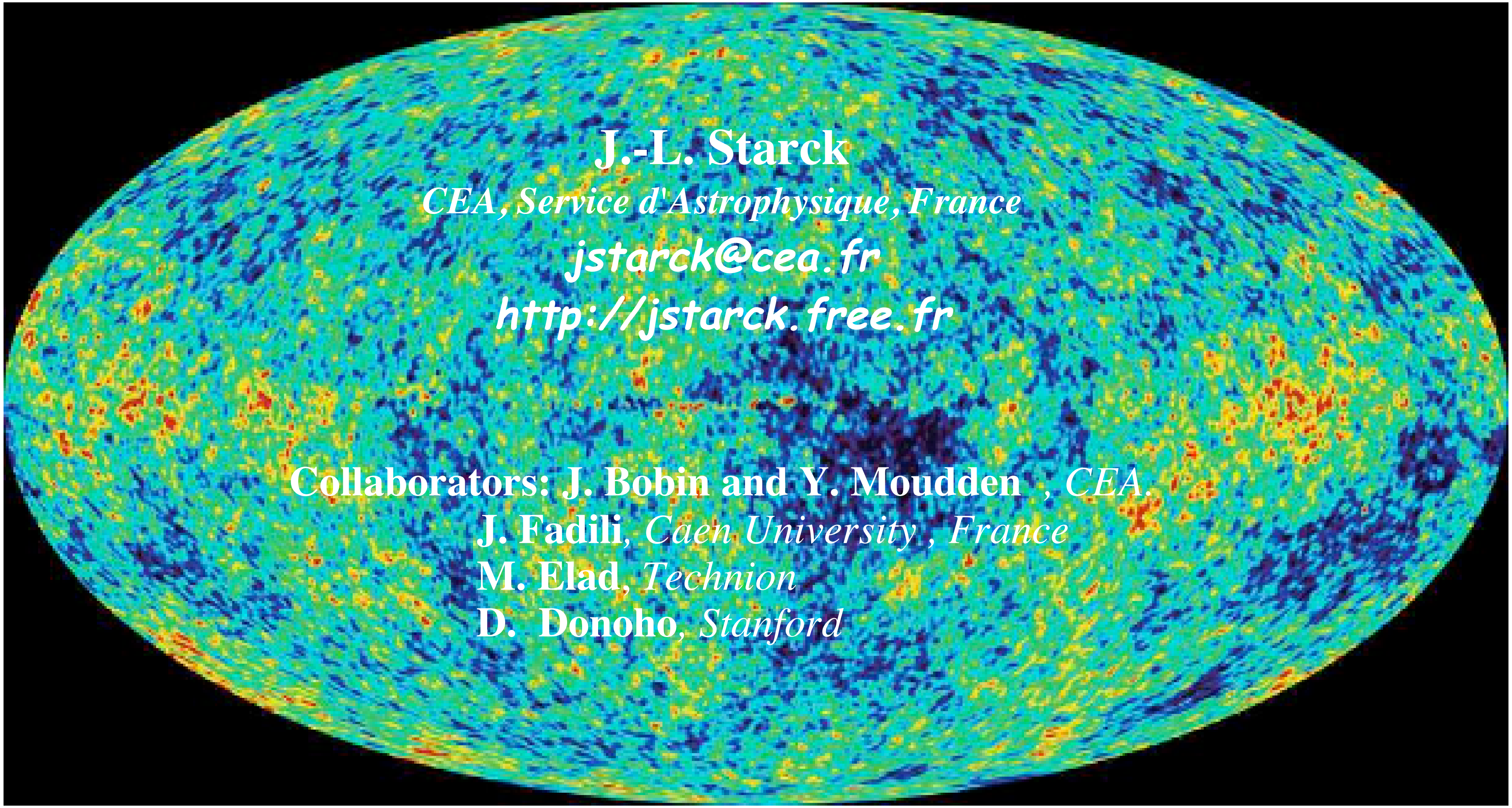
Morphological Component Analysis: From Images to Hyperspectral Data



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D. Donoho, *Stanford*

- Introduction: Sparsity

- Morphological Diversity

  The Morphological Component Analysis (MCA) and its applications

- Multichannel/Hyperspectral Data (GMCA)

- Application for PLANCK
  - Extension of Multiscale Methods on the Sphere
  - Extension of GMCA on the Sphere

What is a good representation for data?

- Computational harmonic analysis seeks representations of a signal as linear combinations of basis, frame, dictionary, element :

$$f = \sum_k a_k \, \mathbf{b}_k$$

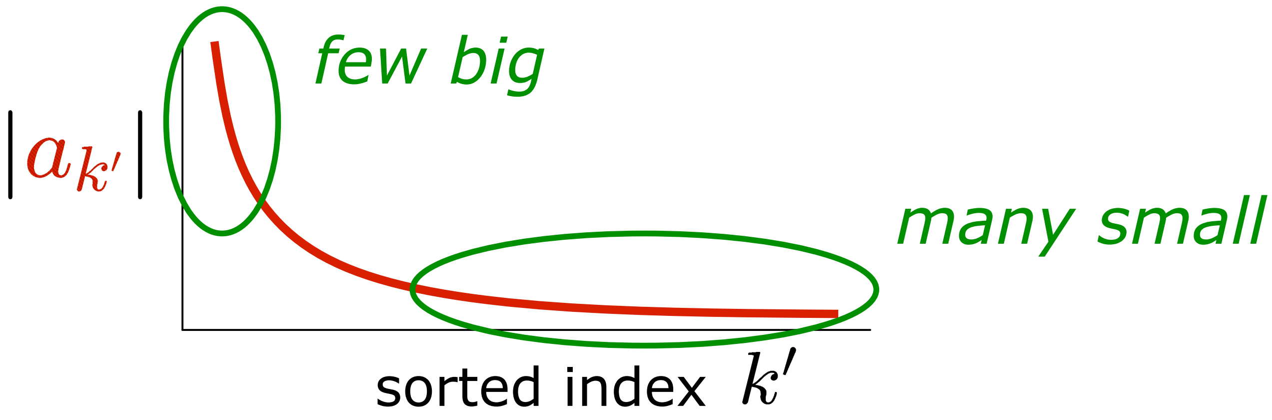
coefficients basis, frame

- Fast calculation of the coefficients a_k
- Analyze the signal through the statistical properties of the coefficients
- Approximation theory uses the sparsity of the coefficients.

Seeking sparse and generic representations

Non-linear approximation curve (reconstruction error versus nbr of coeff)

- Sparsity



- Why do we need sparsity?
  - data compression
  - Feature extraction, detection
  - Image restoration

Truncated Fourier series give very good approximations to smooth functions, but

–Provides poor representation of non stationary signals or image.

–Provides poor representations of discontinuous objects (Gibbs effect)

JPEG / JPEG2000

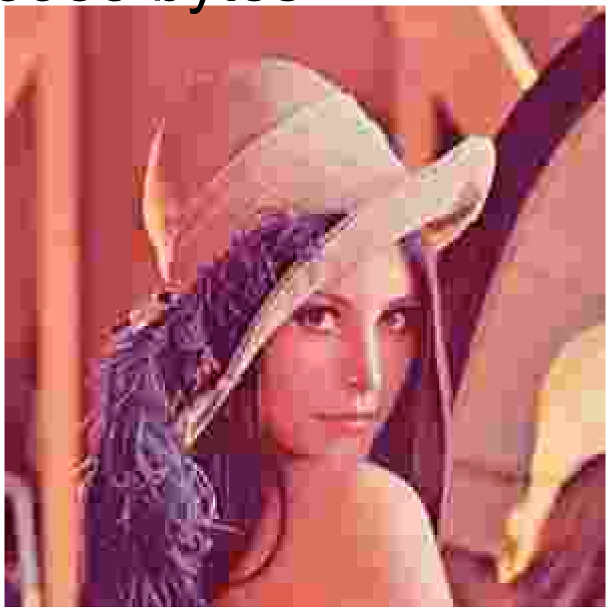
Original
BMP

300x300x24

270056
bytes

JPEG 1:68

3983 bytes

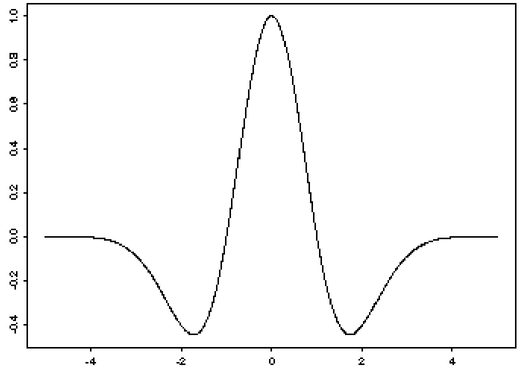


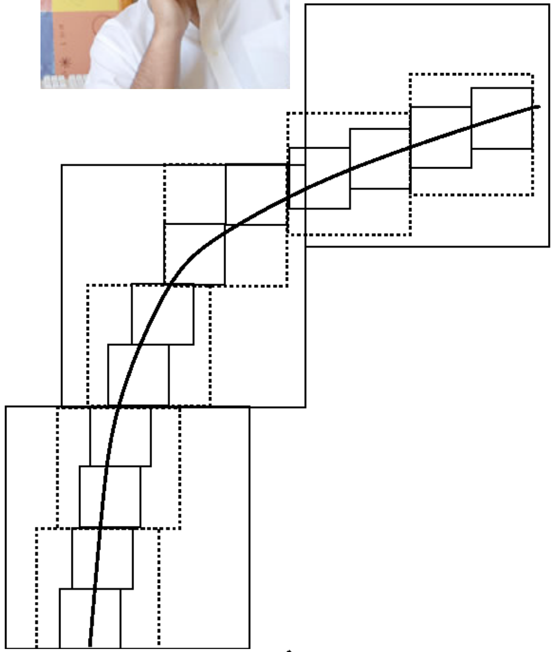
JPEG2000
1:70

3876 bytes

The Curvelet Transform (1999)

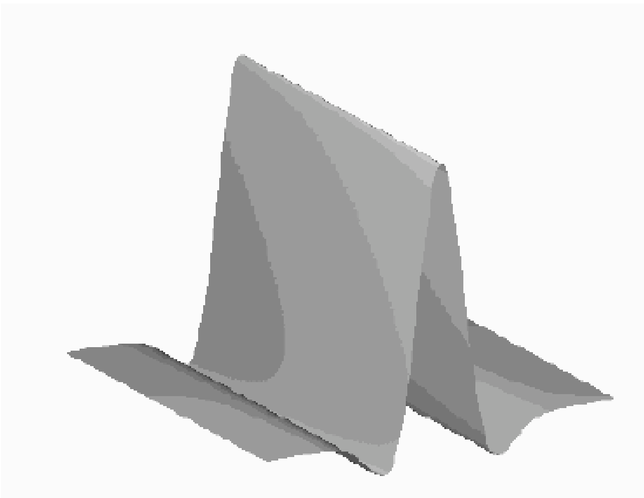
Wavelet

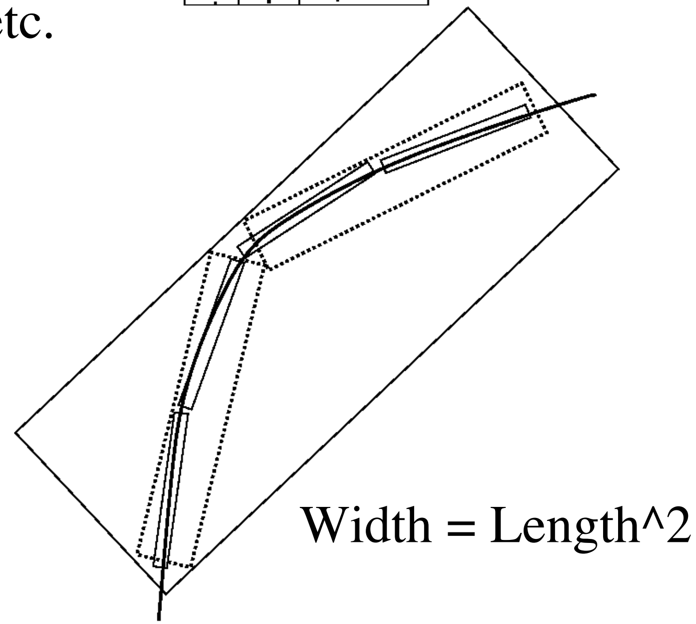




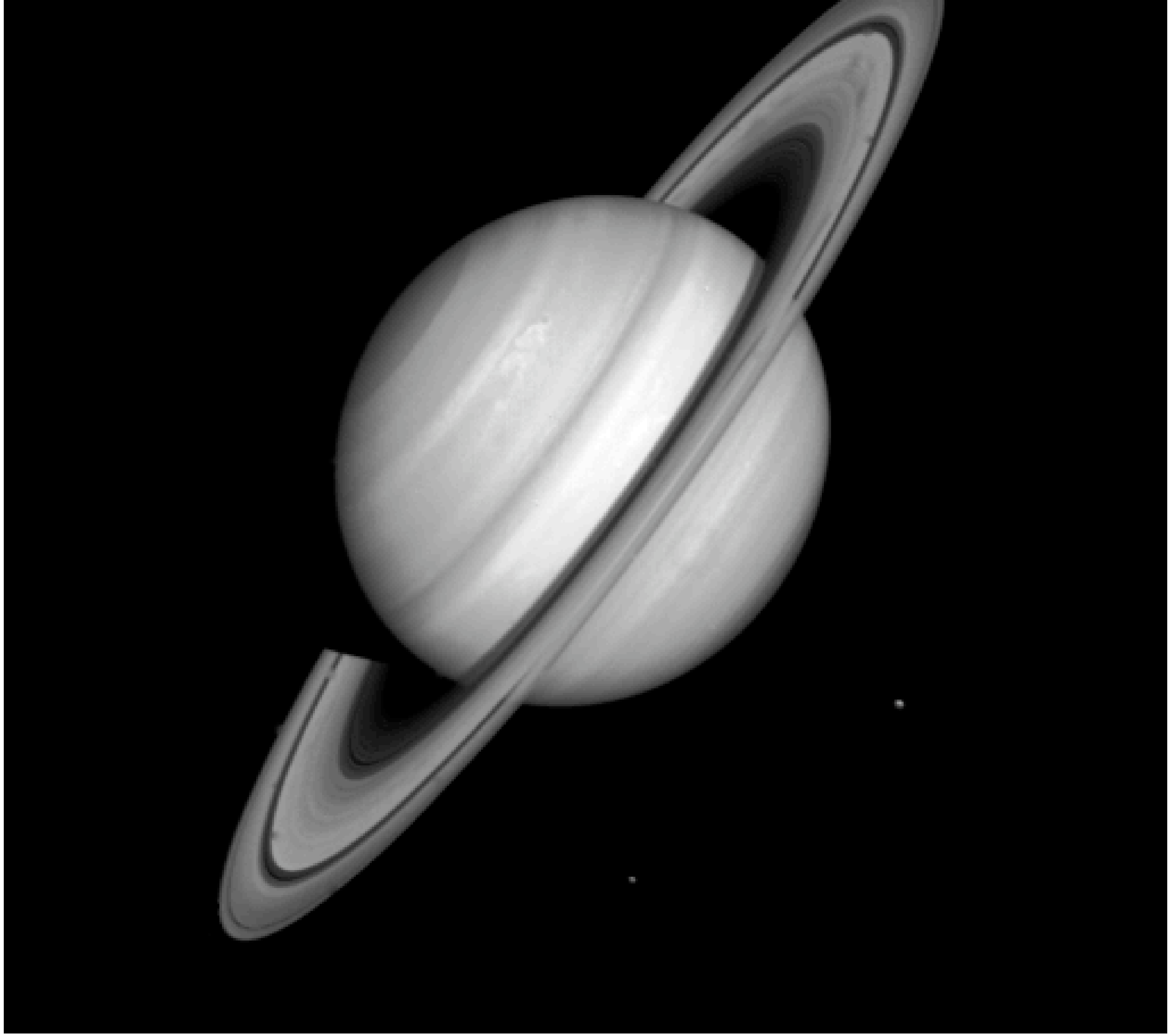
ridgelets, curvelets, contourlets, bandelettes, etc.

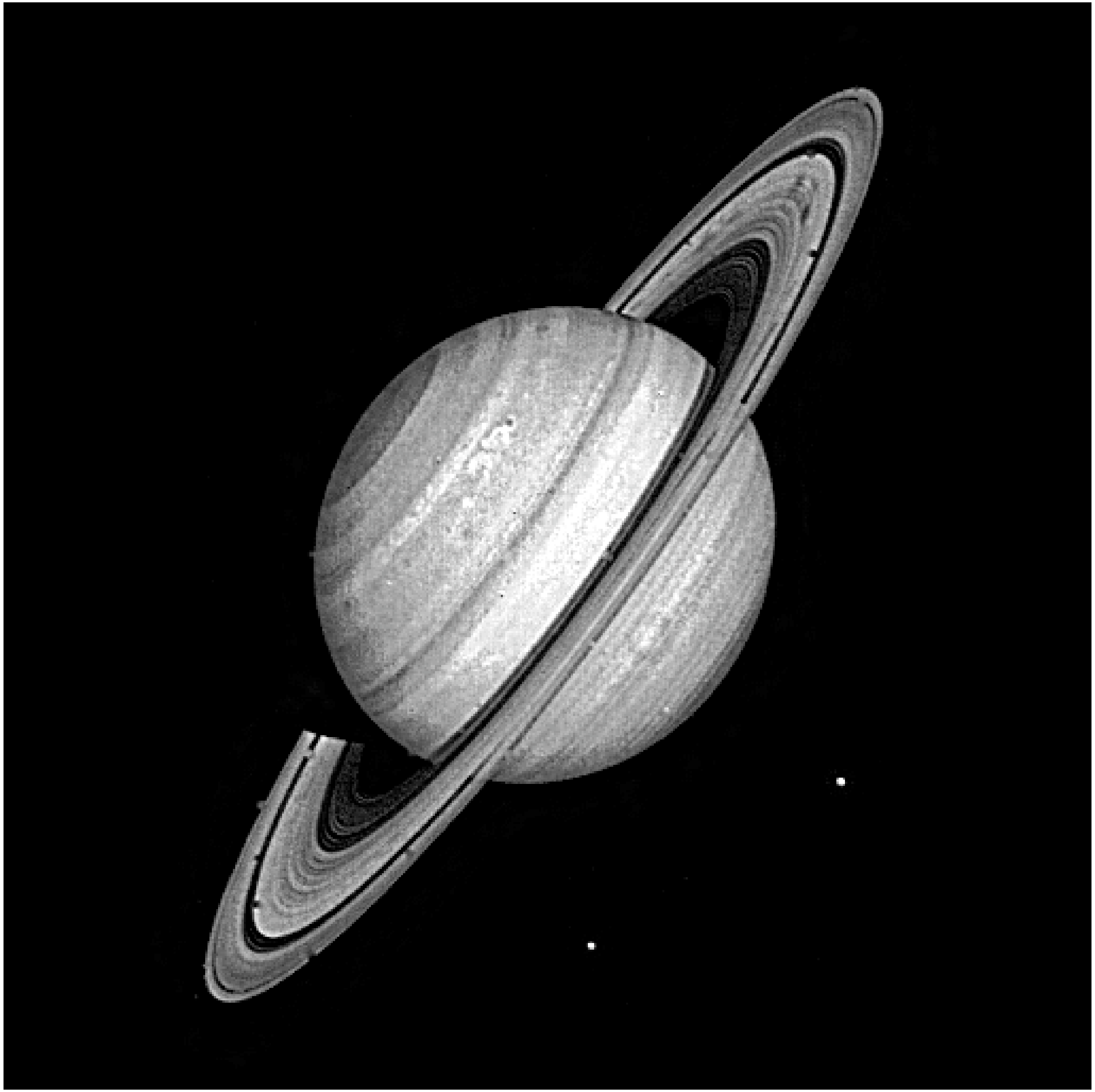
Curvelet





Contrast Enhancement





Multiscale Transforms

Critical Sampling

(bi-) Orthogonal WT
Lifting scheme construction
Wavelet Packets
Mirror Basis

Redundant Transforms

Pyramidal decomposition (Burt and Adelson)
Undecimated Wavelet Transform
Isotropic Undecimated Wavelet Transform
Complex Wavelet Transform
Steerable Wavelet Transform
Dyadic Wavelet Transform
Nonlinear Pyramidal decomposition (Median)

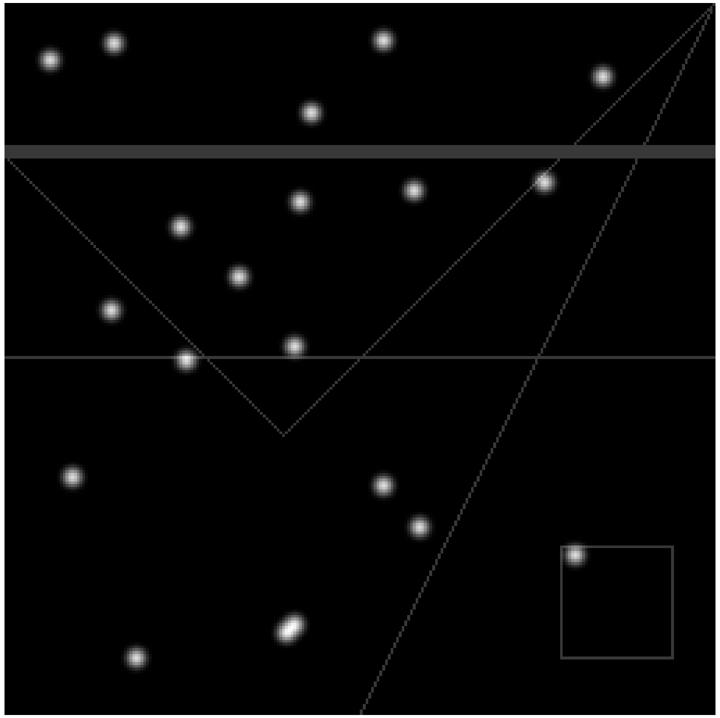
New Multiscale Construction

Contourlet
Bandelet
Finite Ridgelet Transform
Platelet
(W-)Edgelet
Adaptive Wavelet

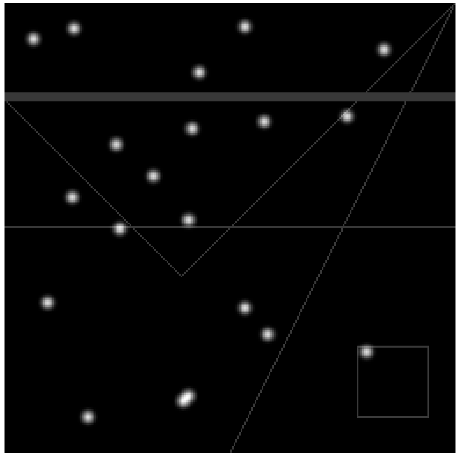
Ridgelet
Curvelet (Several implementations)
Wave Atom

A difficult issue

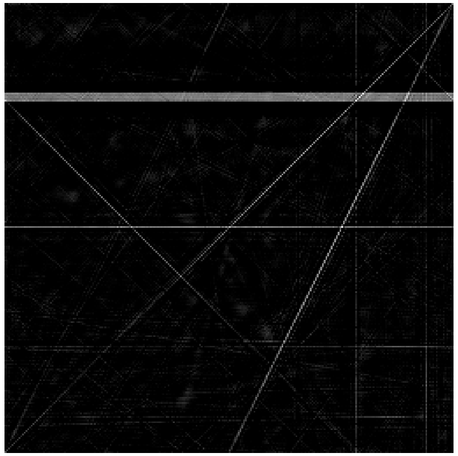
Is there any representation that well represents the following image ?



Going further

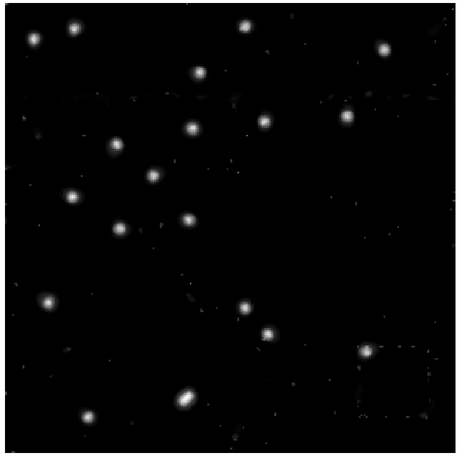


=



Lines

+



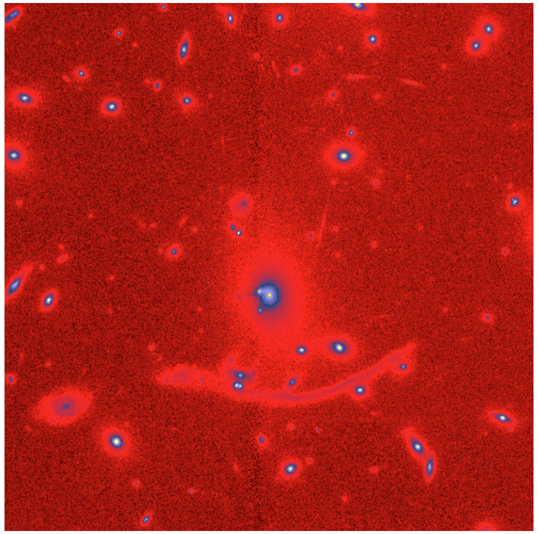
Gaussians

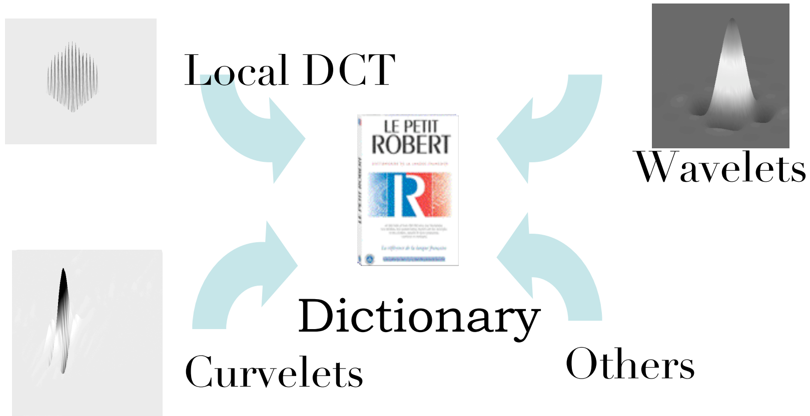
Curvelets

Wavelets

Redundant Representations

Morphological Diversity



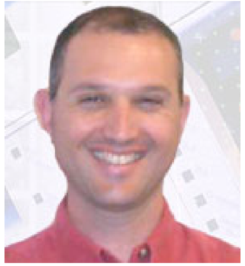


$$\phi = [\phi_1, \ldots, \phi_L], \quad \alpha = \{\alpha_1, \ldots, \alpha_L\}, \quad s = \phi\alpha = \sum_{k=1}^{L} \phi_k \alpha_k$$

Model:

$$s = \sum_{k=1}^{L} s_k + n$$

and s_k ($s_k = \phi_k \alpha_k$) is sparse in ϕ_k .



New Perspectives

Morphological Component Analysis (MCA)

- ***Redundant Multiscale Transforms and their Application for Morphological Component Analysis***, *Advances in Imaging and Electron Physics, 132, 2004.*
- ***Image Decomposition Via the Combination of Sparse Representation and a Variational Approach***, *IEEE Trans. on Image Proces., 14, 10, pp 1570--1582, 2005*
- ***Morphological Component Analysis: an adaptive thresholding strategy***, IEEE Trans. on Image Processing, Vol 16, No 11, pp 2675--2681, 2007..

$$J(s_1,\ldots,s_L) = \left\| s - \sum_{k=1}^{L} s_k \right\|_2^2 + \lambda \sum_{k=1}^{L} \left\| T_k s_k \right\|_p$$

Morphological Component Analysis (MCA)

$$J(s_1,\ldots,s_L) = \left\| s - \sum_{k=1}^{L} s_k \right\|_2^2 + \lambda \sum_{k=1}^{L} \left\| T_k s_k \right\|_p$$

. Initialize all s_k to zero

. Iterate j=1,...,Niter

- Iterate k=1,..,L

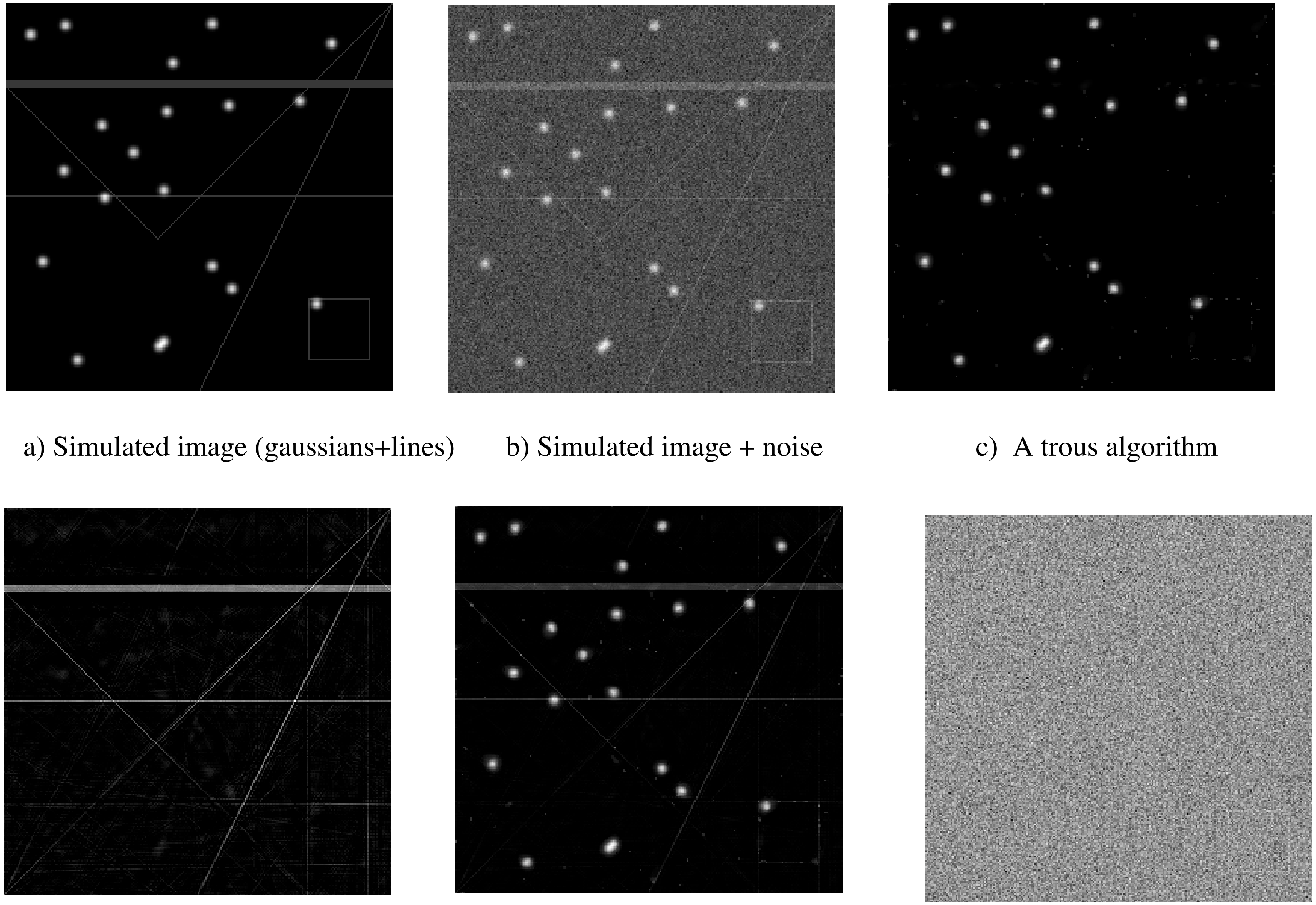
Update the kth part of the current solution by fixing all other parts and minimizing:

$$J(s_k) = \left\| s - \sum_{i=1, i \neq k}^{L} s_i - s_k \right\|_2^2 + \lambda^{(j)} \left\| T_k s_k \right\|_p$$

Which is obtained by a simple **hard**/soft thresholding of : $s_r = s - \sum_{i=1, i \neq k}^{L} s_i$

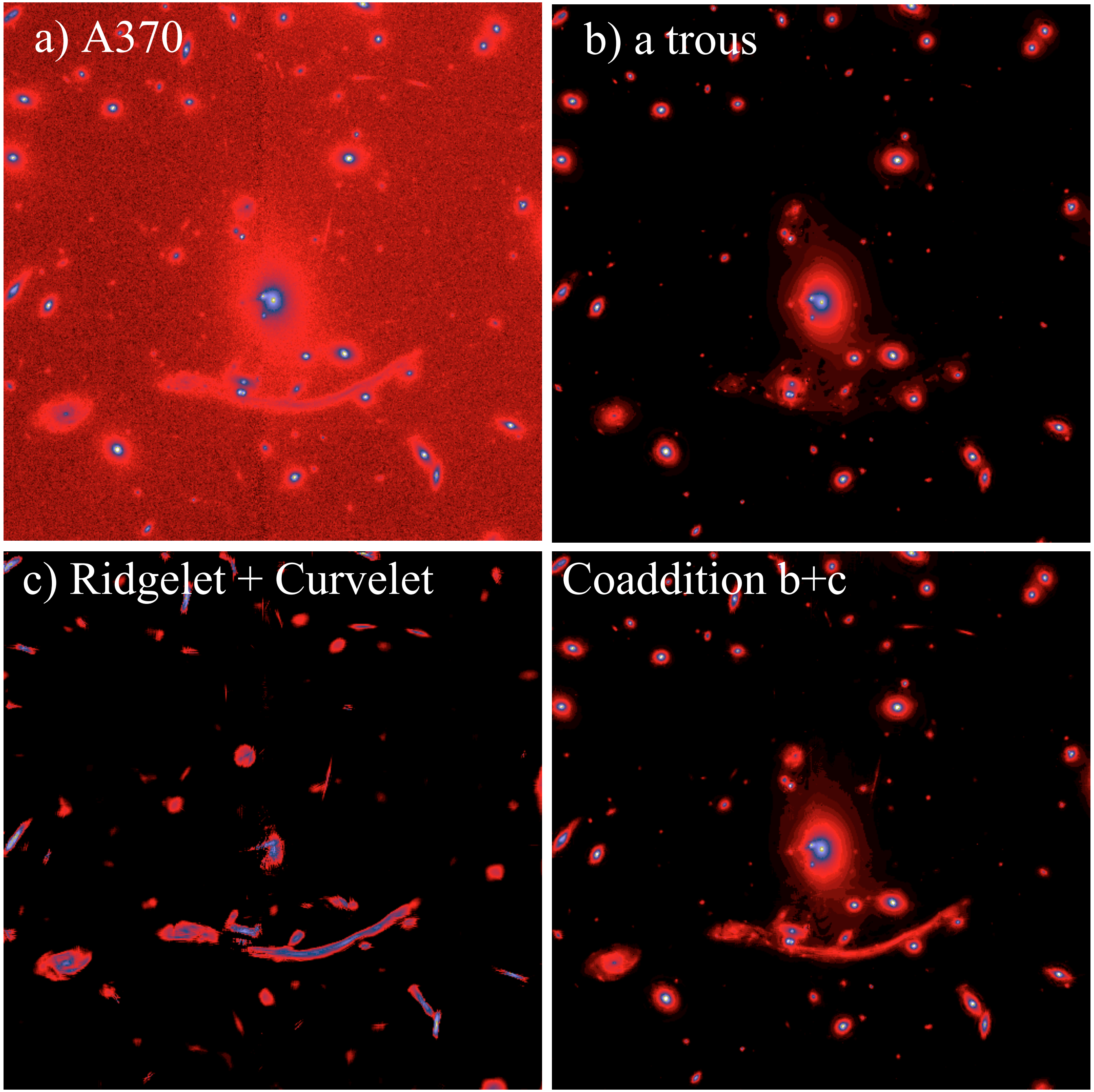
- Decrease the threshold $\lambda^{(j)}$

$$MIN_{s_1,s_2}\left(\left\|Ws_1\right\|_p + \left\|Cs_2\right\|_p\right) \text{ subject to } \left\|s-(s_1+s_2)\right\|_2^2 < \varepsilon$$



a) Simulated image (gaussians+lines) b) Simulated image + noise c) A trous algorithm

d) Curvelet transform e) coaddition c+d f) residual = e-b

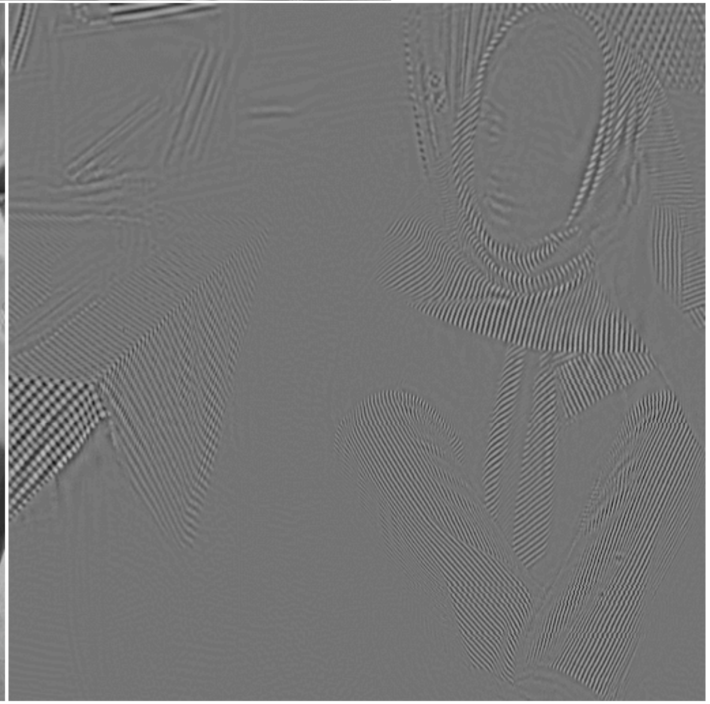


a) A370
b) a trous
c) Ridgelet + Curvelet
Coaddition b+c

Separation of Texture from Piecewise Smooth Content

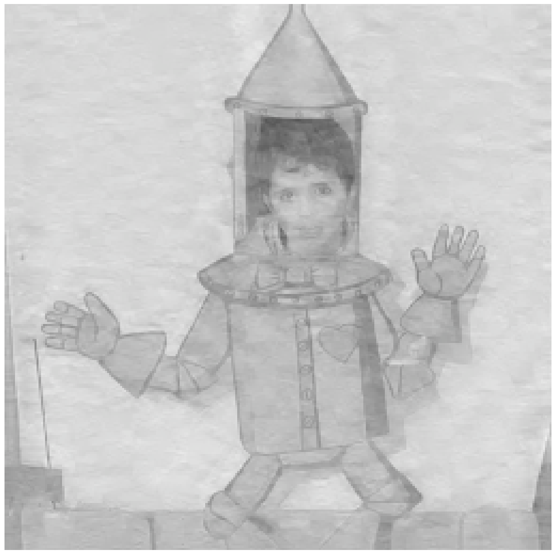
<u>The separation task</u>: decomposition of an image into a texture and a natural (piecewise smooth) scene part.

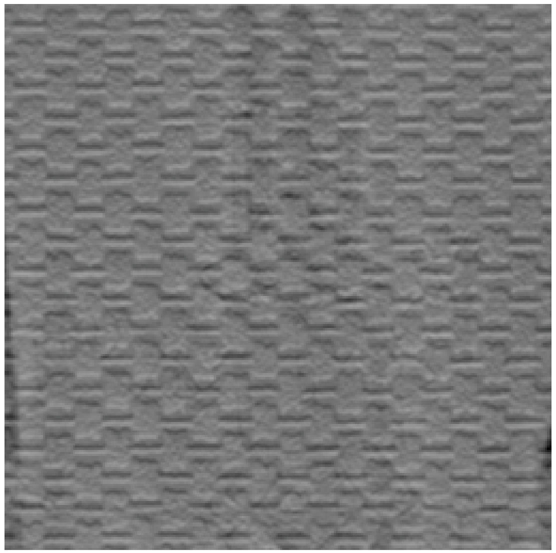
•***Image Decomposition Via the Combination of Sparse Representation and a Variational Approach***, *IEEE Trans. on Image Proces., 14, 10, pp 1570--1582, 2005*



Texture Separation using MCA: Curvelet + DCT

X_n





X_t

Edge Detection

Interpolation of Missing Data

$$J(s_1,\ldots,s_L) = \left\| M\left(s - \sum_{k=1}^{L} s_k\right) \right\|_2^2 + \lambda \sum_{k=1}^{L} \left\| T_k s_k \right\|_p$$

Where M is the mask: M(i,j) = 0 ==> missing data
M(i,j) = 1 ==> good data

If the data are composed of a piecewise smooth component + texture

$$J(X_t, X_n) = \left\| M(X - X_t - X_n) \right\|_2^2 + \lambda\left(\left\| \mathbf{C} X_n \right\|_1 + \left\| \mathbf{D} X_t \right\|_1 \right) + \gamma \,\mathrm{TV}(X_n)$$

•*M. Elad, J.-L. Starck, D.L. Donoho, P. Querre, "Simultaneous Cartoon and Texture Image Inpainting using Morphological Component Analysis (MCA)", ACHA, Vol. 19, pp. 340-358, November 2005.*
•*M.J. Fadili, J.-L. Starck and F. Murtagh, "Inpainting and Zooming using Sparse Representations", in press.*

. Initialize all s_k to zero

. Iterate j=1,...,Niter

- Iterate k=1,..,L

  - Update the kth part of the current solution by fixing all other parts and minimizing:

$$J(s_k) = \left\| M(s - \sum_{i=1, i \neq k}^{L} s_i - s_k) \right\|_2^2 + \lambda \left\| T_k s_k \right\|_1$$

Which is obtained by a simple soft thresholding of :

$$s_r = M(s - \sum_{i=1, i \neq k}^{L} s_i)$$

20%

50%

80%

Morpho-Spectral Diversity

Data: $X = [x_1, \ldots, x_m]$

Source: $S = [s_1, \ldots, s_n]$

$$X = [x_1, \ldots, x_m] = AS$$

$$x_l = \sum_{i=1}^{n} a_{i,l} s_i$$

$$\min_\alpha \|\alpha\|_p \text{ s.t } \mathbf{X} = \sum_{\gamma \in \Gamma} \alpha_\gamma \psi_\gamma$$

$\mathbf{\Phi_A} = [\mathbf{\Phi_{A,1}}, \mathbf{\Phi_{A,2}}]$

$\mathbf{\Phi_S}$

Spatial Dictionary with
Spectral Dictionary

$$\mathbf{\Psi} = [\mathbf{\Phi_{A,1}} \otimes \mathbf{\Phi_S}, \mathbf{\Phi_{A,2}} \otimes \mathbf{\Phi_S}]$$

Generalized MCA (GMCA)

•J. Bobin, Y. Moudden, J.-L. Starck and M. Elad, "***Morphological Diversity and Source Separation***", IEEE Transaction on Signal Processing , Vol 13, 7, pp 409--412, 2006.

•J. Bobin, J.-L. Starck, M.J. Fadili, and Y. Moudden, "***Sparsity, Morphological Diversity and Blind Source Separation***" , Vol 16, No 11, pp 2675--2681, 2007.

Source: $S = [s_1, \ldots, s_n]$ Data: $X = [x_1, \ldots, x_m] = AS$

We now assume that the sources are linear combinations of morphological components :

$$s_i = \sum_{k=1}^{K} c_{i,k} \quad \text{such that} \quad \alpha_{i,k} = T_{i,k} c_{i,k} \text{ sparse}$$

$$\Longrightarrow \qquad X_l = \sum_{i=1}^{n} A_{i,l} s_i = \sum_{i=1}^{n} A_{i,l} \sum_{k=1}^{K} c_{i,k}$$

$$\phi = [[\phi_{1,1}, \ldots, \phi_{1,K}], \ldots, [\phi_{n,1}, \ldots, \phi_{n,K}]], \quad \alpha = S\phi^t = [[\alpha_{1,1}, \ldots, \alpha_{1,K}], \ldots, [\alpha_{n,1}, \ldots, \alpha_{n,K}]]$$

GMCA aims at solving the following minimization:

$$\min_{A, c_{1,1}, \ldots, c_{1,K}, \ldots, c_{n,1}, \ldots, c_{n,K}} = \sum_{l=1}^{m} \left\| X_l - \sum_{i=1}^{n} A_{i,l} \sum_{k=1}^{K} c_{i,k} \right\|_2^2 + \lambda \sum_{i=1}^{n} \sum_{k=1}^{K} \left\| T_{i,k} c_{i,k} \right\|_p$$

The GMCA Algorithm

. Initialize all c_k to zero, $\lambda_1 = \max(\alpha), \delta = \max(\alpha)/\mathrm{Niter}$

. Iterate t=1,...,Niter

- Iterate i=1,..,NbrSource

  Defining a multichannel residual $\mathbf{D_i}$: $$D_i = X - \sum_{i' \neq i} a^{i'} s_{i'}$$

  Iterate k=1,..,K_k

  - Least square estimate of $c_{i,k}$: $$l_{i,k} = \frac{1}{a^{i^T} a^i} a^{i^T} (D_i - a^i \sum_{k' \neq k} c_{i,k'})$$

  - Minimize: $$J(\tilde{l}_{i,k}) = \left\| l_{i,k} - \tilde{l}_{i,k} \right\|_2^2 + \lambda_t \left\| T_{i,k} \tilde{l}_{i,k} \right\|_1$$

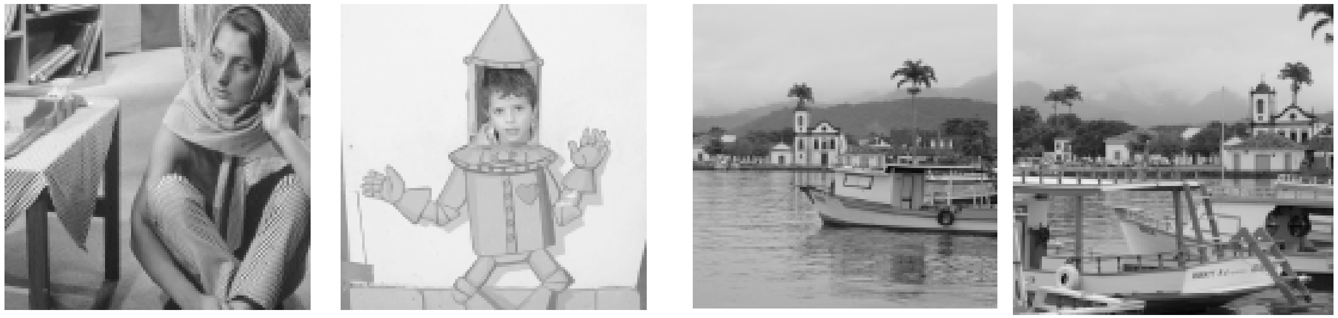
    which is obtained by a simple hard/soft thresholding of $l_{i,k}$

  $$s_k = \sum_i l_{k,i}$$

- $S = [s_1, \ldots, s_K]^t$
- Estimation of the matrix A: $A = XS^t (SS^t)^{-1}$
- Decrease $\lambda_{t+1} = \lambda_t - \delta$

BSS experiment : Noiseless case

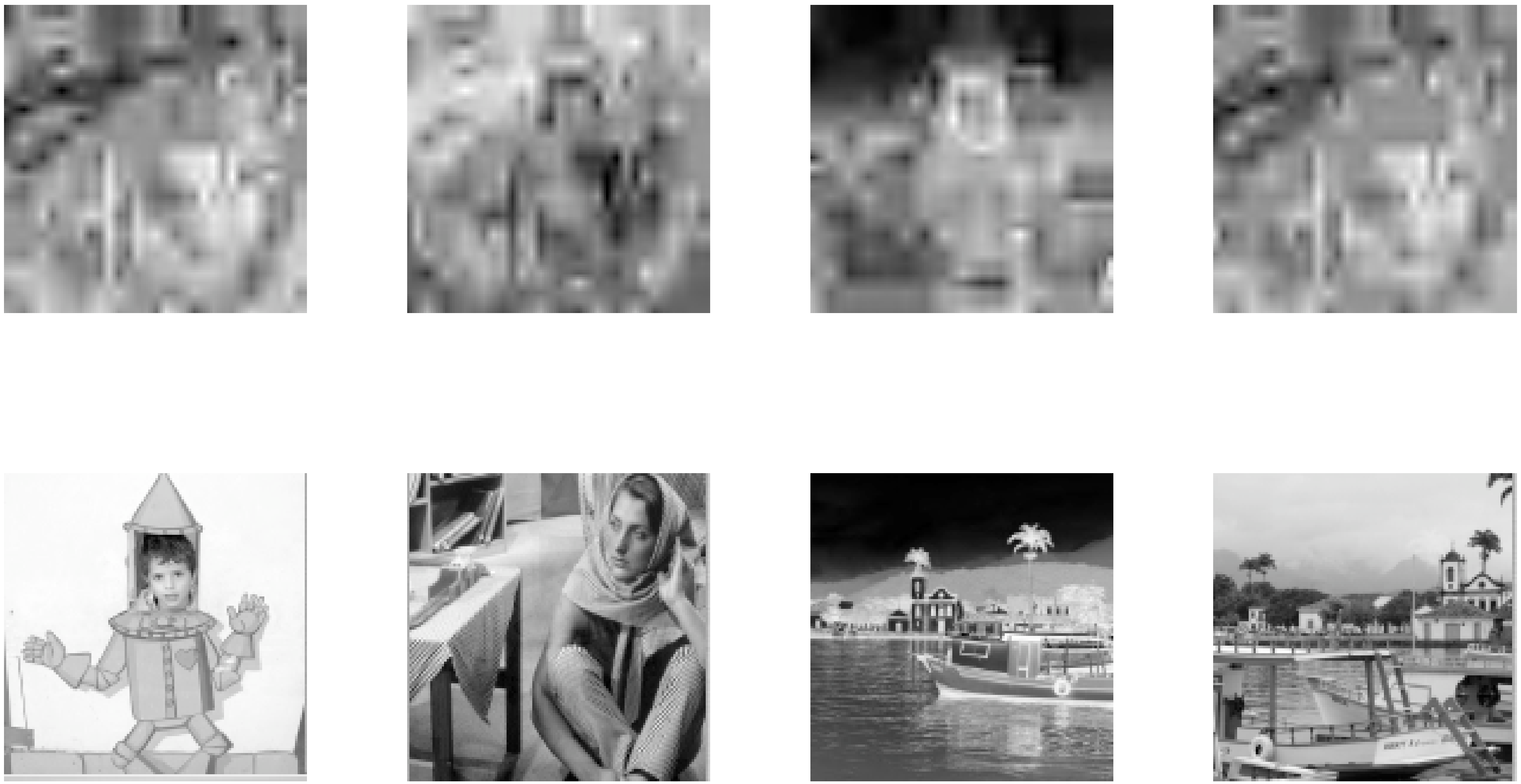
Original Sources

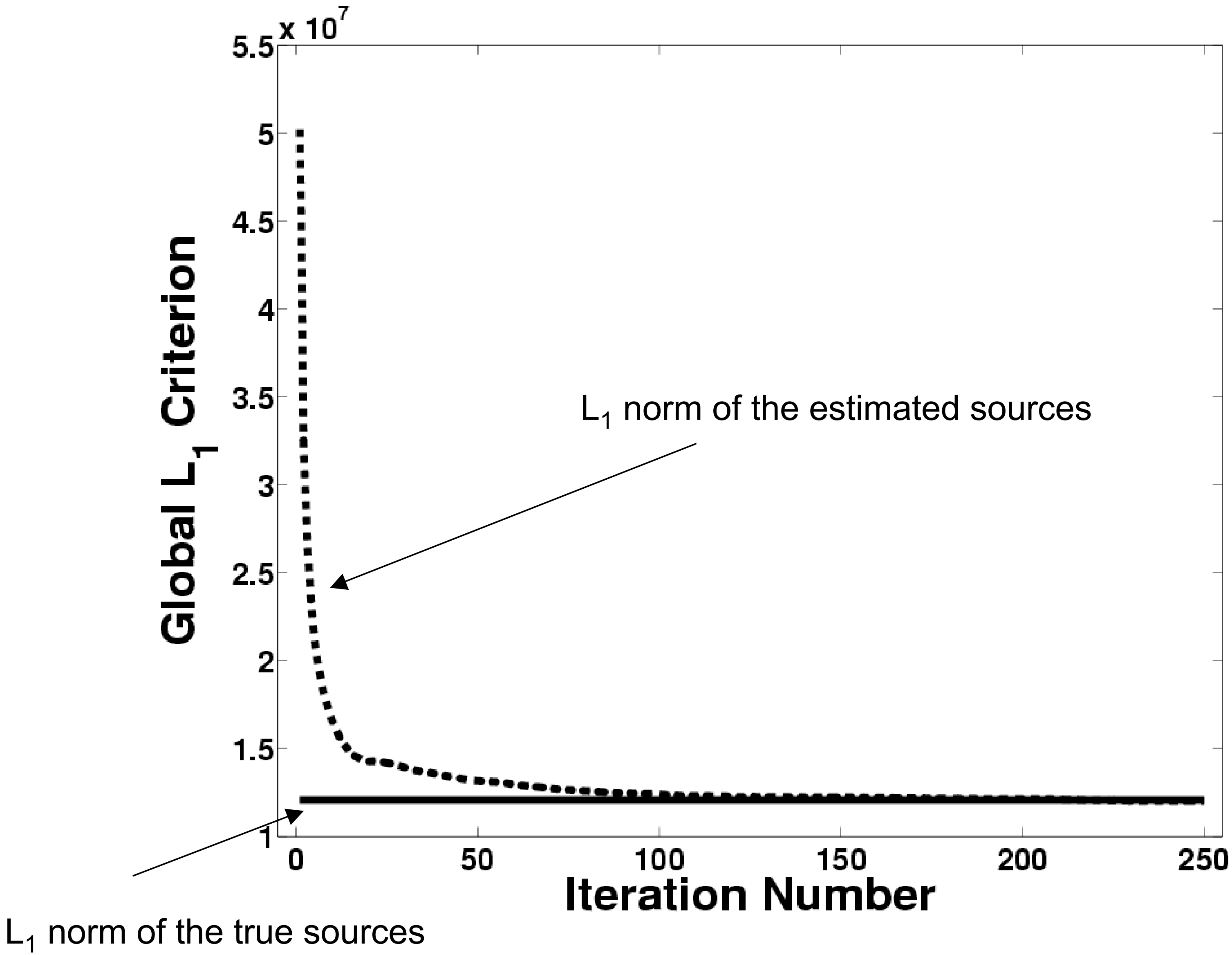


2 of 4 Mixtures

Noiseless experiment, 4 random mixtures, 4 sources

Results

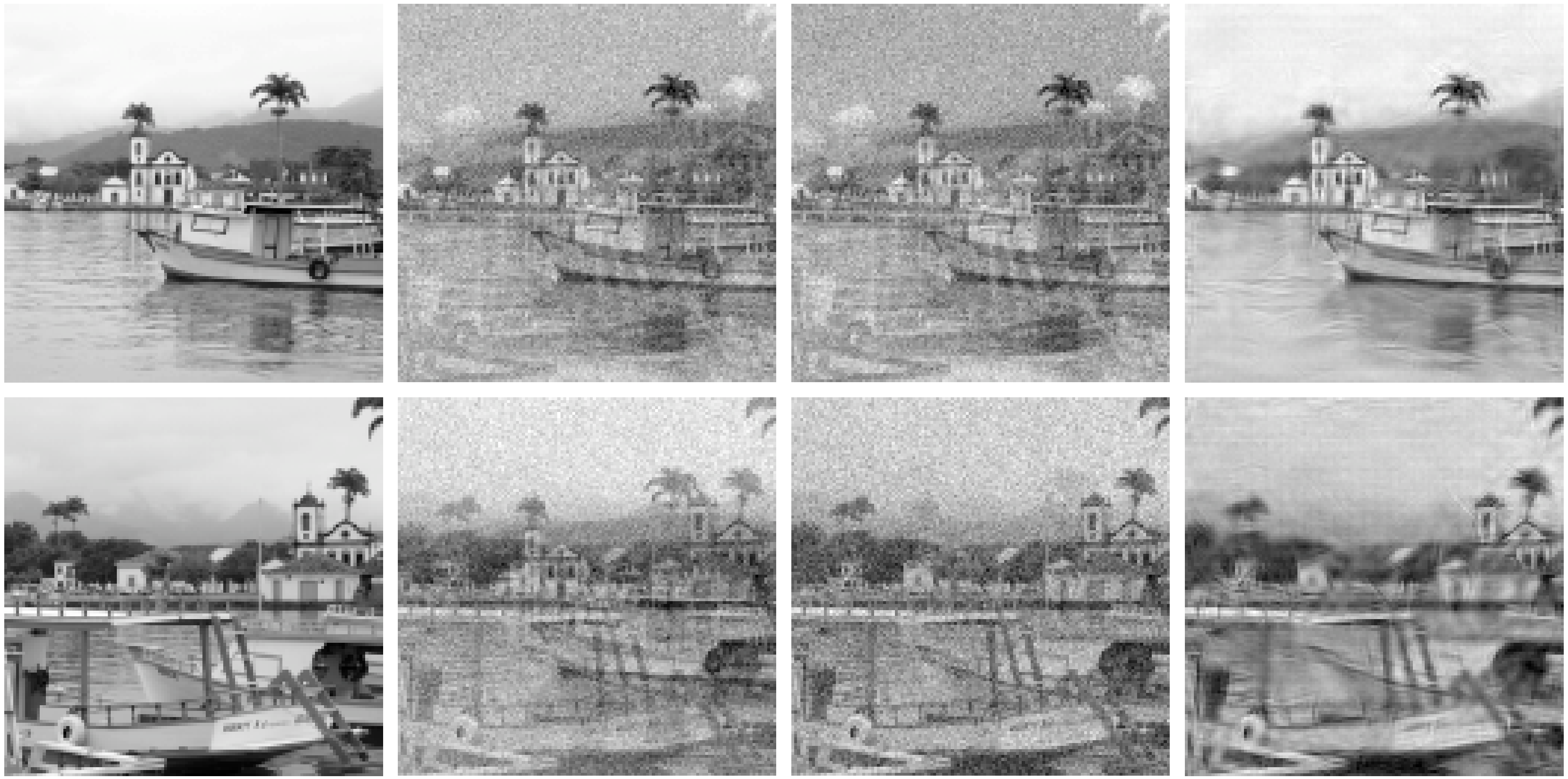




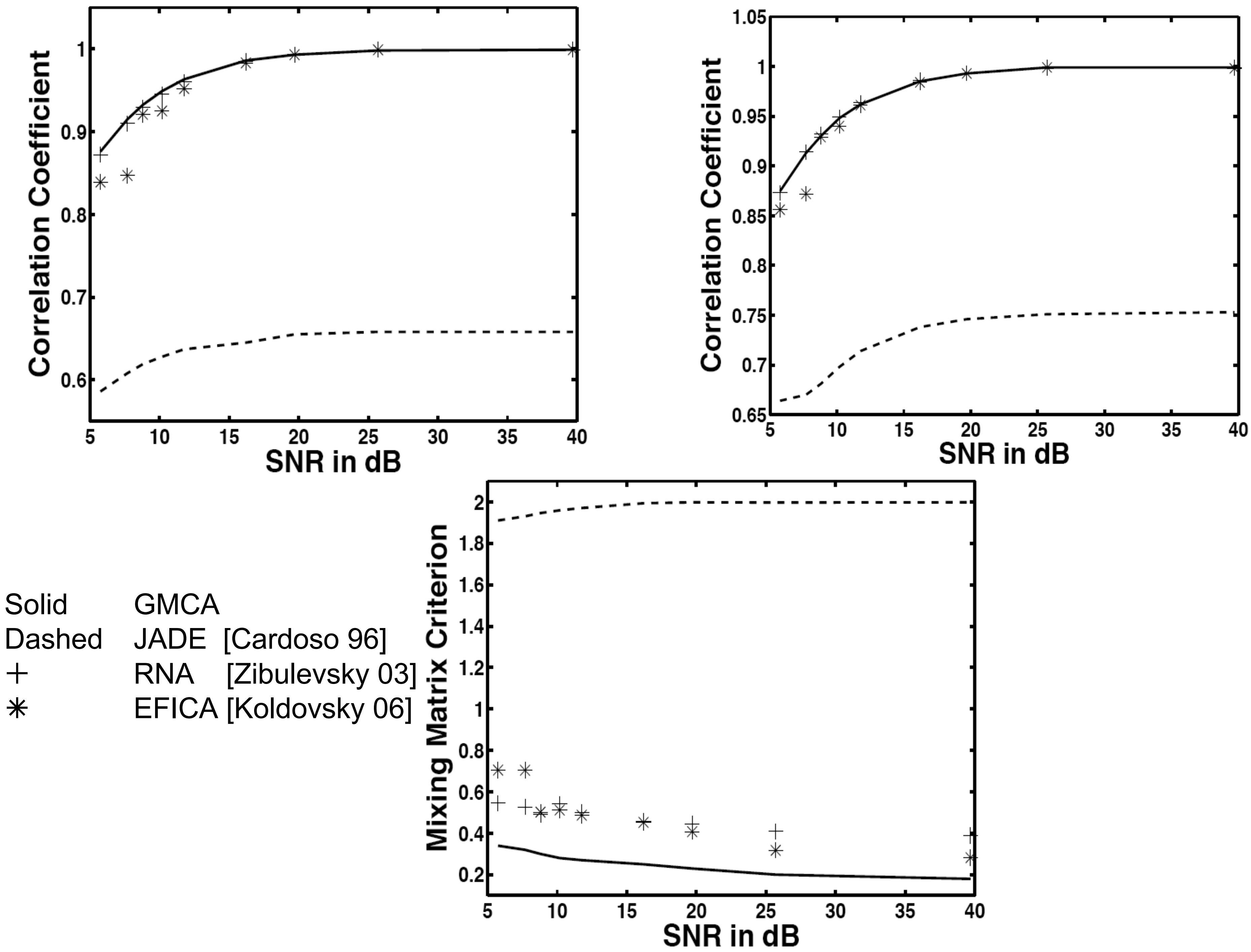
x 10^7
5.5
5
4.5
4
3.5
3
2.5
2
1.5
1
0
50
100
150
200
250
Global L1 Criterion
Iteration Number
L1 norm of the estimated sources
L1 norm of the true sources

BSS experiment : Noisy case

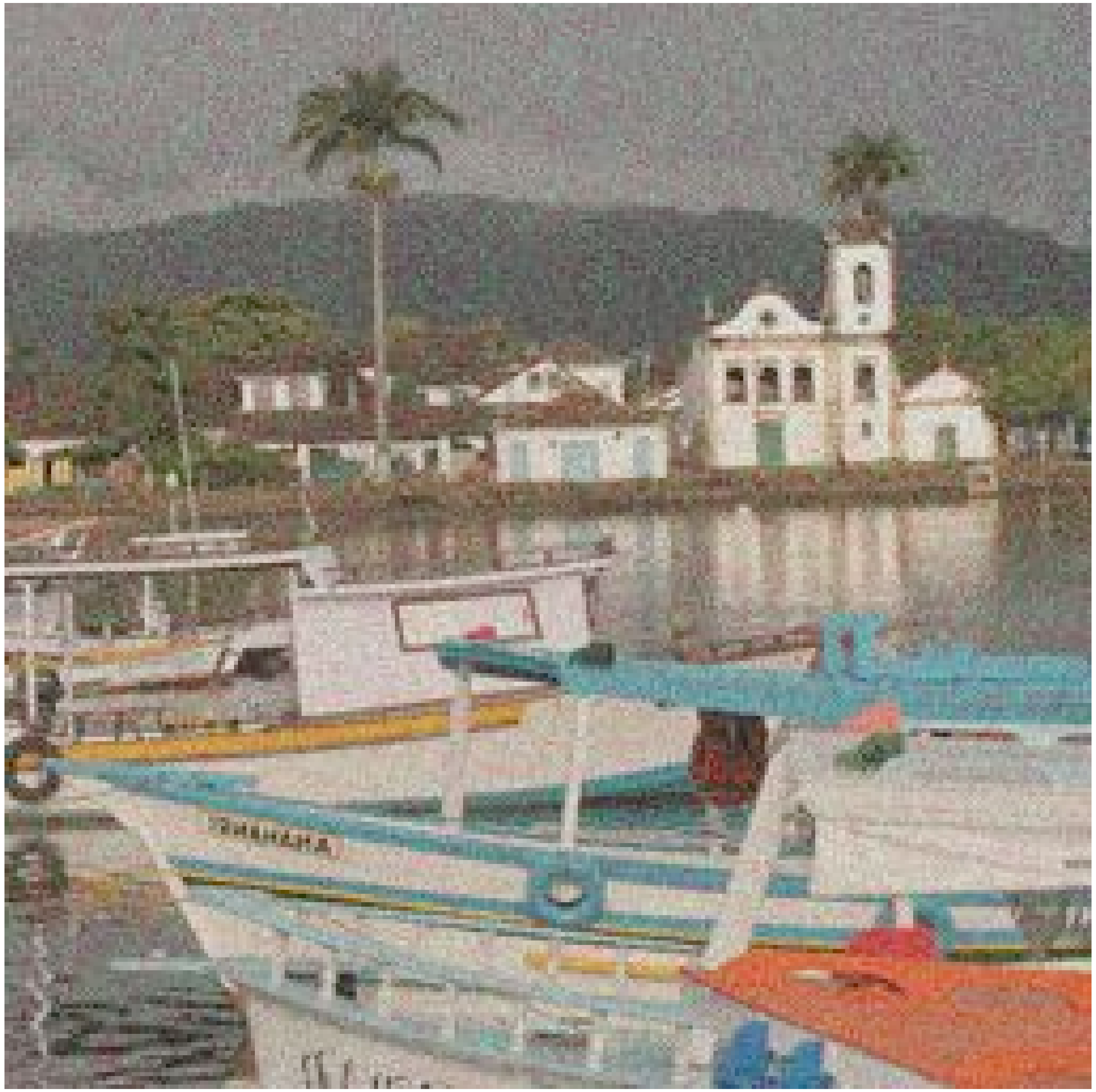
2 mixtures SNR = 10.4dB Dictionary: Curvelets + DCT



Sources Mixtures JADE GMCA


Correlation Coefficient
SNR in dB
Correlation Coefficient
SNR in dB
Mixing Matrix Criterion
SNR in dB
Solid GMCA
Dashed JADE [Cardoso 96]
RNA [Zibulevsky 03]
EFICA [Koldovsky 06]

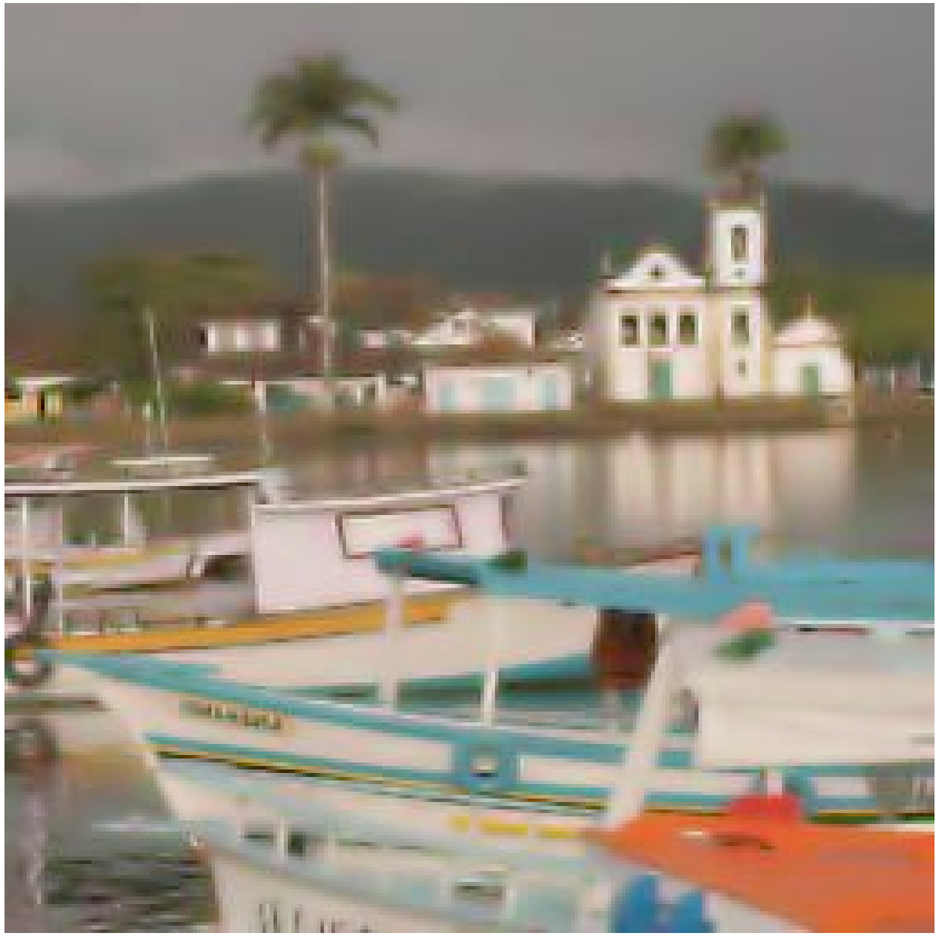
Color image denoising



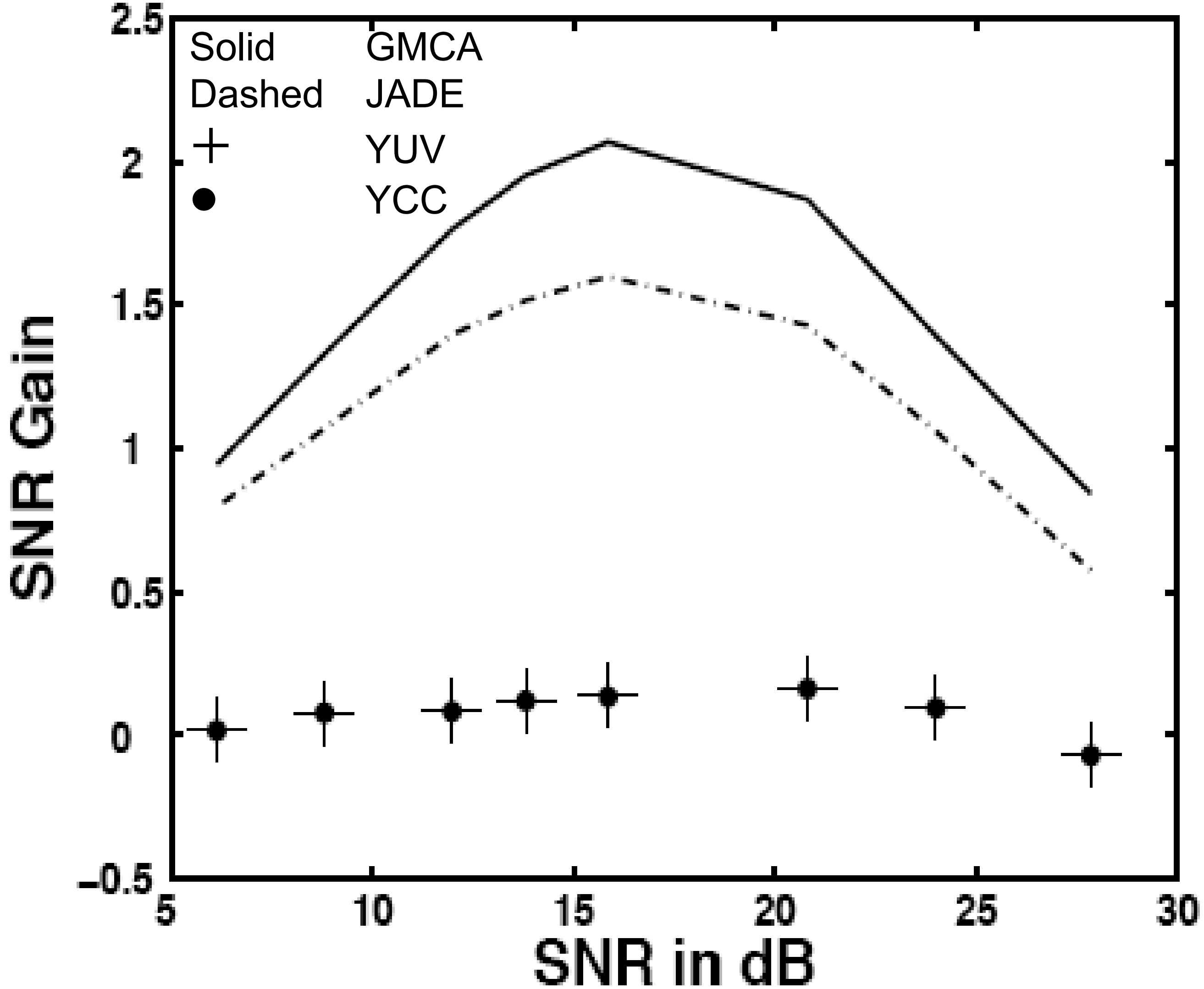
Noisy 15dB

Results

In RGB space



In GMCA space



SNR improvement compared to denoising in the RGB color space.

Multichannel Inpainting using GMCA

GMCA inpainting aims at solving the following minimization:

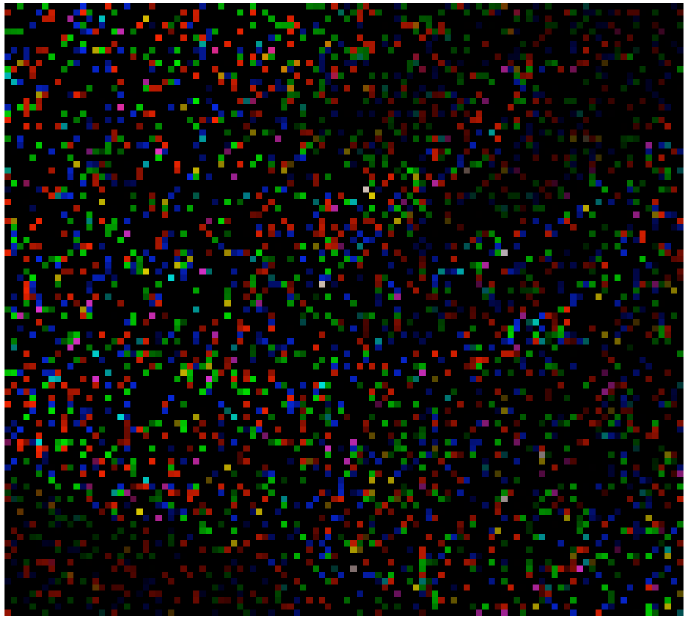
$$\min_{A, c_{1,1}, \ldots, c_{1,K}, \ldots, c_{n,1}, \ldots, c_{n,K}} = \sum_{l=1}^{m} \left\| M_l \left(X_l - \sum_{i=1}^{n} A_{i,l} \sum_{k=1}^{K} c_{i,k} \right) \right\|_2^2 + \lambda \sum_{i=1}^{n} \sum_{k=1}^{K} \left\| T_{i,k} c_{i,k} \right\|_p$$

Where M is the mask: $M_l(i,j) = 0$ ==> missing data

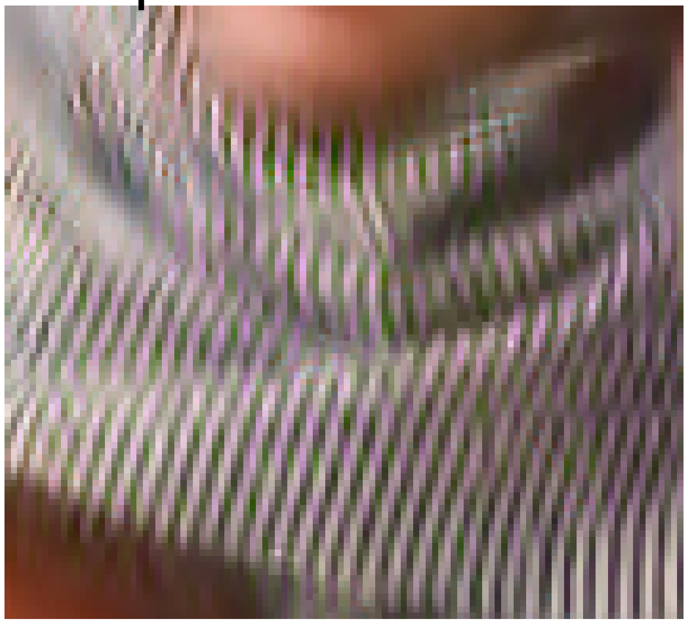
$M_l(i,j) = 1$ ==> good data

Original

Masked



Inpainted in GMCA



3 color channels

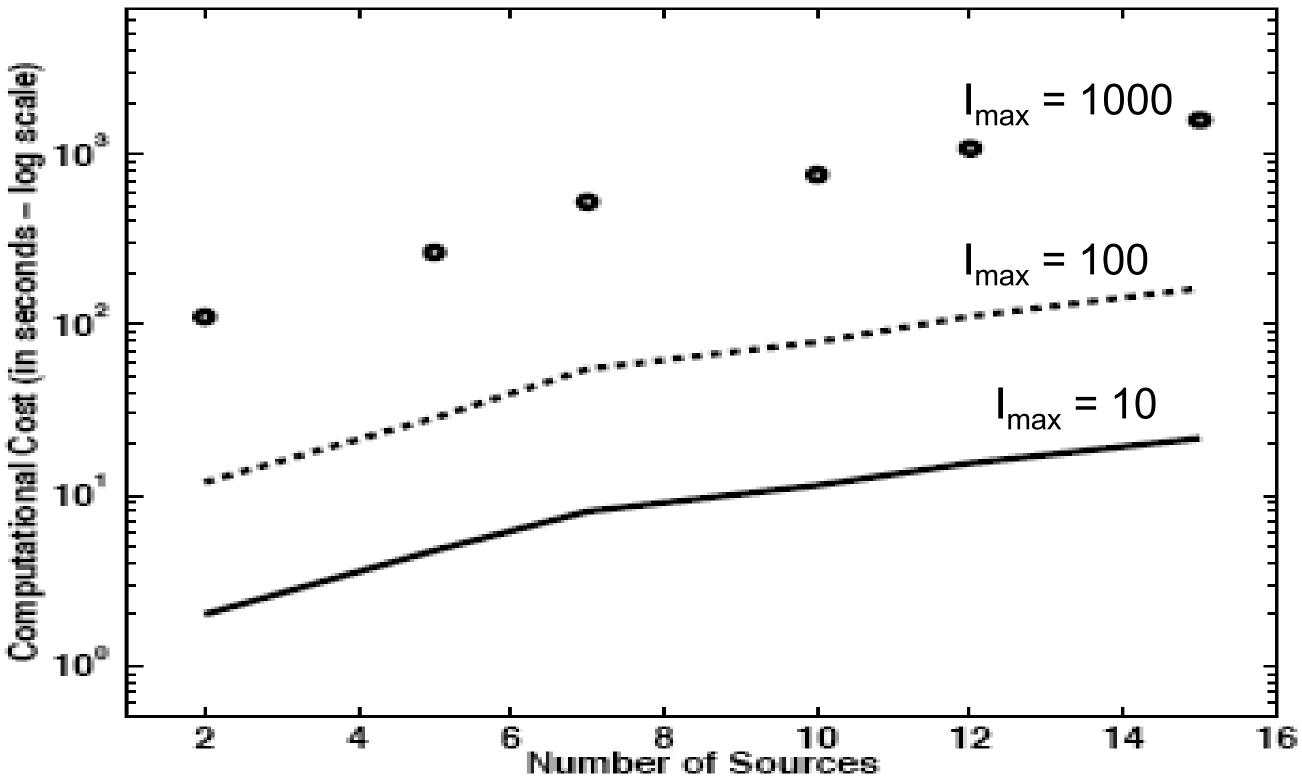
Dictionary
Curvelets + LDCT

Restoring audio signals

2 channels (stereo)   Dictionary Wavelets + LDCT

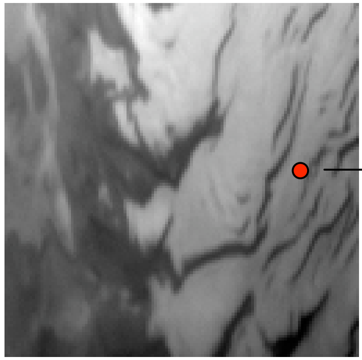
Original   Masked   Restored

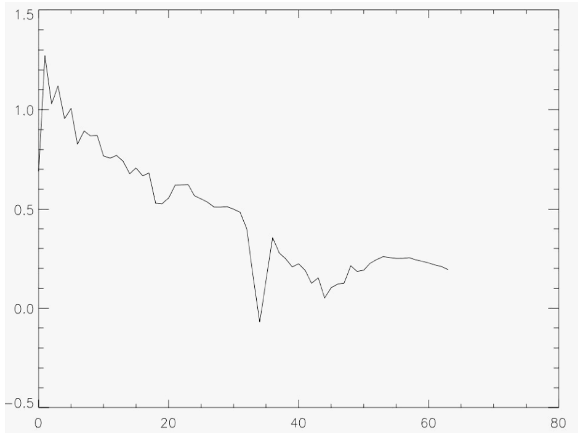
GMCA with more sources: Higher dimension and computational cost



Hyper-spectral Data

Morphological Diversity and Sparsity for Multichannel Data Restoration,
Journal of Mathematical Imaging and Vision, submitted.





We want to add also constraint on the spectrum of each pixel:

$$\min_{A,S} = \sum_{l=1}^{m} \left\| X - AS \right\|_2^2 + \lambda \sum_{i=1}^{n} \left\| a^i s_i \right\|_p$$

We add a sparse constraint on each hyper-spectral cube related to each couple (source, spectrum).

$$\min_{A, c_{1,1}, \ldots, c_{1,K}, \ldots, c_{n,1}, \ldots, c_{n,K}} = \sum_{l=1}^{m} \left\| M_l \left(X_l - \sum_{i=1}^{n} A_{i,l} \sum_{k=1}^{K} c_{i,k} \right) \right\|_2^2 + \lambda \sum_{i=1}^{n} \sum_{k=1}^{K} \left\| \phi^{(1D)} A_i (T_{i,k} c_{i,k})^t \right\|_1$$

$\phi^{(1D)} = W^{1D}$ is a natural choice for hyperspectral data.

Positivity on sources and/or spectra is also an interesting contraint to consider.

The GMCA Algorithm with Constraint on the mixing matrix

. Initialize all c_k to zero, $\lambda_1 = \max(\alpha), \delta = \max(\alpha)/\mathrm{Niter}$

. Iterate t=1,...,Niter

- Iterate i=1,..,NbrSource

  Defining a multichannel residual $\mathbf{D_i}$: $D_i = X - \sum_{i' \neq i} a^{i'} s_{i'}$

  Iterate k=1,..,K

  - Least square estimate of $c_{i,k}$ $l_{i,k} = \frac{1}{{a^{i}}^T a^i} {a^i}^T (D_i - a^i \sum_{k' \neq k} c_{i,k'})$
  - Minimize: $J(\tilde{l}_{i,k}) = \left\| l_{i,k} - \tilde{l}_{i,k} \right\|_2^2 + \mu_t \left\| T_{i,k} \tilde{l}_{i,k} \right\|_1$ with $\mu_t = \lambda_t \left\| \phi^{1D} a^i \right\|_1$

  which is obtained by a simple hard/soft thresholding of $l_{i,k}$

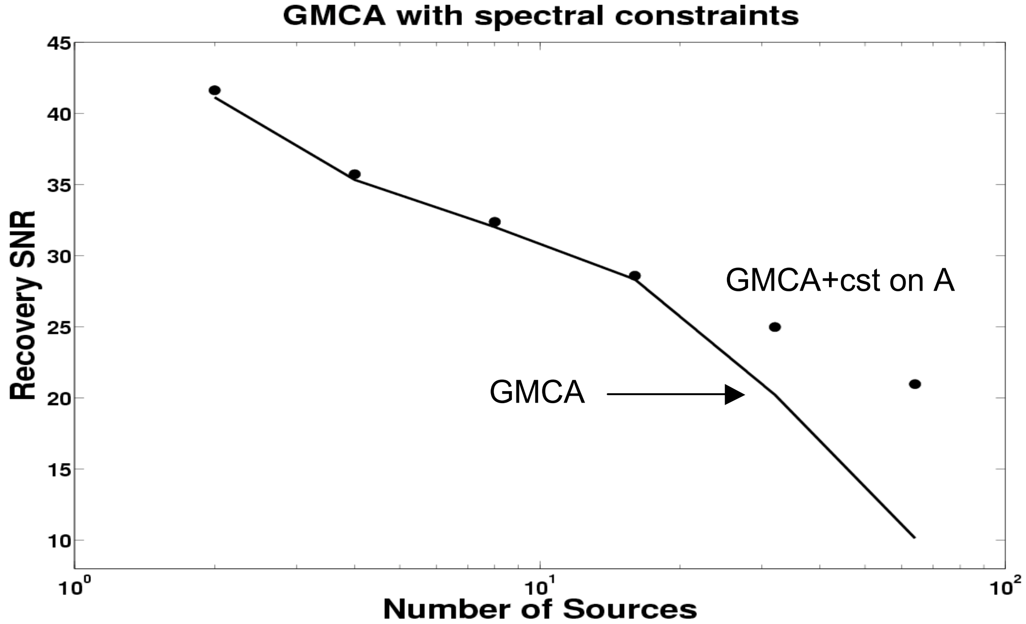
  $s_i = \sum_k l_{i,k}$

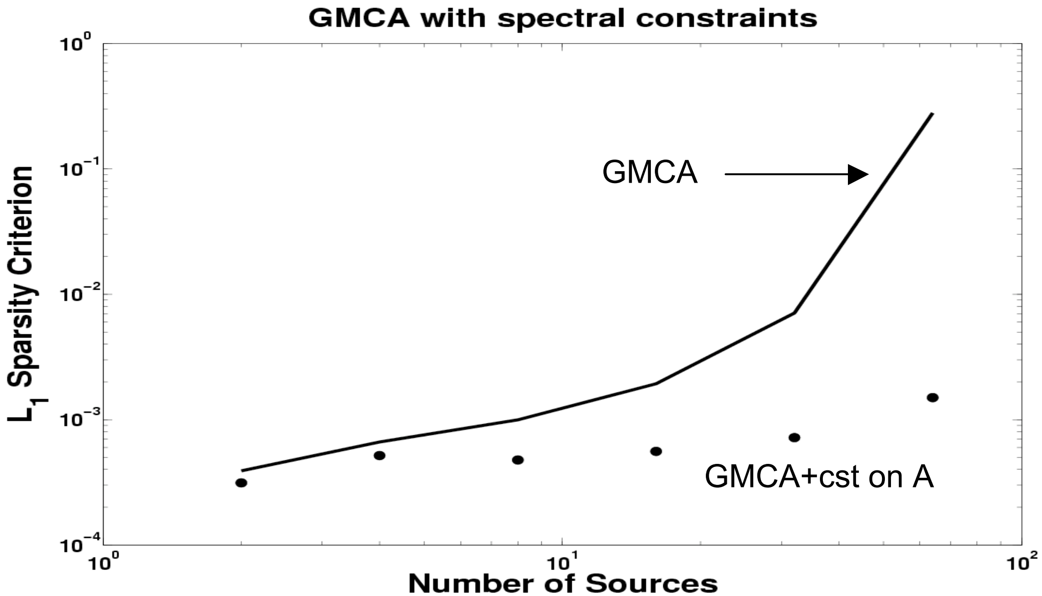
  - Least square estimate of a^i : $l_i = \frac{1}{s_i s_i^T} (D_i - a^i s_i) s_i^T$
  - Minimize: $J(\tilde{l}_i) = \left\| l_i - \tilde{l}_i \right\|_2^2 + \mu_t \left\| \phi^{1D} \tilde{l}_i \right\|_1$ with $\mu_t = \lambda_t \sum_k \left\| T_{i,k} c_{i,k} \right\|_1$

  which is obtained by a simple hard/soft thresholding of l_i

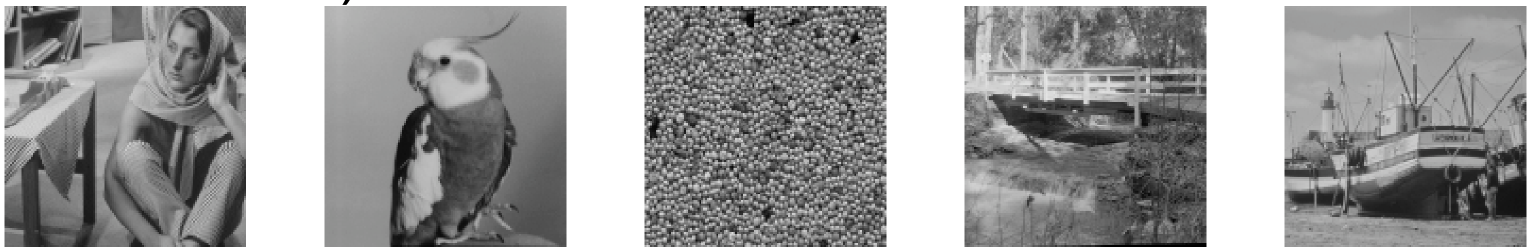
- Decrease $\lambda_{t+1} = \lambda_t - \delta$

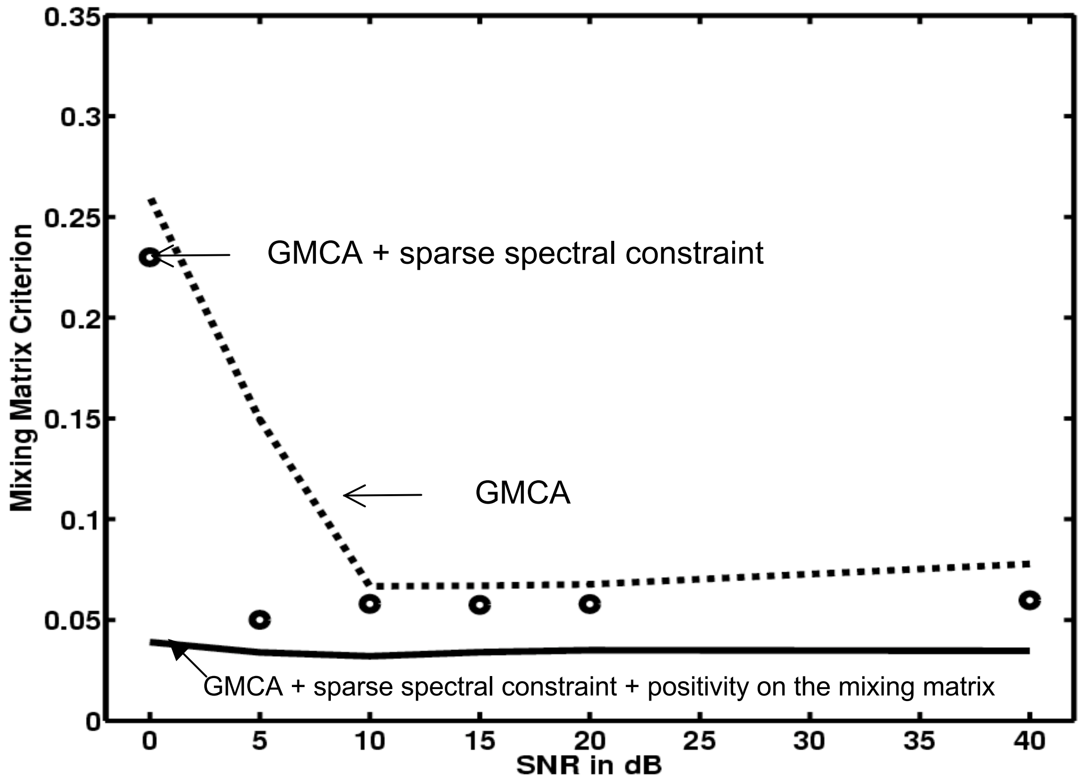
Average over 100 realizations of random sources (with 2048 samples and 128 channels) and matrices with Laplacian distributions





Five natural images, 128 channels using a sparse matrix in the wavelet domain (i.e. a Bernoulli-Gaussian distribution of its wavelet coefficients).





Optimal Number of Sources

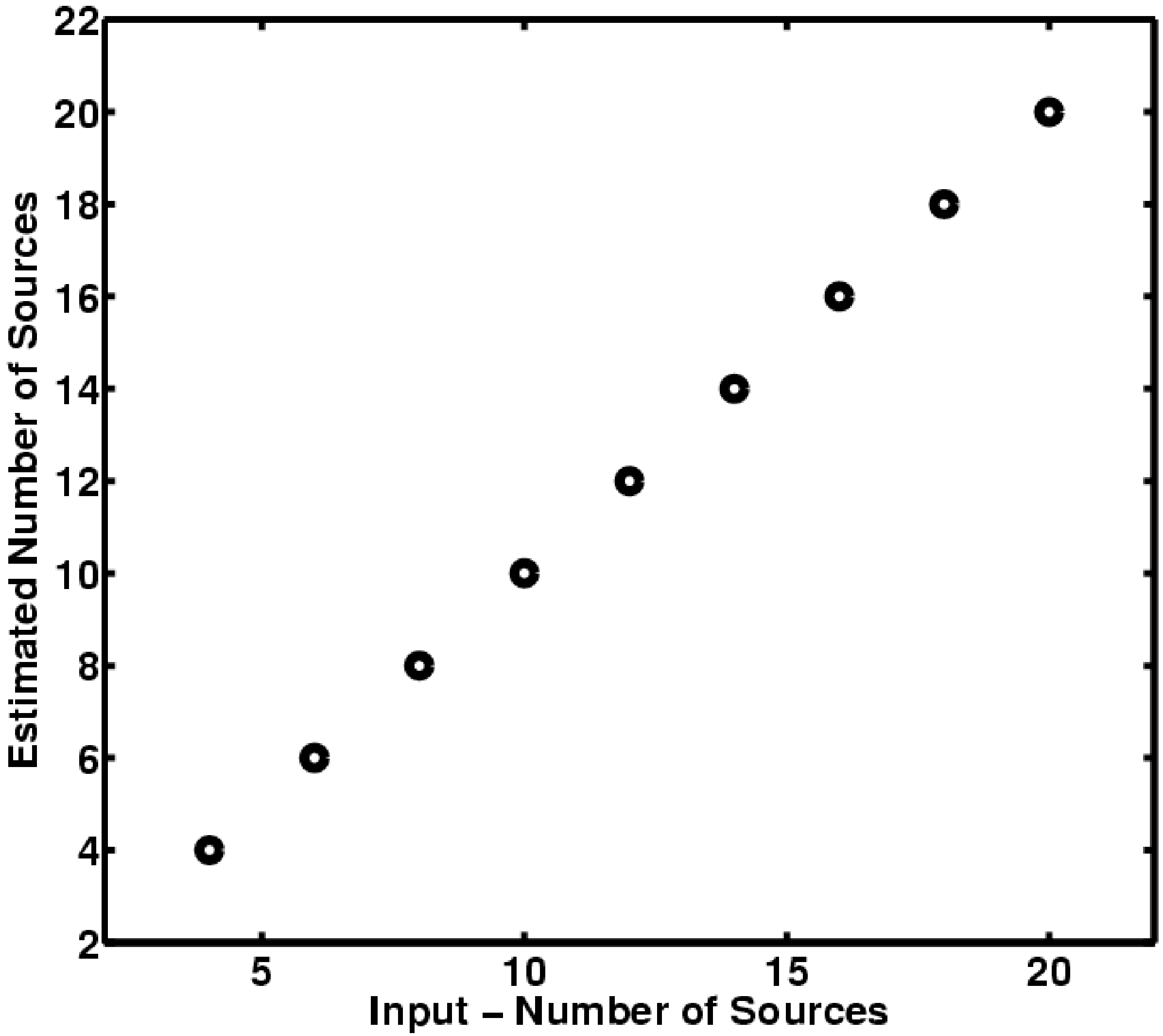
What is the number of sources ?

$$\min_{n_s} \min_{A,S} = \|X - AS\|_2^2 + P(A,S)$$

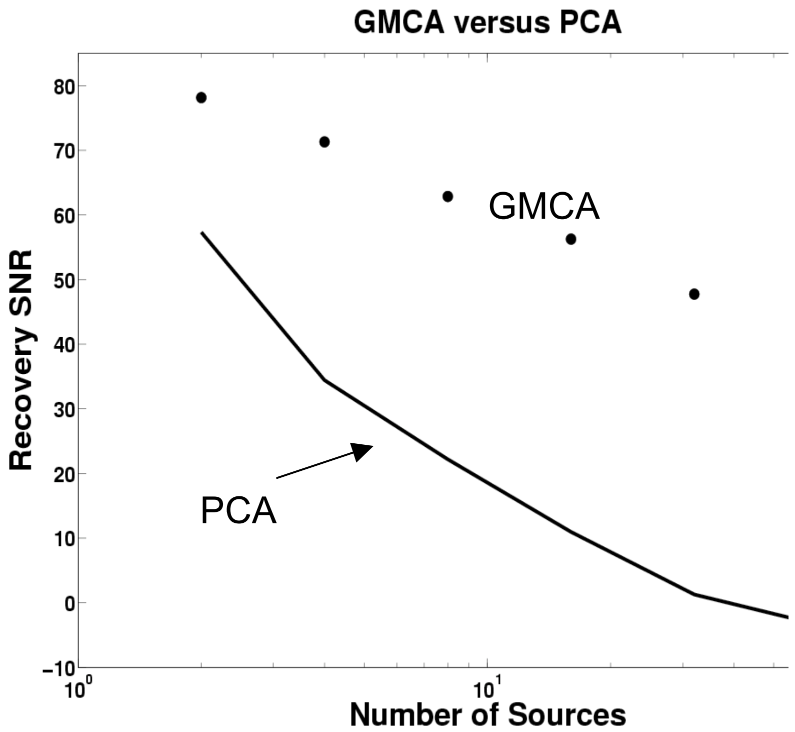
Principle: Increase the number of sources until the hyper-spectral residual is below a given level:

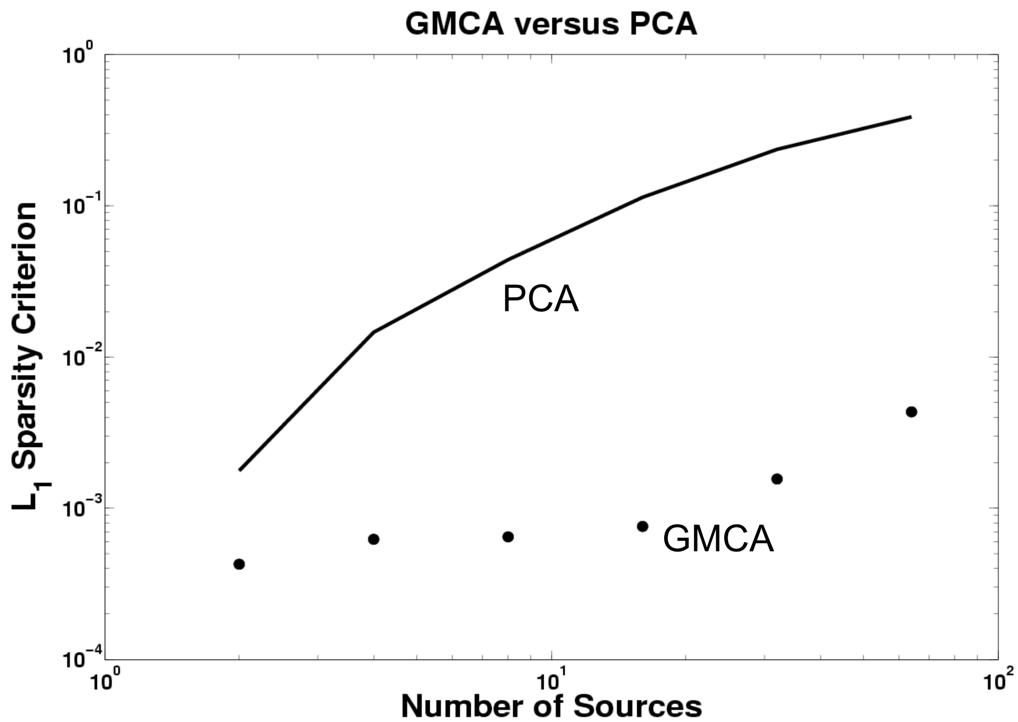
$$\|X - AS\|_2^2 < \sigma$$

64 channels with 256 samples, average over 25 experiments, the standard deviation is null. The sources are random with a Laplacian distribution, and the mixing matrix is random with a Gaussian distribution.



64 channels with 2048 samples, average over 25 experiments, the standard deviation is null. The sources are random with a Laplacian distribution, and the matrix is random with a Gaussian distribution.

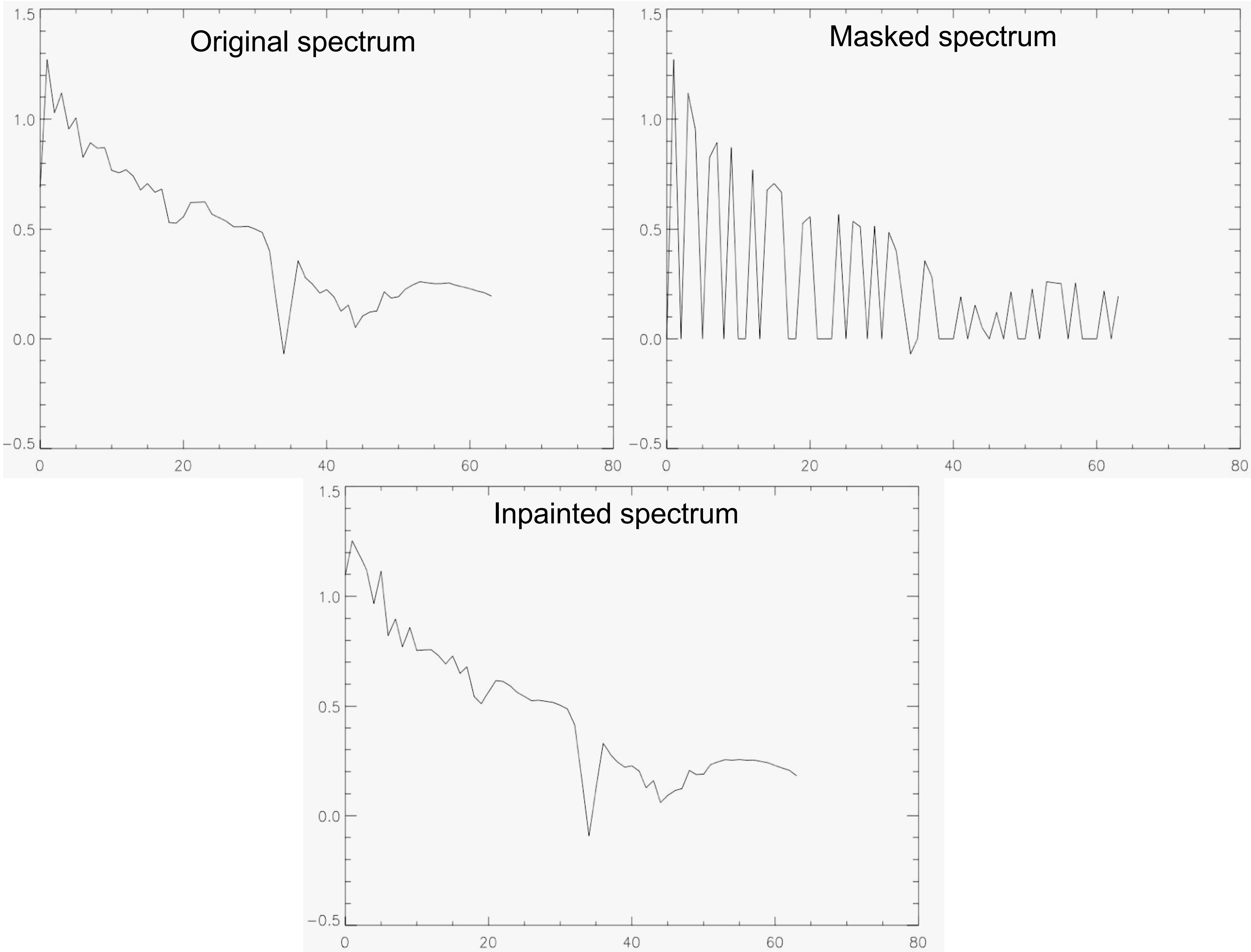




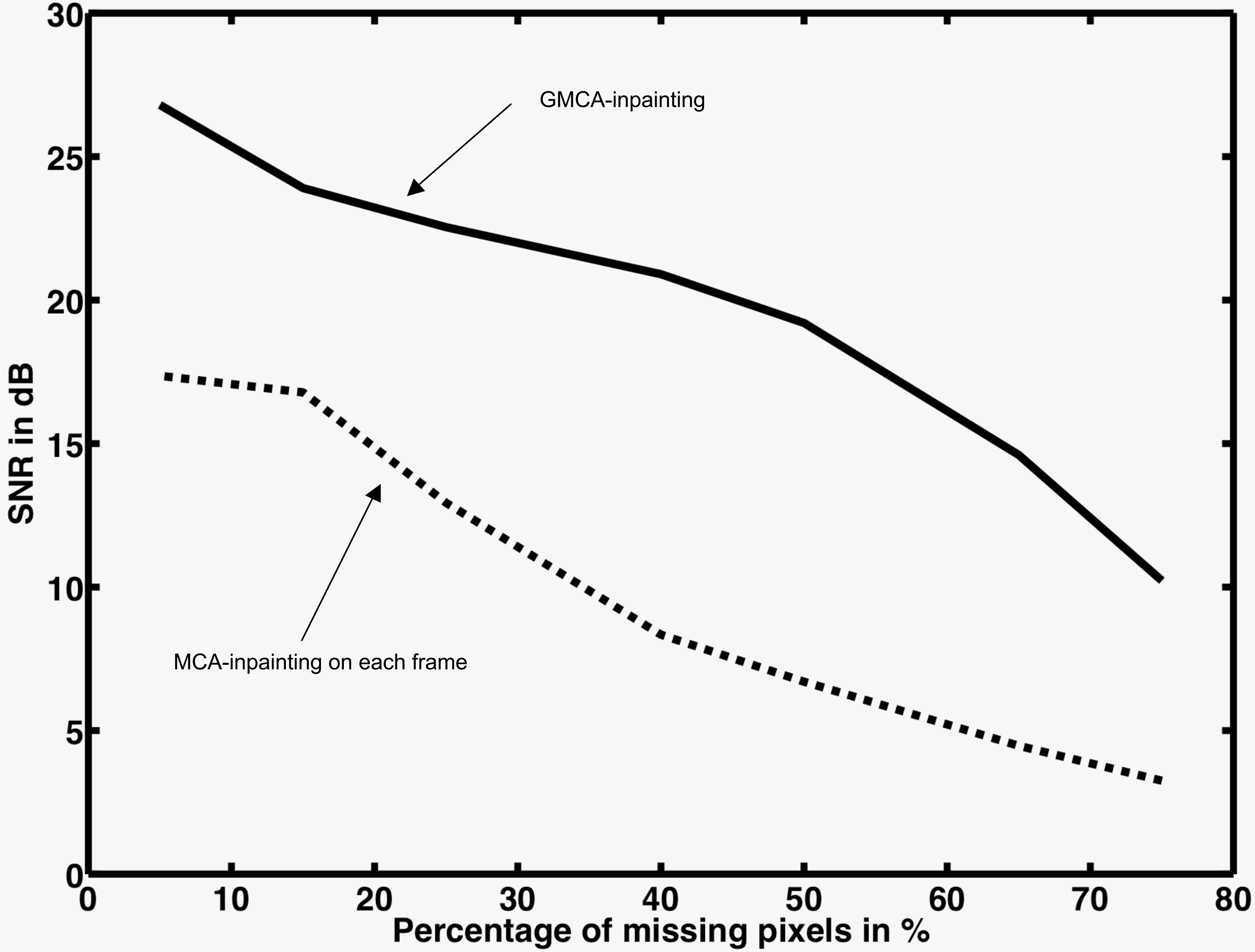
Inpainting hyperspectral data

Omega Camera on Mars Orbiter: 128 x128 x 64 channels

50% missing
pixels

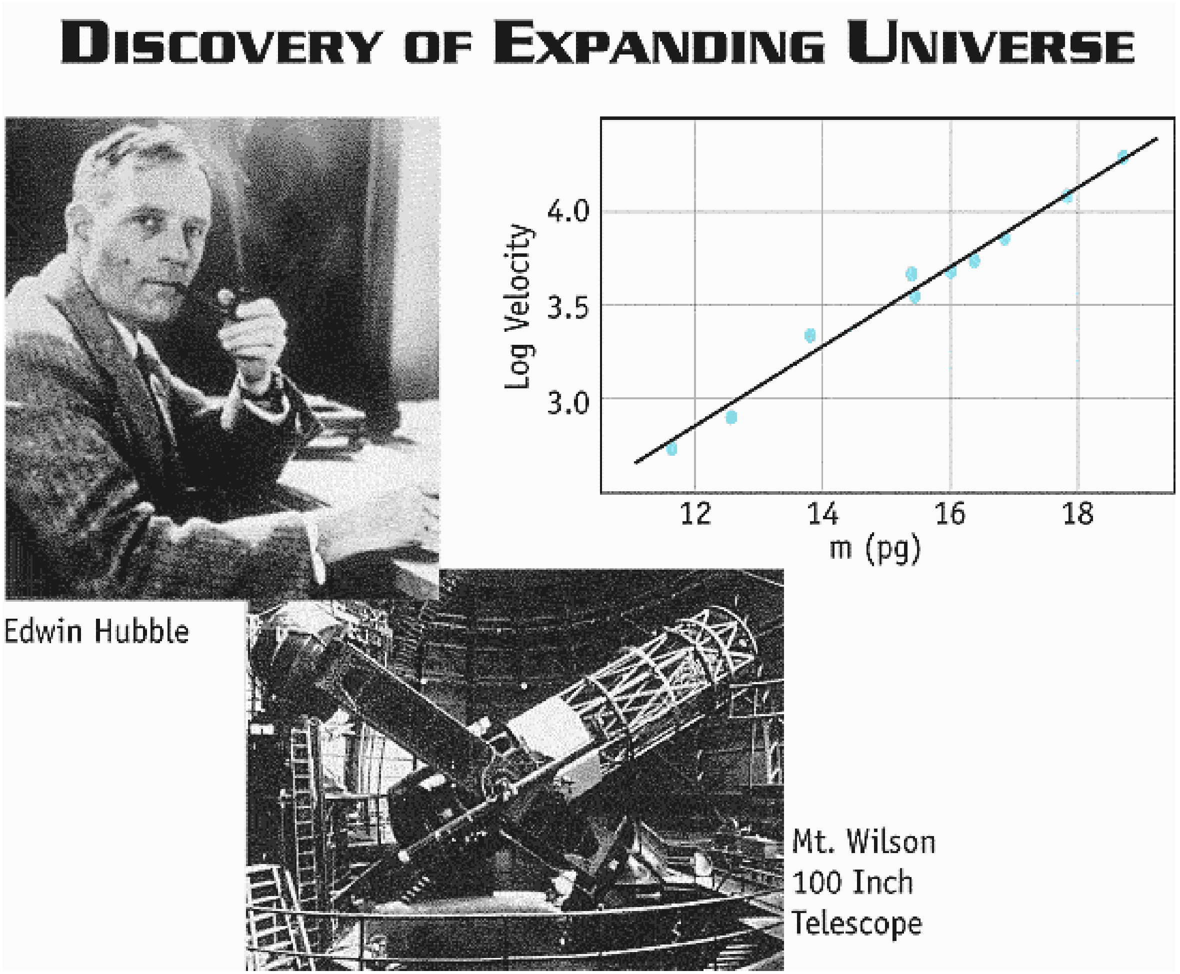


Original spectrum
Masked spectrum
Inpainted spectrum
1.5
1.0
0.5
0.0
−0.5
0
20
40
60
80



GMCA-inpainting
MCA-inpainting on each frame
SNR in dB
Percentage of missing pixels in %
0
5
10
15
20
25
30
0
10
20
30
40
50
60
70
80

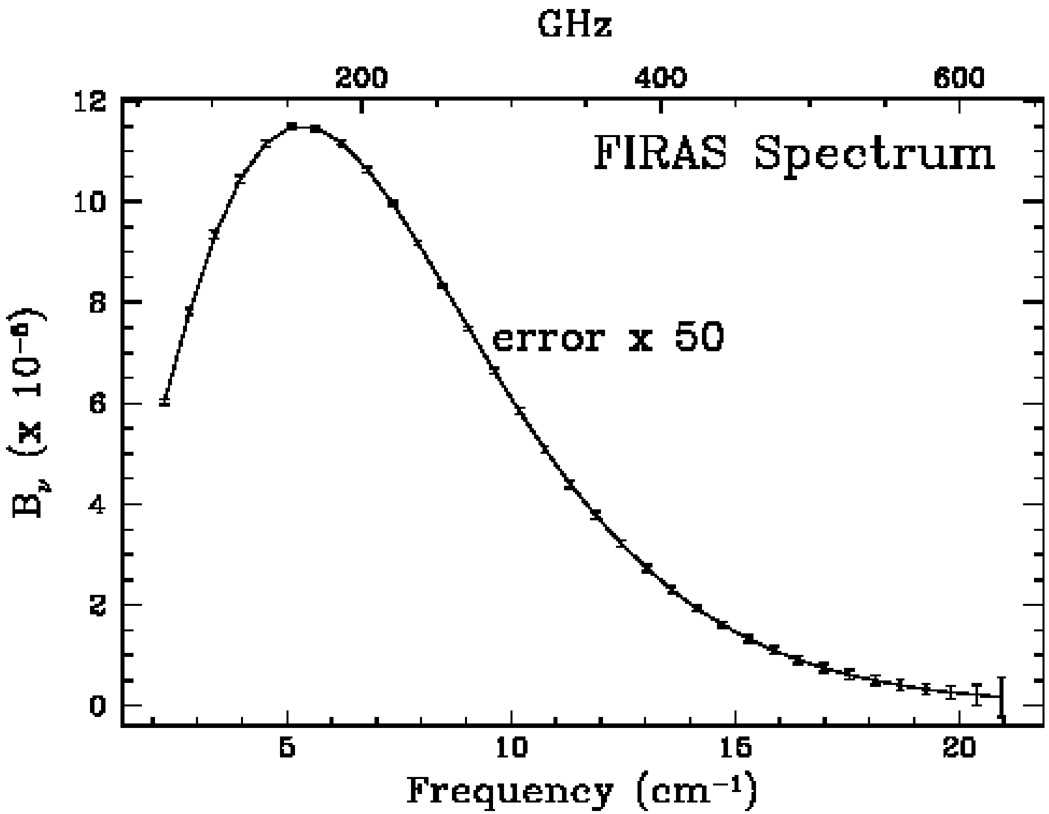
The Big Bang


DISCOVERY OF EXPANDING UNIVERSE
Log Velocity
4.0
3.5
3.0
12
14
16
18
m (pg)
Edwin Hubble
Mt. Wilson
100 Inch
Telescope

The Cosmic Microwave Background

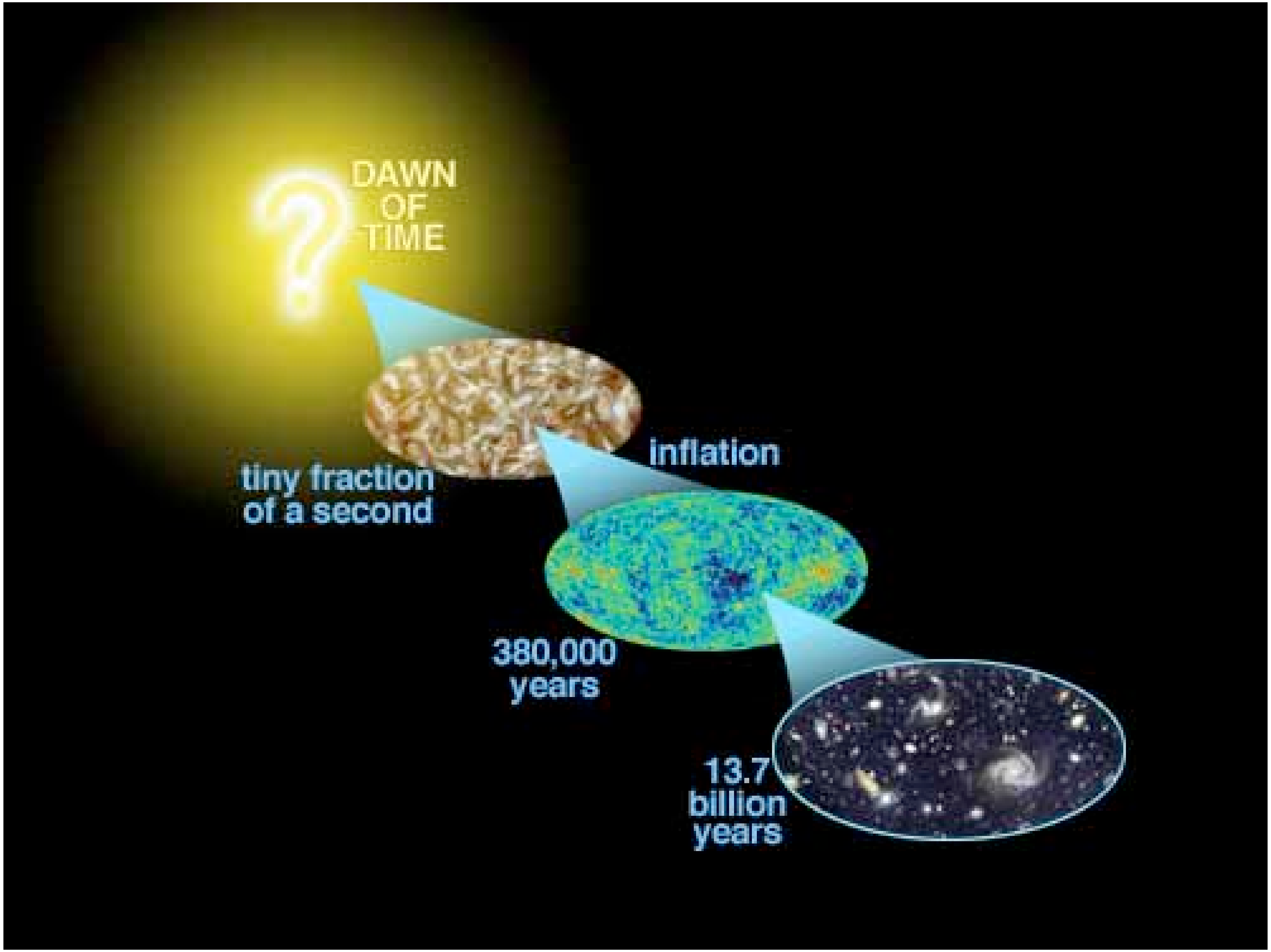
- The Universe is filled with a blackbody at a temperature of 3K.
- Predicted by Gamow in 1948
- Observed for the first time by Pienzas and Wilson (1965)
- Confirmed by COBE (1990)

The Cosmic Microwave Background

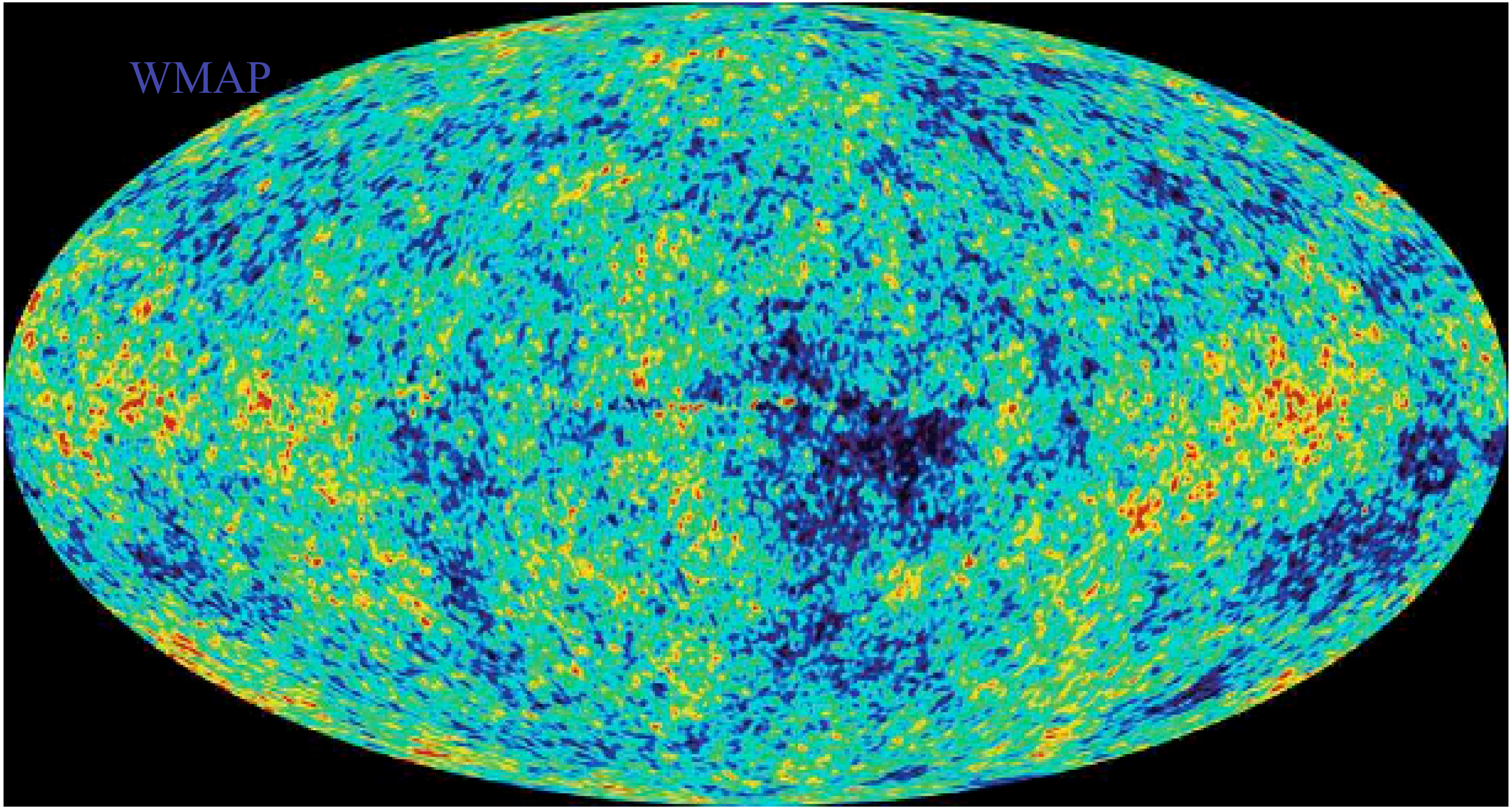


$T = 2.728 \pm 0.004$ K

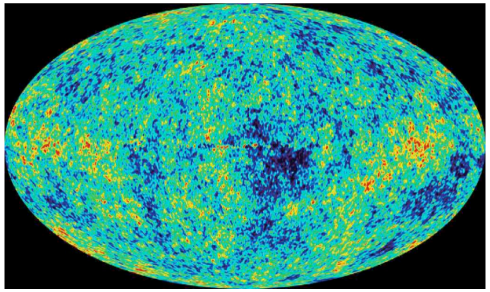
CMB electromagnetic spectrum


DAWN
OF
TIME
inflation
tiny fraction
of a second
380,000
years
13.7
billion
years

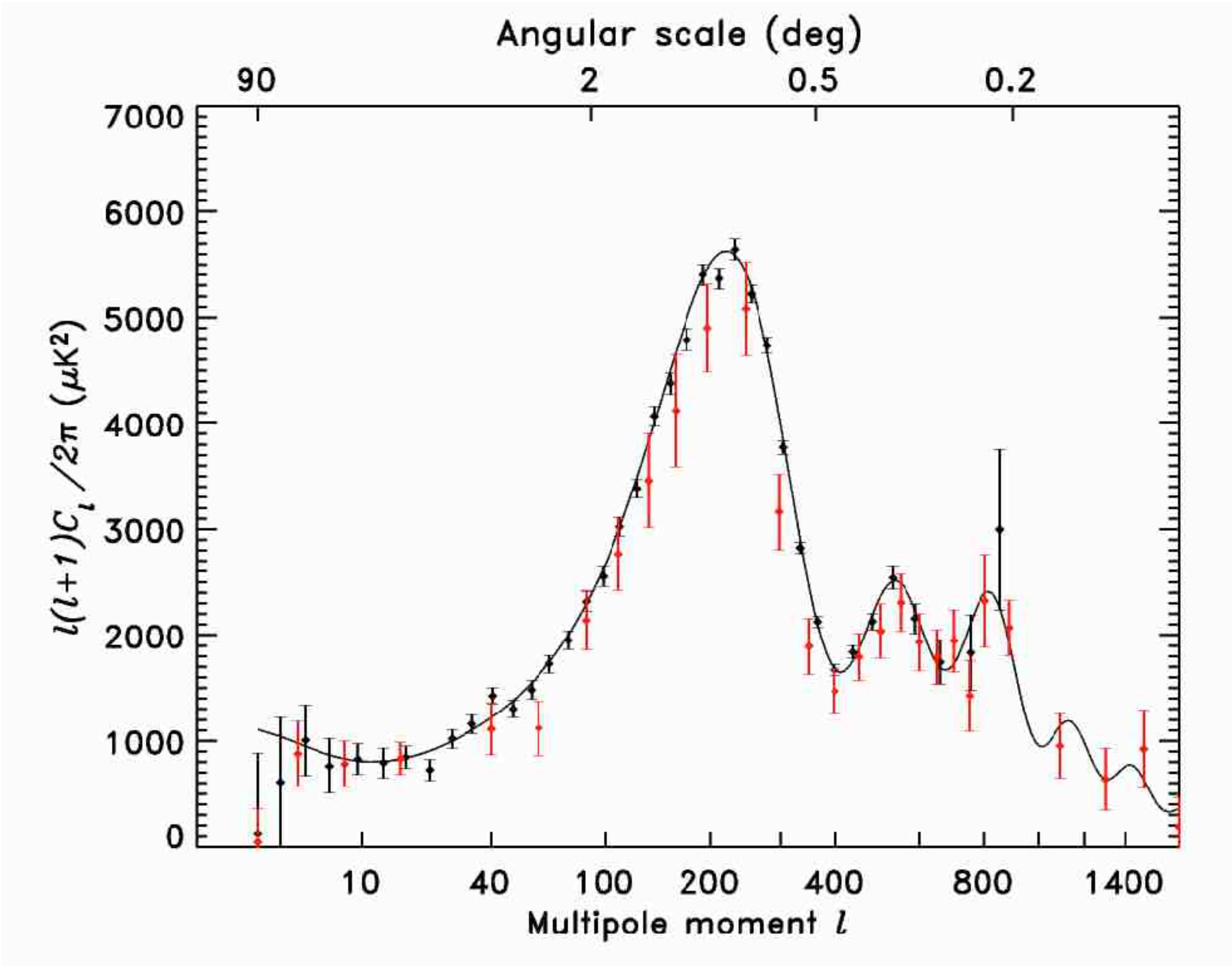
The CMB exhibits Fluctuations



The Cosmic Microwave Background (CMB) is a relic radiation (with a temperature equals to 2.726 Kelvin) emitted 13 billion years ago when the Universe was about 370000 years old.



Power spectrum of WMAP



Remarkably consistent with earlier data

PLANCK PROJECT

Successor of WMAP (better resolution, better sensitivity, more channels)

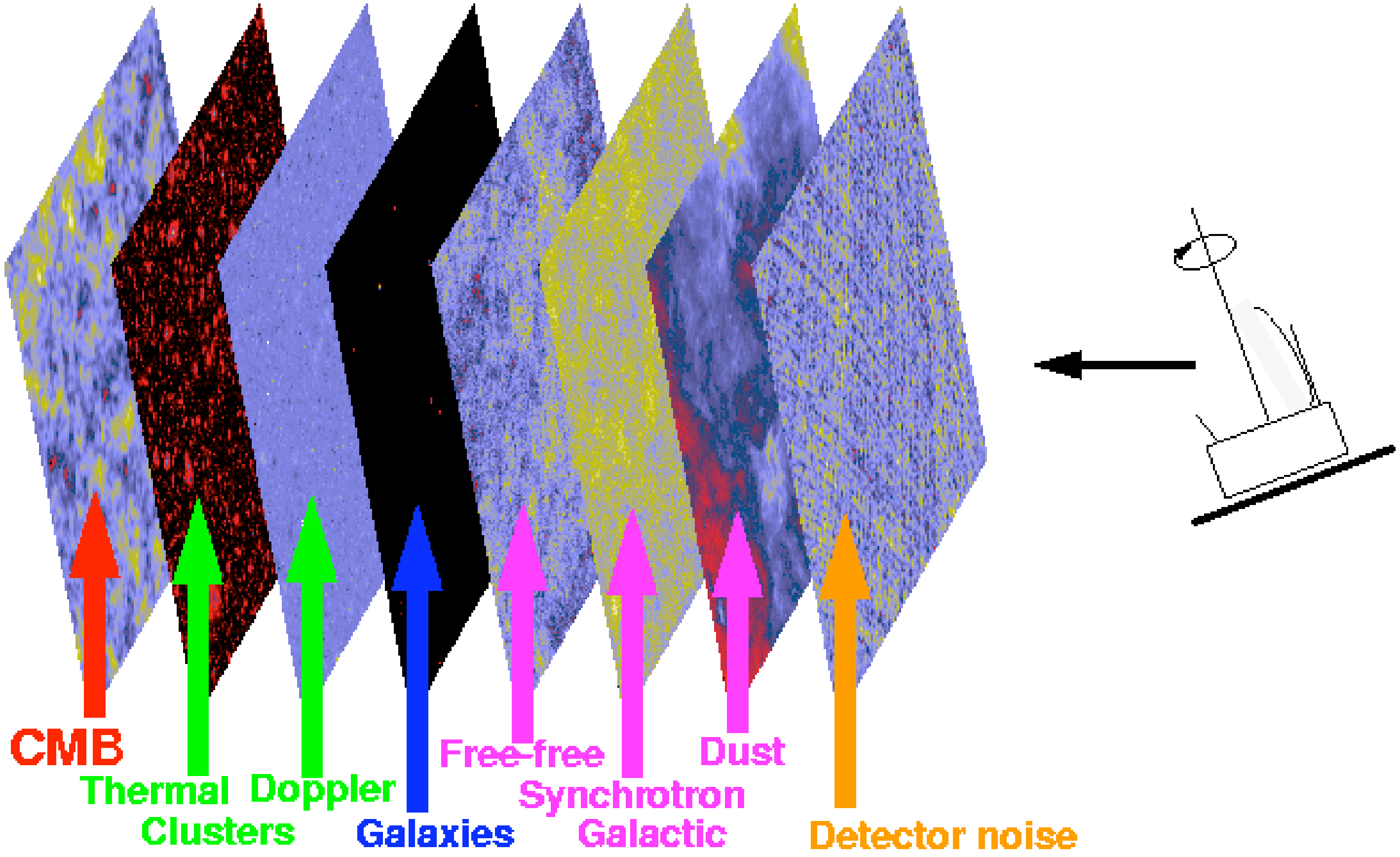
Launch in 2008

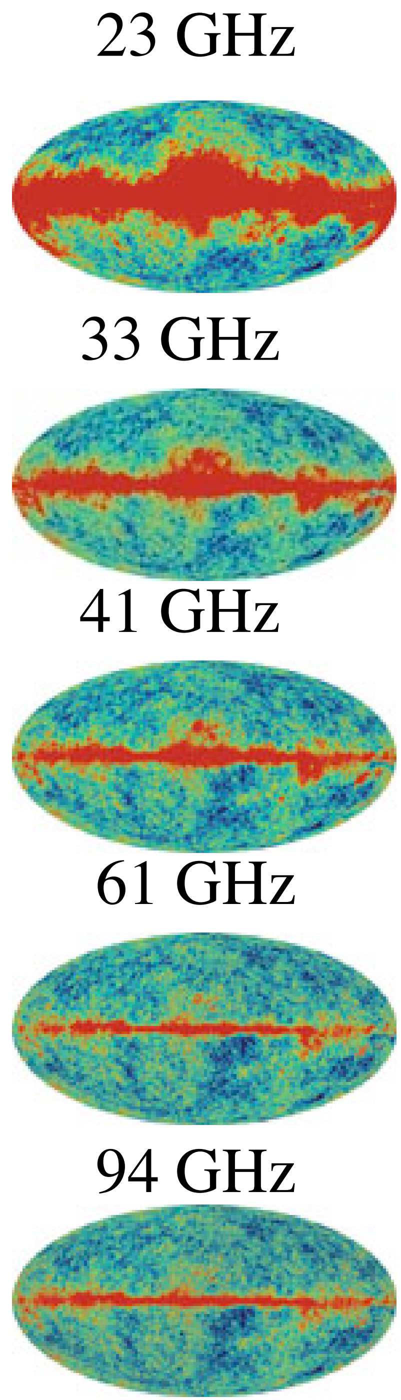
Two instruments LFI and HFI

Nine maps at 30,44,70,100,143,217,353,545,857 GHz

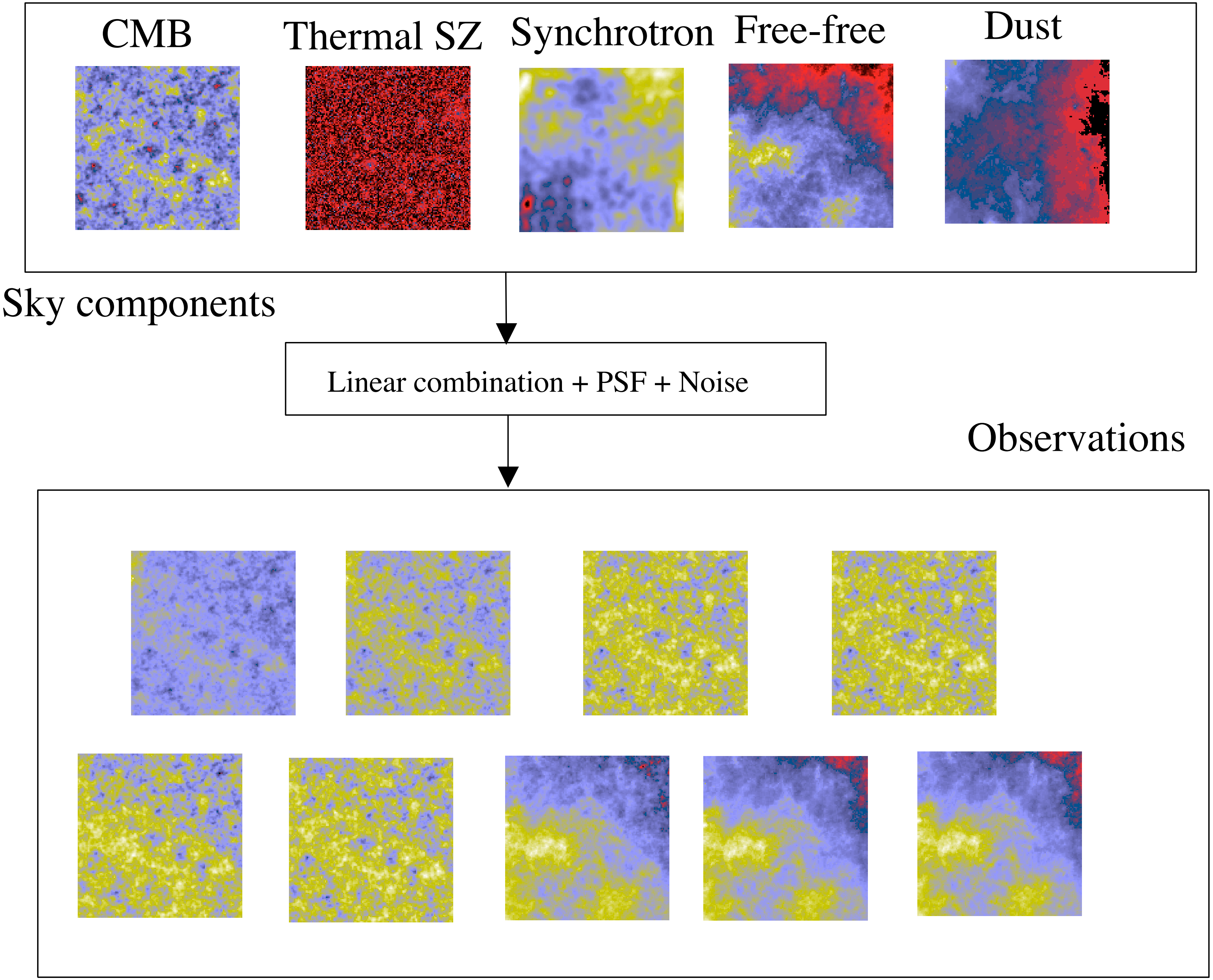
Angular resolutions: 33', 24', 14', 10', 7.1', 5', 5', 5', 5'

Size of each map = 9 x 12 x 2048^2

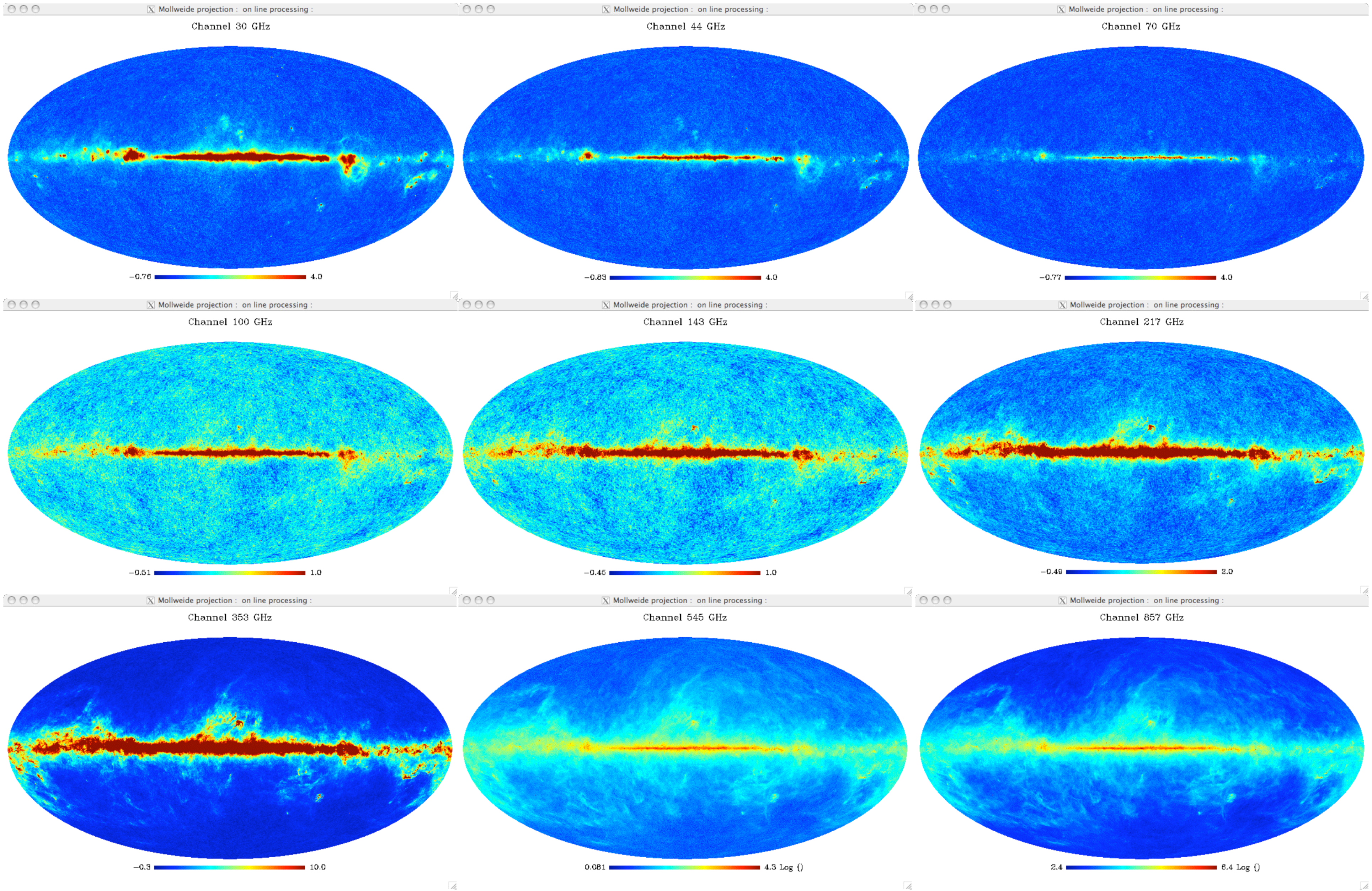


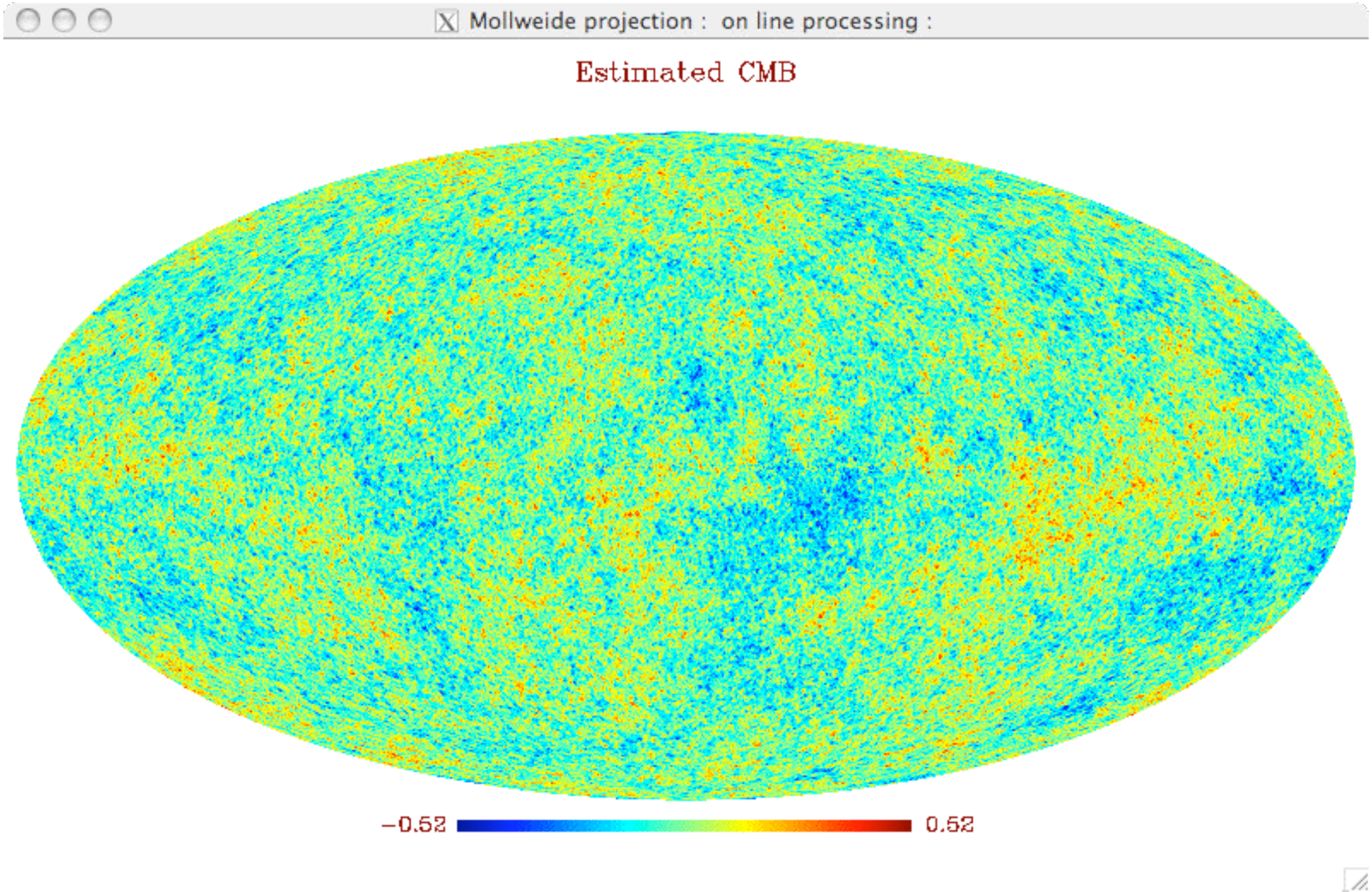


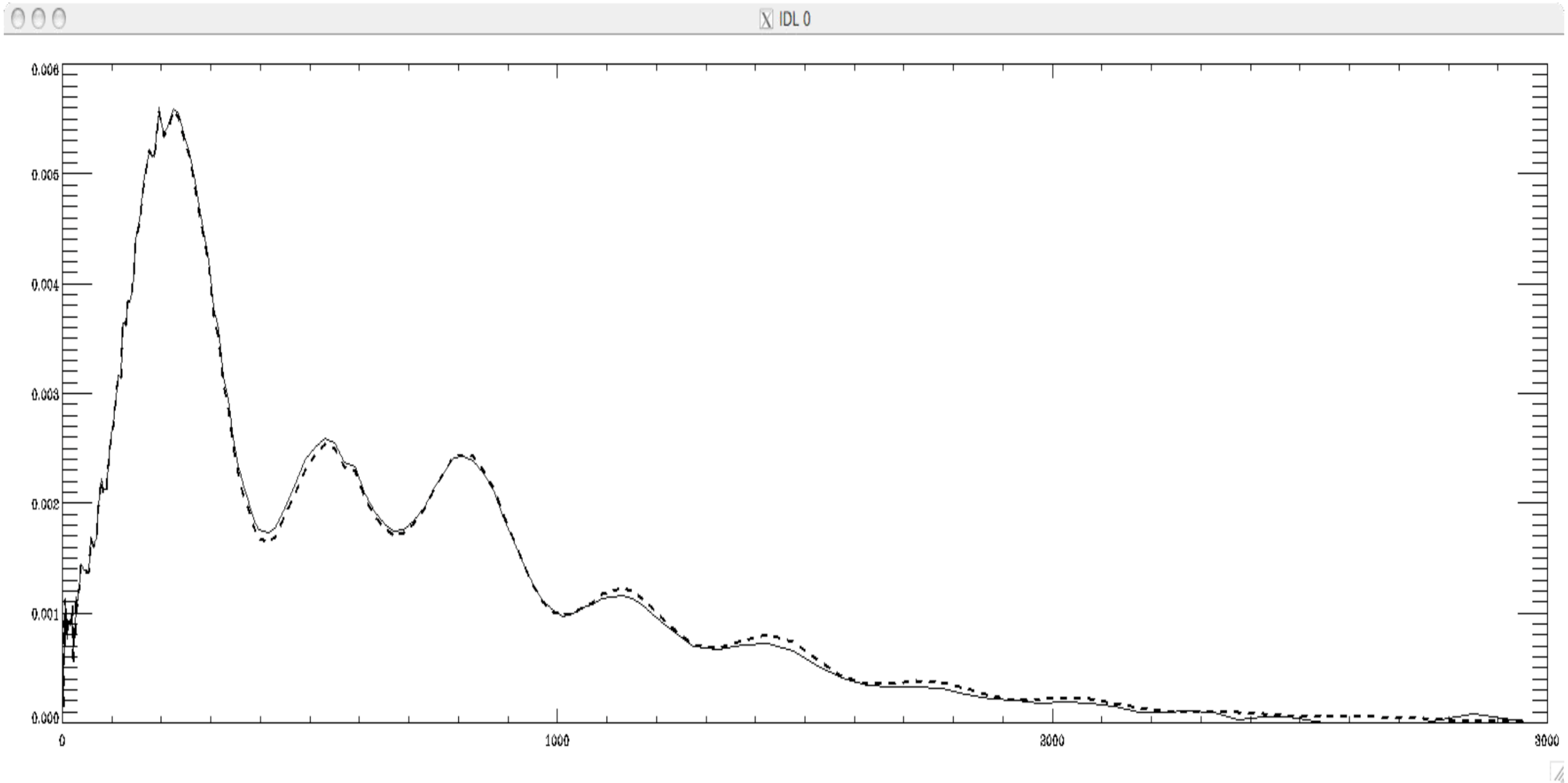
Synchrotron emission due to cosmic rays electrons accelerated into galactic magnetic fields

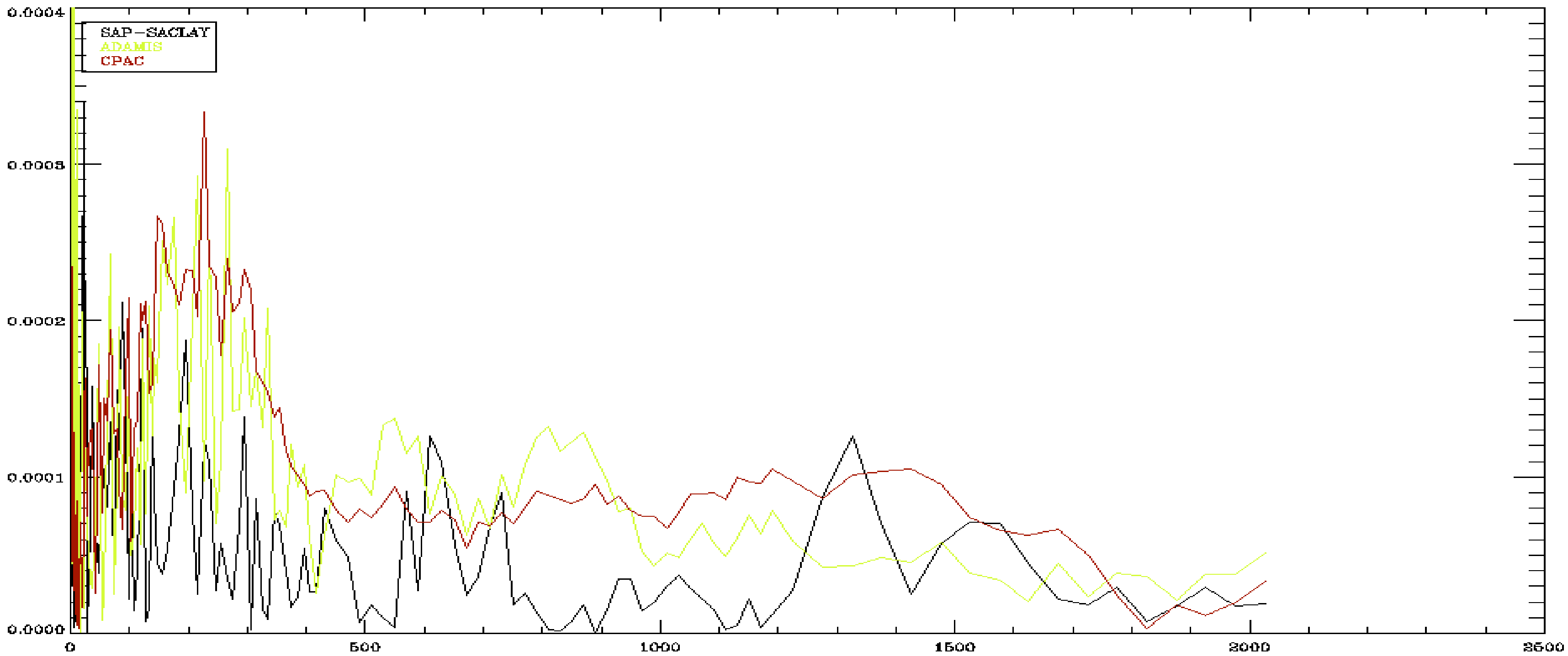

CMB
Thermal SZ
Synchrotron
Free-free
Dust
Sky components
Linear combination + PSF + Noise
Observations

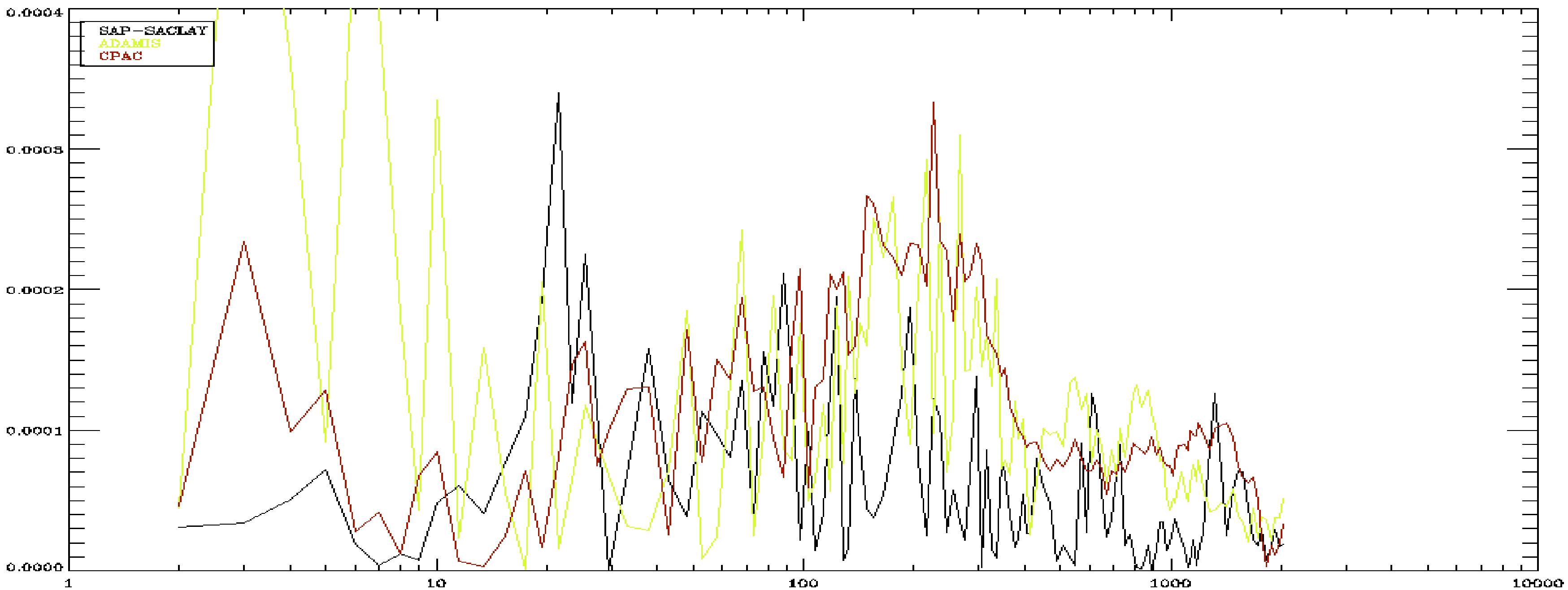
PLANCK Simulated Data



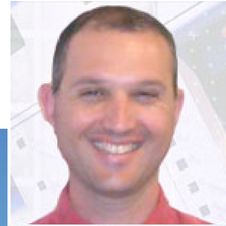

Mollweide projection : on line processing :
Estimated CMB
−0.52
0.52


IDL 0
0.006
0.005
0.004
0.003
0.002
0.001
0.000
0
1000
2000
3000


SAP–SACLAY
ADAMIS
CPAC
0.0004
0.0003
0.0002
0.0001
0.0000
0
500
1000
1500
2000
2500


SAP–SACLAY
ADAMIS
CPAC
0.0004
0.0003
0.0002
0.0001
0.0000
1
10
100
1000
10000

New Perspectives Based on Union of Overcomplete Dictionaries: *The Morphological Diversity*



Denoising and Deconvolution

Very High Quality Image Restoration, SPIE, Vol 4478, 2001.
Wavelets and Curvelets for Image Deconvolution: a Combined Approach, *Signal Processing, 83, 10, pp 2279-2283, 2003.*

Morphological Component Analysis (MCA) + Inpainting

•***Redundant Multiscale Transforms and their Application for Morphological Component Analysis***, *Advances in Imaging and Electron Physics, 132, 2004.*
•***Image Decomposition Via the Combination of Sparse Representation and a Variational Approach***, *IEEE Trans. on Image Proces., 14, 10, pp 1570--1582, 2005.*
•***Morphological Component Analysis: an adaptive thresholding strategy***, *IEEE Trans. on Image Proces.*, Vol 16, No 11, pp 2675--2681, 2007.
•***Simultaneous Cartoon and Texture Image Inpainting using Morphological Component Analysis (MCA),*** *ACHA, 2005.*
•***Inpainting and Zooming using Sparse Representations***, The Computer Journal , *in press*.

MCAlab available at: http://www.greyc.ensicaen.fr/~jfadili

Extension to Multichannel/Hyperspectral Data

• ***Morphological Diversity and Source Separation***", IEEE Trans. on Signal Processing , Vol 13, 7, pp 409--412, 2006.
•***Sparsity, Morphological Diversity and Blind Source Separation***" , IEEE Trans. on Image Processing , Vol 16, pp 2662 - 2674, 2007.
• **Morphological Diversity and Sparsity for Multichannel Data Restoration**, J. of Mathematical Imaging and Vision, submitted.

GMCAlab is NOW available at: http://perso.orange.fr/jbobin/gmcalab.html

In order to prepare the next CMB Planck mission:

✓We have extended the wavelet, the ridgelet and the curvelet on the sphere. Software available at: http://jstarck.free.fr/mrs.html
•***Wavelets, Ridgelets and Curvelets on the Sphere***, *Astronomy & Astrophysics*, 446, 1191-1204, 2006.

✓We have extended the MCA-inpainting method on the sphere.
• **Morphological Component Analysis and Inpainting on the Sphere: Application in Physics and Astrophysics**, J. of Fourier Analysis and Applications (JFAA), in press.

✓ We have applied GMCA on PLANCK simulated data.
• **SZ and CMB reconstruction using GMCA**, Statistical Methodology, in press.

Many things remain to be done:

•The linearity is only a first approximation. We need to take into account the fact that the spectral index of the dust component varies with the spatial position.
•GMCA needs to be extended to take into account polarized data.